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IMPEDANCE DESIGN OF EFFICIENT METAMATERIAL SOUND ABSORBERS

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This paper describes the development of efficient sound absorbers based on acoustic metamaterials (AMM). In the presentation, it is proposed to design efficient sound absorbers as approximation to the so-called best absorber at some discrete frequencies of the working frequency region. Two conditions for the best absorber are formulated. The basic absorber structure chosen is a metamaterial structure composed of a number of compactly "coiled-up" resonators with various parameters. A method for computing the necessary absorber parameters and its efficiency is developed. The key point of the method is certain measurable and computable impedance characteristics that allow one to control fulfillment of the necessary conditions of the best absorber. Agreement between the theory and measurement is obtained in laboratory experiment using the metamaterial samples.

Keywords: acoustic metamaterials, sound absorption, the best absorber

1. Introduction

The paper considers designing effective sound absorbers made of acoustic metamaterial (AMM) using the concept of the best absorber [1,2]. The best absorber is such a body that among all possible visco-elastic bodies of the same configuration absorbs maximum energy of incident fields. The main properties of the best absorber are described in terms of the surface impedances [1]. Some of the properties are rather severe and can be realized only as specially designed metamaterial structures. In this paper, a method of designing such absorbers is presented in the form of combination of quarter wavelength resonators (tubes) coiled-up into a labyrinthine space structure. First, a method of computing the main parameters of the resonators as well as efficiency of the absorber is presented where damping is taken into account. Choice of the optimal values of the parameters are discussed. Some results of laboratory experiment on one of the designed and 3–D printed sample are presented which show a good agreement with prediction.

2. The best absorber

By definition, the best absorber is the body of finite dimension that absorbed more energy of incident sound fields than any other linear body of the same configuration. So far, the best absorber is described analytically only with the help of matrices of the input surface impedances, see, e.g. [1].

Such matrix is defined as follows. Let *A* be the interface between the absorber and ambient fluid. Representing *A* as a set of N small surface elements ΔA_n, the pressure and normal velocity continuously distributed on *A* can then be represented on *A* as N-vectors. The absorber input impedance NxN-matrix *Z* describes linear relation between the N-vector of normal velocity V_n and N-vector of forces $p\Delta A_n$. Similar matrix Z_{rad} with respect to interface *A* can be defined for the ambient fluid. It is usually called the matrix of the radiation impedances of the absorber.

The main theoretical result obtained for the best absorber reads: to be the best absorber a body should have the input surface impedance matrix *Z* equal to Hermitian conjugate of the of the radiation impedance matrix Z=Z_{rad}. By separating real and imaginary parts, this equation is equivalent to two real valued symmetric matrix equations

$$X + X_{rad} = 0, \quad R = R_{rad}\ . \tag{1}$$

The first condition – equality to zero of the sum of the reactance matrices – means that the best absorber should be of the resonant type. The second Eq. (1) – equality of the resistance matrices – physically means that maximum of the absorbed energy is achieved only when it becomes equal to the energy absorbed by the ambient medium. This second condition of the best absorber seems very special and poor understandable, but it works well in electrodynamics and was mentioned in some particular situation in acoustics (e.g. [3]). While conducting experiments, the present authors were convinced many times in its validity.

3. Quarter wavelength resonator

The basic element of the absorber under design is considered as a quarter wavelength resonator (a tube with one closed and one open ends). It is one of the simplest elements of the resonant type available to analytical description of its longitudinal vibration including the energy losses. If L is a length of the tube then its first resonance frequency is equal to

$$f = \frac{c_0}{4L} \tag{2}$$

where c_0=333 [ms^{-1}] is the speed of sound.

The energy losses take place mostly near tube walls and are caused by viscosity and heat transfer. Exact solution taking into account these effects was obtained by G. Kirchhoff in 1898. After that approximate and more simple solutions were derived by many authors. Here equations of the book [3] is used where it was shown that viscous losses are equivalent to complex density and heat transfer leads to complex compressibility. Using the theory, for the complex wave number of the wave travelling in the tube one can write:

$$k = k_0 + i\gamma,$$
$$\gamma(\omega) = \frac{1}{2c}\frac{l}{\sigma}\left[\sqrt{\frac{\nu\omega}{2}} + (\frac{C_p}{C_v} - 1)\sqrt{\frac{\chi\omega}{2}}\right]. \tag{3}$$

Here $k = \omega/c_0$, l and σ are length and area of the tube cross-section, $C_p/C_v = 1.4$ is the specific heat ratio, $\nu = 1.5 \cdot 10^{-5}$ [m^2s^{-1}] is the kinematic viscosity, $\chi = 3 \cdot 10^{-5}$[m^2s^{-1}] is the thermal conductivity. Using Eq. (3), one can find the input impedance of the finite tube at the open end with respect to external force as:

$$Z_{tube} = i\rho c\sigma \cdot \cot(kL) \tag{4}$$

4. Effective impedance of absorber

If the absorber has outer surface *S* and consist of *N* quarter-wavelength tubes whose open ends are situated at *S* (Fig. 1), then effective surface impedance of the absorber is equal:

$$Z_{eff} = Z_{tube}\frac{S}{\sigma N} \quad ,$$

where Z_{tube} is given in Eq. (4).

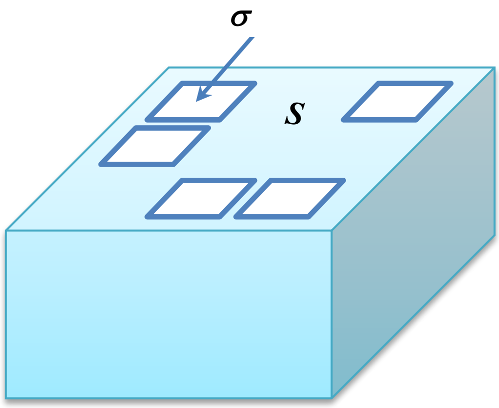

Figure 1: Outer surface of the absorber.

More generally, if there are *M* types of quarter-wavelength tubes tuned on *M* different frequencies and the absorber contains N_m tubes of type *m*, then effective impedance of the absorber can be shown to be equal:

$$Z_{eff} = \left[\sum_{m=1}^{M}\frac{1}{Z_{tube}^{(m)}}\frac{\sigma_m N_m}{S}\right]^{-1}.$$

5. Impedance designing

The process of designing of the absorber is reduced to choice of several parameters (M, N_m, L_m, σ_m) that provide the best fulfilment of the requirements (1). The first requirement is satisfied at *M* given resonance frequencies determined by the length of the tubes, see Eq. (2). The second requirement rewritten in the form

$$Re(Z_{eff}) = R_{rad}$$

which also must be satisfied at *M* resonance frequencies have several solutions. One of this solution may be chosen according to other criteria (e.g. minimum absorber volume). The radiation resistance of the absorber R_{rad} is determined by the acoustic environment (preliminary measured or computed) and therefore considered as known.

Fig. 2 presents by solid line the predicted efficiency of the absorber designed for the Kundt's tube (for which R_{rad} always equals to ρc) containing two types of tubes tuned for frequencies f_1=200 Hz ($N_1 = 5$) and f_2=400 Hz ($N_2 = 6$). Such a sample was fabricated by 3-D printing, where the tubes were represented in the form of zigzags for the sake of volume economy. Measured efficiency of the sample is shown in Fig. 2 by diamonds. It is seen that accordance between the theory and measurements is acceptable. By choosing proper resonance frequencies and numbers of tubes and using the presented design procedure one can develop almost ideal sound absorber in any given frequency range.

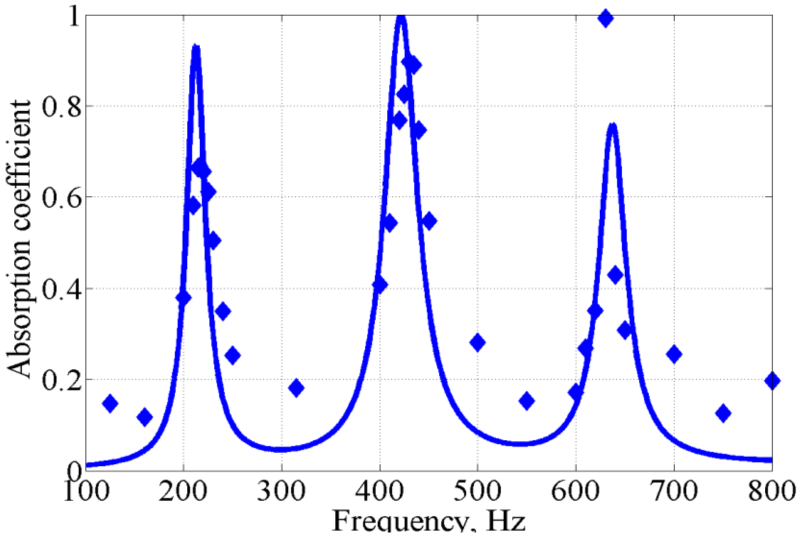

Figure 2: Frequency dependences of absorption coefficient: theory (solid line) and experiment.

6. Summary

A design procedure of efficient sound absorber based on the concept of the best absorber is presented. It is reduced to implementation of some simple impedance relations following from the general theory. The procedure is validated in laboratory experiment.

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