DESIGN OF MICRO STRUCTURE OF POROELASTIC MATERIAL TO CONTROL SOUND ABSORPTION BY HOMOGENIZATION METHOD

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In this paper, we apply the multi-scale analysis developed by one of the author and predict sound absorption coefficients for normal incidence from the microscopic structures. Poroelastic sound absorbing media are generally classified into foam material and fibrous material. Here we capture the characteristics of micro structure of polyurethane foam and fibrous felt from the observation by SEM such as the pore size, thin membranes of polygonal faces and fiber diameter, and construct unit cell models assuming that sound absorbing materials have a geometric periodicity in microscopic point of view. We obtained practically good agreements between measured and calculated sound absorption coefficients.
Keywords: Microscopic structure, Poroelastic, Sound absorption, Homogenization, Material design

1. Introduction

Noise reduction is one of the major issues in automotive engineering to secure quietness in passenger compartments. One possible measure is to absorb sound by utilizing poroelastic media, e.g., floor carpets and dash insulators. In the design process of a vehicle, it is necessary to predict the macroscopic performance of poroelastic media, such as their sound absorption coefficients. Poroelastic material is composed of solid and fluid phases and the macroscopic performance of a poroelastic material is governed by the characteristics of each phase. Since those characteristics depend significantly on the microscopic geometry of the poroelastic material, predicting the macroscopic performance from the microscopic geometry would be essential for profound understanding of the physical behavior involved.

Macroscopic properties and governing equations can also be derived from the microscopic geometry by using the homogenization theory based on the method of asymptotic expansions, assuming that geometric periodicity exists on the micro-scale. Auliault et al. [1] considered a macroscopic description of rigid porous media saturated with an incompressible viscous fluid and derived a macroscopic permeability tensor that they verified experimentally. Terada et al. [2] studied the macroscopic characteristics of deformable poroelastic media saturated with an incompressible viscous fluid and presented numerical results to show the practical applicability of their approach. Lafarge et al. [3], Boutin et al. [4], and Lee [5] derived the macroscopic models of sound propagation through rigid porous media. Air contained in pores was modeled as a compressible viscous fluid, and the thermal dissipation from the fluid phase to the solid phase was also taken into account. Levy [6], and Burridge and Keller [7] derived the macroscopic governing equations of deformable poroelastic media saturated with a compressible viscous fluid. However, the thermal dissipation from the fluid phase into the solid phase was not taken into account and accordingly, the bulk modulus of the fluid phase was assumed to be constant. Although sound absorption in poroelastic media is a typical multiphysics phenomenon where the behavior of the elastic solid, the compressible viscous fluid and the fluid temperature must be all considered at the same time, the studies mentioned above deal with only some of the physics observed in sound-absorbing poroelastic media.

Therefore, in the study presented here, we apply a general and complete model that describes the macroscopic properties and the governing equations of sound-absorbing poroelastic media using the mathematical homogenization method. This model takes into account the motions of the elastic solid and compressible viscous fluid, and the distribution of temperature in the fluid.

The remainder of this paper is organized as follows. Section 2 gives a brief description about the homogenization method for sound-absorbing poroelastic material. Section 3 presents the application of the homogenization approach to fibrous poroelastic materials and investigate the performance and the relationship between air flow resistivity and fiber diameter. Section 4 concludes this study.

2. Homogenization of sound-absorbing poroelastic material

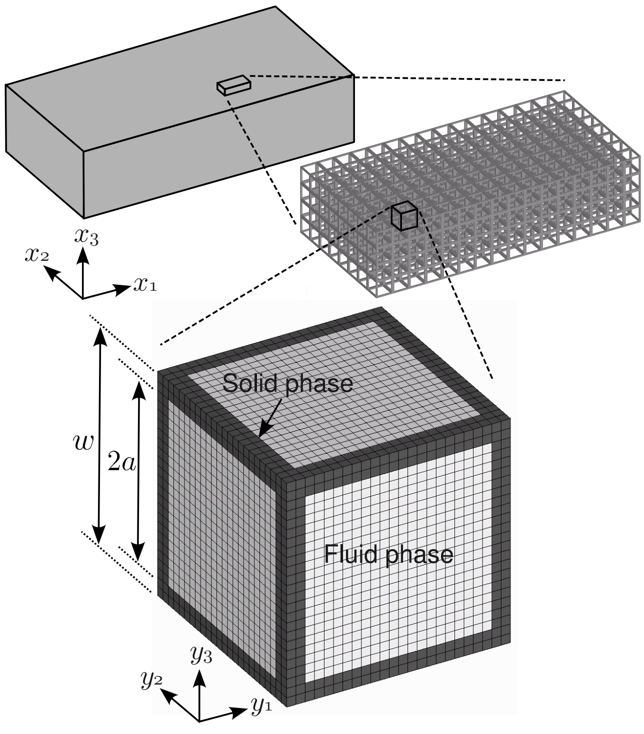

Figure 1: Schematic view of homogenization of sound-absorbing poroelastic material.

Here we give a brief description of the homogenization method applied in this study. Please refer to a published paper [8] by one of the author for the theory and the procedures of the homogenization of sound-absorbing poroelastic material.

We assume that the solid phase is composed of linear elastic material and that the fluid phase is saturated with a compressible viscous fluid. The domain of the fluid phase is assumed to be connected throughout the material. The solid phase is governed by the linear equations of elasticity and the fluid phase by linearized Navier-Stokes equation since infinitesimal harmonic motions are assumed. The fluid phase is also governed by the Fourier's law for thermal conduction assuming that the solid phase can maintain isothermal conditions. The mass conservation law and the state equation of gas are also included. The continuity conditions for the velocity, the strain and the temperature are imposed at the boundary between the solid and fluid phases.

We assume a solution in the asymptotically expanded form for physical variables such as displacements of the solid phase and pressure of the fluid phase. Then the expanded forms of are substituted into the governing equations described above. By collecting terms with same orders, boundary value problems for microscopic scale are derived. These problems can be numerically solved by using finite element method. Furthermore, by averaging the solutions, we can obtain macroscopic material properties that can be used in macroscopic boundary value problems.

3. Application to fibrous poroelastic material

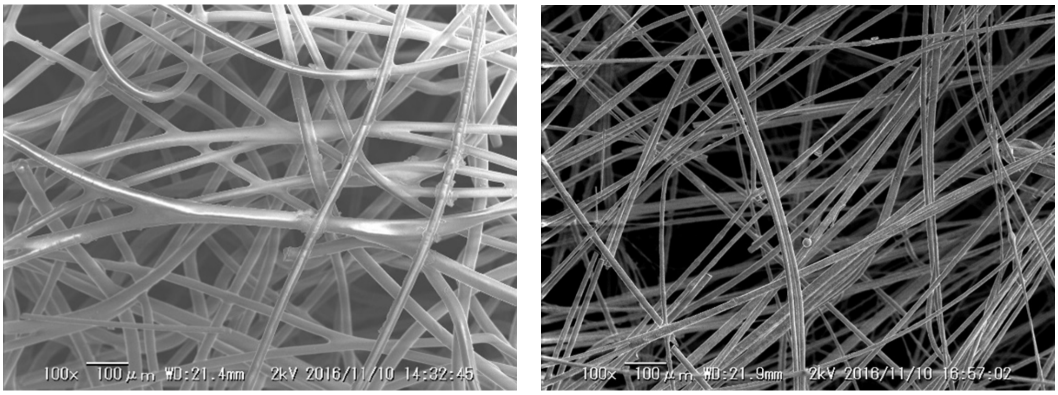

Figure 2: SEM images of typical fibrous poroelastic materials.

We apply the method to a fibrous felt as shown in Fig. 2. Fiber diameter generally ranges from 1 μm to several hundred μm. This type of material usually expects high absorption coefficient above 1 kHz.

3.1 Effect of fibrous diameter

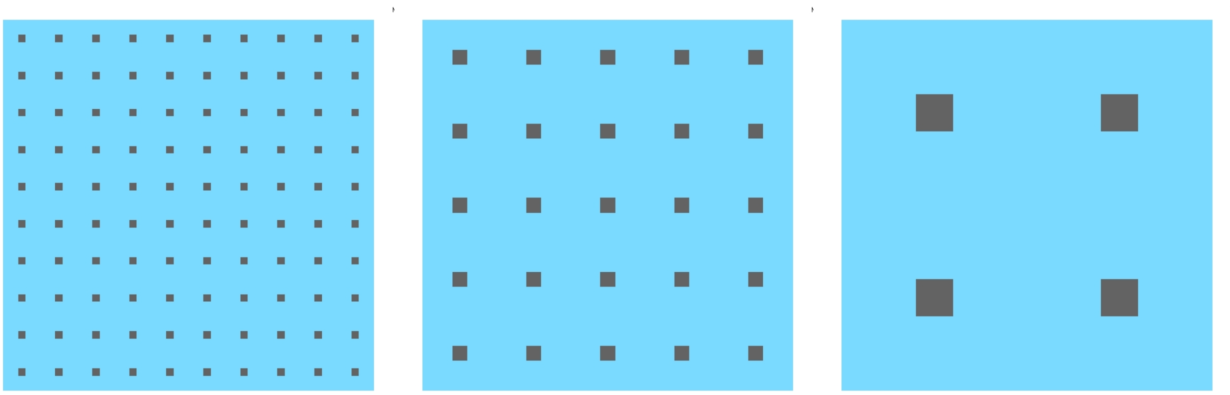

Figure 3: Uniform fibrous poroelastic material of constant porosity 0.96 with various fiber diameter: fiber diameter is 4 μm, 8 μm, and 20 μm.

Two dimensional unit cell models for fibrous felt are considered. Fiber is assumed to be located periodically and uniformly as shown in Fig.3 and porosity is constant as 0.96. Fiber diameter ranges from 0.2 μm to 100 μm that corresponds to unit cell size ranges from 1.0 μm to 500 μm. For simplicity rectangular fiber section is also assumed. Polypropylene is selected as fiber material. Young's

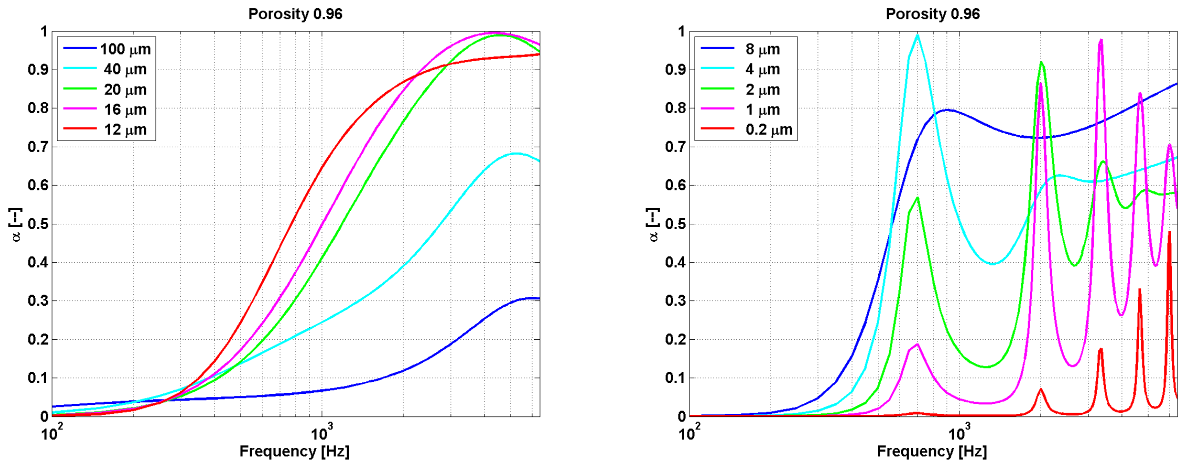

Figure 4: Absorption coefficients for various fiber size.

modulus and mass density are 1.35 GPa and 900 kg/m^3, respectively. Poisson's ratio is 0.40 and the loss factor is set to 0.30. Unit cell is discretized by 40^2 cubic 1st order elements.

Sound absorption coefficients are calculated as shown Fig.4 by applying the homogenization method based on the asymptotic approach. Fiber size around 15 μm is found to exhibit relatively high absorption from 2 to 6 kHz.

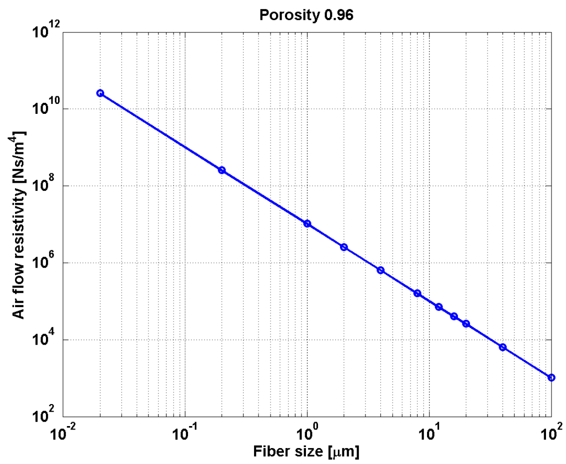

Figure 5: Relationship between air flow resistivity and fiber size.

Air flow resistivity tensor σ_{ij} can be directly calculated in the homogenization process. Generally, Air flow resistivity tensor is defined as $\sigma_{ij} v_i = -\dfrac{\partial p}{\partial x_j}$. On the other hand, the mean velocity averaged over a unit cell can be expressed using the homogenization method as $\langle w_i^{(0)} \rangle_{Y^f} = \langle \xi_i^j(\boldsymbol{y}) \rangle_{Y^f} \left(-\dfrac{\partial p^{(0)}}{\partial x_i} \right)$. Comparing with these equations, air flow resistivity is obtained as follows:

$$\sigma_{ij} = \langle \xi_i^j(\boldsymbol{y}) \rangle_{Y^f}^{-1} \tag{1}$$

In the calculation of air flow resistivity tensor, quasi static condition is applied by imposing very small excitation frequency of 0.001 Hz. From Fig.5, logarithm of fiber size and logarithm of air flow resistivity is found to be proportional. Since fibers are distributed uniformly, the distance between fibers is also proportional to air flow resistivity.

Once air flow resistivities are obtained, empirical Delany - Bazley model can be applied for a model of fibrous poroelastic material. From Fig.6, poroelastic materials with fiber diameter larger

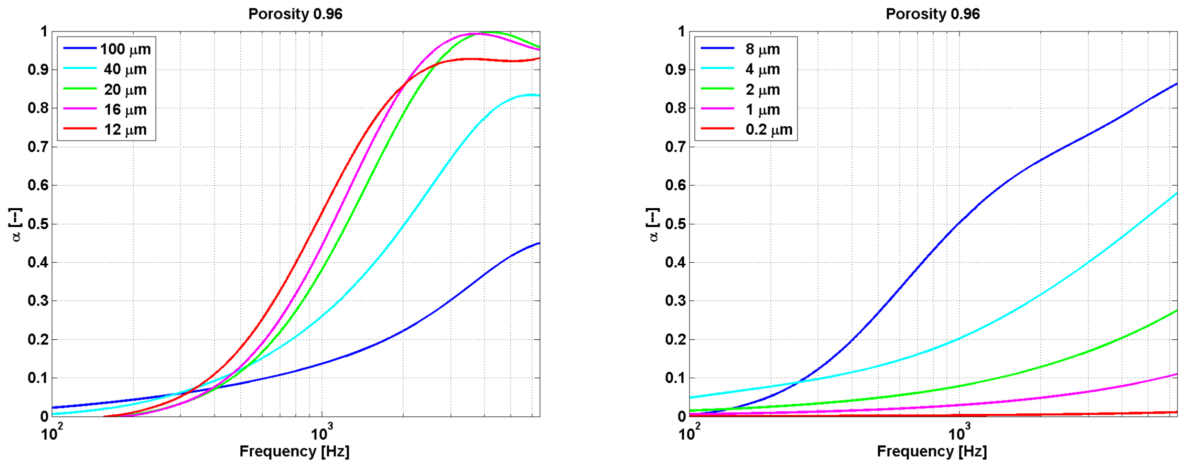

Figure 6: Absorption coefficients by Delany-Bazley model.

than 10 μm which have air flow resistivity less than 100 kNs/m^4 can be described well by Delany-Bazley model. For fibers narrower than 10 μm, several peaks appear in absorption spectrum.

3.2 Effect of compression

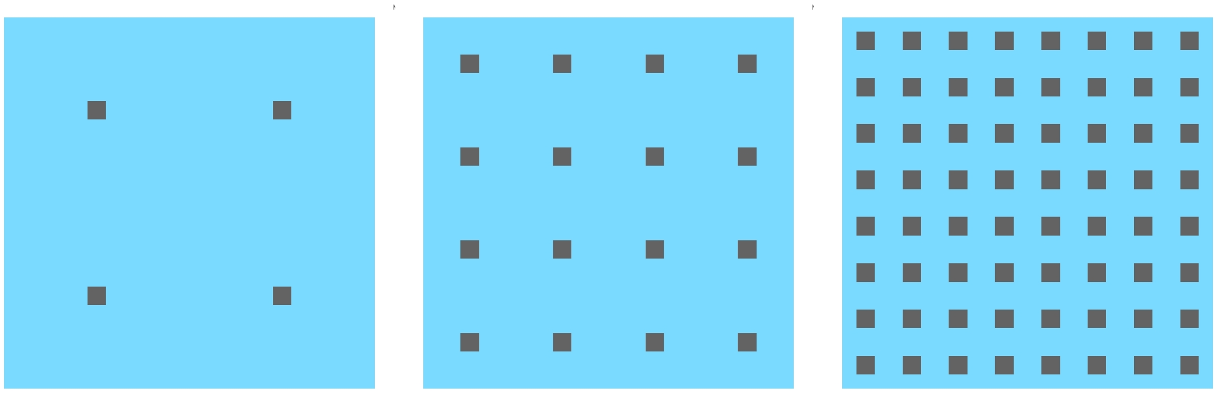

Figure 7: Compressed fibrous material of fiber diameter 20 μm with variable porosity (2D): porosity is 0.99, 0.96, and 0.84.

Next we consider the effect of compression of fibrous felt which is usually applied in manufacturing processes. 2-dimensional fiber models are prepared with constant fiber size of 20 μm. The size of unit cell is varied from 50 μm to 400 μm as shown in Fig. 7. Here fibrous materials are assumed to be compressed uniformly in each directions, although they are generally compressed in one direction. As shown in Fig. 8, 3-dimensional fiber models are also constructed whose orientations are only $0°$ and $90°$ for simplicity.

Sound absorption coefficients are calculated as shown Fig.9 by applying the homogenization method based on the asymptotic approach. Fibrous material with porosity from 0.92 to 0.98 can be described by Delany-Bazley model.

Since fiber size is kept constant unlike the models in the previous section, porosity can be an alternative parameter for material characterization. However, neither ϕ nor $1-\phi$ is proportional to air flow resistivity. The width of air flow path can be considered to be an important factor as viscous dissipation is generally significant. Thus, we now define a new parameter as the mean fiber distance d_f. When we consider a 2-dimensional unit cell model that has n rectangular fibers of the size d within rectangular region of a^2, porosity is expressed as

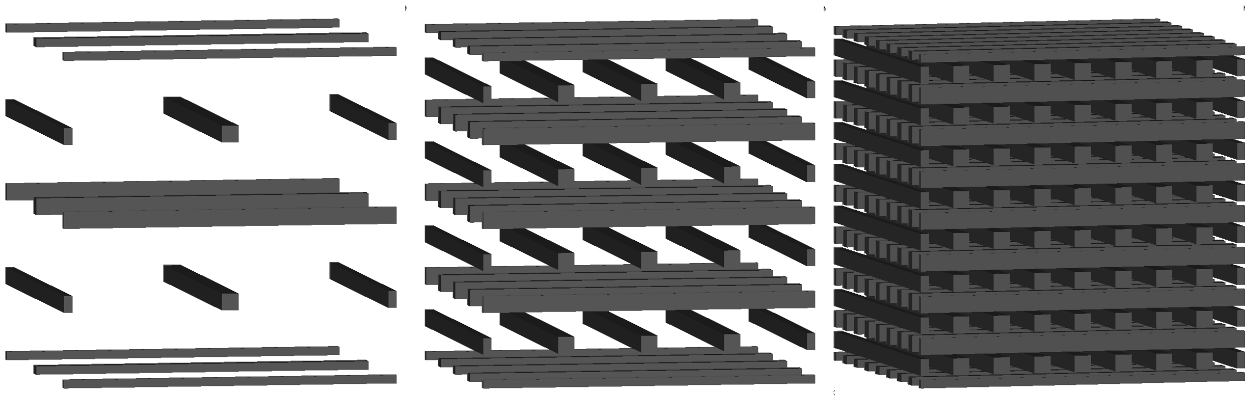

Figure 8: Compressed fibrous material of fiber diameter 20 μm with variable porosity (3D): porosity is 0.98, 0.92, and 0.68.

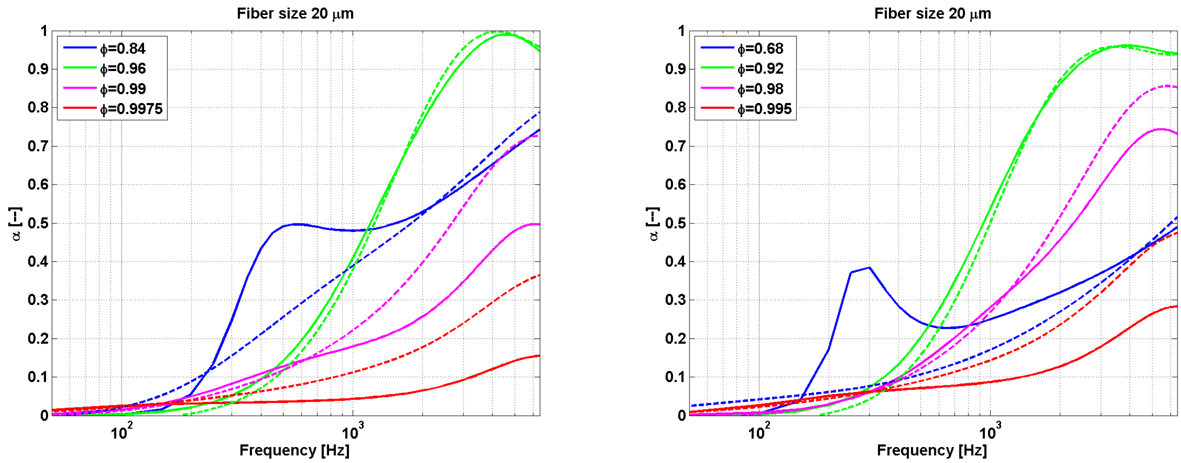

Figure 9: Sound absorption coefficients by homogenization approach and Delany-Bazley model.

$$\phi = 1 - \frac{nd^2}{a^2} \tag{2}$$

The mean fiber distance d_f can be expressed as

$$d_f = \sqrt{\frac{a^2}{n}} - d = \left(\frac{1}{\sqrt{1-\phi}} - 1 \right) d \tag{3}$$

From Fig. 10, logarithm of the mean fiber distance d_f is well proportional to logarithm of air flow resistivity for both 2-dimensional and 3-dimensional models.

4. Conclusions

The homogenization method for sound-absorbing poroelastic material are applied for fibrous poroelastic material. Predicted sound absorptions for fibrous materials that has air flow resistivity less than 100 kNs/m^4 agree well with those by empirical Delany-Bazley model. The mean fiber distance d_f is newly defined here and is found to be well proportional to air flow resistivity for both 2-dimensional and 3-dimensional models.

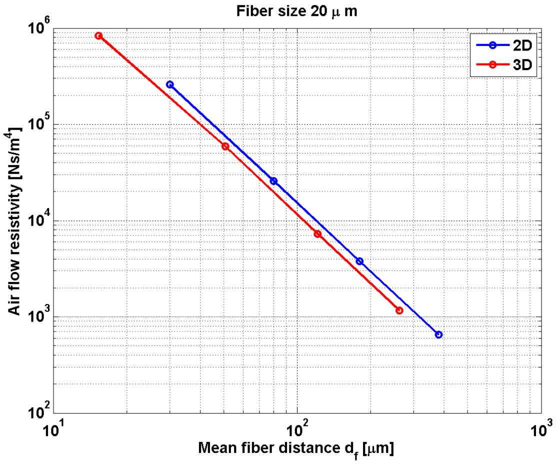

Figure 10: Relationship between air flow resistivity and mean fiber distance.

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