ACOUSTIC/MATHEMATICAL MODELLING FOR AIRCRAFT SIMULATOR DESIGN

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Introduction. A mathematical model of interior cockpit noise for a jet aircraft has been developed using the measured noise data for aircraft operation on ground and in flight. The results of detailed modelling were used with success for the practical design of a simulator showing good agreement between experimental and simulated noise data.

Basic experimental measurements used for the model development. Two sets of measured interior cockpit noise data were used as basic information for the mathematical model development: 1) interior cockpit values of engine noise (on ground) for a set of different RPM values, as well as the noise of auxiliary devices such as valves, pumps, inverter, emergency system etc.; 2) noise audible inside the cockpit in flight, for different flight conditions within a given range of altitudes H(K ft) and flight Mach values (M∞); measurements were performed for stabilized flight.

Main principles of mathematical modelling. The investigation of the measured spectrum shapes for different operation conditions showed some typical features of spectrum dynamic behaviour:

1) The spectrum shape does not remain constant for different operation conditions. However, the frequency position of its maximum value tends to be constant.

2) The auxiliary devices such as valves, pumps, inverter etc. are characterized mainly by the presence or absence of specific discrete frequencies in the noise spectrum at the moment that the corresponding device is turned on (off). These discrete frequencies generally change their position and peak value (relative to the spectrum) as the functions of time variables: current time t, turn-on and turn-off moments (T_1 and T_2) and of durations of transient processes.
3) The noise spectrum of the auxiliary device is generally masked by engine noise spectrum (if engine on) while the specific discrete frequencies of the device are still present in the spectrum.

4) The case of engine operation on ground is characterized by the presence of several discrete frequencies that have a strong tendency to shift into the high-frequency range with RPM increase. The frequency positions of these discrete can be determined with the help of linear functions of RPM, while the values of discrete turned out to be non-linear (and not monotonous) functions of RPM.

Based upon these specific features of spectrum change, several types of control functions were determined as parts of the mathematical model. A control function value may determine: 1) frequency position of discrete frequency; or 2) the value of discrete frequency peak over the mean spectrum value, or 3) the value (in dB) of $S_{PL_{max}}$ for the spectrum; or 4) the spectrum shape (values in dB for octave bands) relative to $S_{PL_{max}}$. A control function may be a function of RPM (operation on ground), or a function of $H$ and $M_{\infty}$ (flight conditions), or a function of time variables for transient turn-on and turn-off processes.

The control function for spectrum shape relative to $S_{PL_{max}}$ are actually spectrum pattern curves of the form that slightly varies as a function of operational conditions. The control function for spectrum shape in flight was developed based upon a previous investigations of aircraft interior noise (Ref. [1]). The empirically determined relation between $S_{PL_{max}}$ and the values of $H$ and $M_{\infty}$ made it possible to find a simple linear form of the control function of $S_{PL_{max}}$:

$$S_{PL_{max}} = E_1 + E_2 H + E_3 M_{\infty}$$

The coefficients $E_i$ are specific to the aircraft type. More complicated relations (linear functions of $H$, $M_{\infty}$, and $H M_{\infty}$) may be obtained (Ref. [1]) for other aircraft types.

The set of control functions used for simulation. In several cases the experimental functions of noise that must be simulated showed rather simple shape of a smoothed rectangular pulse like the one shown in Fig.1. The dependence of Fig.1 describes the behaviour of $S_{PL_{max}}$ for the inverter noise spectrum (engine off). The corresponding control function can be determined as a sum of two "smoothed step functions" (Eq.(2)):

$$S_{PL_{max}} = N\left[1 + e^{-K_1(t-T_{on}-0.5\Delta t_1)}\right]^{-1} - N\left[1 + e^{-K_2(t-T_{off}-0.5\Delta t_2)}\right]^{-1}$$

The coefficients $K_1, K_2$ of Eq.(2) are to be determined empirically due to the transient shape using measured data. Smoothed step functions were used with success for simulation of more complicated empirical relations containing smoothed step or multi-step parts.
An example of simulation for the case of (engine-on-ground) varying value discrete frequency peaks is shown in Fig. 2. The empirical relations of this type were simulated with the help of a sum of inverse second-order polynomials of RPM as follows:

$$\Delta \text{SPL} = \sum_{i=1}^{3} \left[ a_{i,1} + a_{i,2} \text{RPM} + a_{i,3} \text{(RPM)}^2 \right]^{-1}$$ (3)

where \( \text{SPL} \) is the discrete peak value relative to the mean spectrum value and the coefficients \( a_{i,j} \) were determined empirically to fit the shape of the measured curve.

In some cases the linear fractional functions of exponents showed the best agreement with the measured data. The values of "wide discreted" (i.e. wide peaks of approximately 100 - 150 Hz width), relative to the mean spectrum value were generally well simulated by the following control functions:

$$\Delta \text{SPL} = A_0 + \sum_{j=1}^{n} \frac{A_j + C_j e^{-b_j \text{(RPM-b)}_j}}{1 + e^{-b_j \text{(RPM-b)}_j}}$$ (4)

\( n \) is generally equal to 2 or 3 and some of \( C_j \) coefficients may be equal to zero.

To ensure the continuity of control functions for \( \text{SPL}_{\text{max}} \) during the transition processes between flight conditions and landing, an additional term of the following form has been entered:

$$\left( F_1 + F_2 H + F_3 H^2 \right)^{-1}$$ (5)

where \( H \) is altitude in K ft, and the coefficients \( F_1 \) are to be determined empirically based upon measured data.

In case of engine operation on the ground, the control function for \( \text{SPL}_{\text{max}} \) was obtained as a Lagrange polynomial of the 5th order plus an additional transition term in the form of the function of Eq.4. This term ensures the continuity of the control function during the transition interval between ground run and takeoff.

Experimental data used for model development. Noise recorded data used in model development contained 16 different flight conditions and 8 conditions of ground operation (for different RPM values). Frequency range of the spectra was generally from 10 Hz up to 11.5 - 12.5 KHz, with the resolution value \( \Delta f = 10 \) Hz. Noise data for auxiliary device operation on ground (engine off) were also available. Narrow band spectra used for discrete tones behavior investigation had resolution value of \( \Delta f = 5Hz \). Fast transition processes of spectrum change with time (that generally take place after a device like valve or pump is turned on (off)) were investigated with the help of a series of successive spectra taken every 0.5 sec or every 0.2 sec. FFT-analyzer
was used with success to investigate and specify the dynamic behaviour of turn-on or turn-off discretes appearing in the spectrum, in the three-dimensional mode (graphs containing frequency position and the value of a discrete as a function of time).

Conclusions. In the development of the mathematical/acoustic model of a jet aircraft interior cockpit noise (for interior noise simulator), a set of specific control functions has been determined. These functions describe the dynamic changes of noise characteristics (spectrum and discrete frequency tones) in different operation conditions. Though the empirically determined coefficients of these functions are certainly specific for an aircraft, the general form of the functions seem to be universal for description of the behaviour of noise characteristics of other aircraft types. The use of the previously developed model for spectrum changes in flight made it possible to minimize the number of experimental noise measurements in different flight conditions.

Reference.

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**Fig.1** Maximum spectrum value for the inverter noise (engine off) as a function of time variables.

**Fig.2** The value of engine noise discrete peak (ground operation) relative to mean spectrum level.