24th INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
23–27 July 2017, London

ACOUSTIC STREAMING IN RESONATOR OF SQUARE CROSS SECTION

Takeru Yano
Osaka University, Department of Mechanical Engineering, Suita, Japan
email: yano@mech.eng.osaka-u.ac.jp

Resonant gas oscillations and related nonlinear phenomena in a resonator of square cross section are studied by numerically solving the system of Navier–Stokes equations for compressible fluid flows. Large-scale and three-dimensional computations of unsteady motions of compressible thermo-viscous fluid are performed with second-order central differences in space and a third-order Runge–Kutta method in time. The gas oscillation is supposed to be excited by a harmonic oscillation of the resonator with the fundamental resonant frequency, and the resulting flow is an acoustic standing wave in the case of sufficiently weak excitation or a gas oscillation with a shock wave in the case of strong excitation. The boundary layer on the wall of resonator is resolved with enough accuracy. We determine the three-dimensional velocity field of acoustic streaming by a time-average of oscillating mass flux density vector both in the inside and outside of the boundary layer.
Keywords: acoustic streaming, standing wave, shock wave

1. Introduction

Gas oscillations at resonance states in systems of gas and (rigid) resonator are large amplitude, leading to the emergence of nonlinear phenomena, if the frequency is not too high. One of the important nonlinear phenomena is acoustic streaming [1, 2, 3, 4, 5], which slowly convey the mass of the gas, and hence also the momentum and energy. The purpose of this paper is simple: we want to know how, and from where to where, the mass of gas is conveyed by the streaming motion in the resonator. Although several contributions have actually been made in theories, experiments, and simulations [6, 7, 8, 9, 10, 11], they are limited to two-dimensional or axisymmetric flows. We therefore perform large-scale three-dimensional computations of unsteady motions of compressible thermo-viscous fluid in a resonator of square cross section.

2. Formulation of problem

A resonator of length L with a uniform square cross section of side a is filled with an ideal gas with specific heat ratio γ. We consider the gas oscillation of the fundamental resonance mode with frequency $c_0/2L$ (c_0 is the speed of sound in an initial undisturbed gas), driven by a sinusoidal shaking of resonator in its axial direction (see Fig. 1(a)). The motion of the gas in the resonator is determined as a solution of initial and boundary value problem of the system of compressible Navier–Stokes equations with external force terms:

$$\frac{\partial \rho}{\partial t} + \frac{\partial \rho v_i}{\partial x_i} = 0 \tag{1}$$

$$\frac{\partial \rho v_i}{\partial t} + \frac{\partial}{\partial x_j}(\rho v_i v_j + p\delta_{ij}) = \frac{1}{Re}\frac{\partial}{\partial x_j}\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\frac{\partial v_k}{\partial x_k}\delta_{ij}\right) + \delta_{i1} Ma\, \rho \sin t \tag{2}$$

$$\frac{\partial E}{\partial t} + \frac{\partial}{\partial x_j}[(E+p)v_j] = \frac{1}{Re}\frac{\partial}{\partial x_j}\left[\left(\frac{\partial v_i}{\partial x_j} + \frac{\partial v_j}{\partial x_i} - \frac{2}{3}\frac{\partial v_k}{\partial x_k}\delta_{ij}\right)v_i\right] + \frac{1}{RePr(\gamma-1)}\frac{\partial^2 T}{\partial x_k^2} + Ma\, \rho v_1 \sin t \tag{3}$$

$$v_i = 0,\ \rho = 1, T = 1,\ p = \gamma^{-1} \text{ everywhere at } t = 0, \tag{4}$$

$$v_i = 0 \text{ and } T = 1 \text{ on the wall of the resonator for } t \geqq 0, \tag{5}$$

where t is the time normalized by $L/c_0\pi$, x_i is the ith component of space coordinates normalized by L/π, ρ is the gas density normalized by ρ_0, v_i is the ith component of the gas velocity normalized by c_0, T is the gas temperature normalized by T_0, $p = \rho T/\gamma$ is the normalized gas pressure, $E = (1/2)\rho v_i^2 + p/(\gamma-1)$ is the normalized total energy per unit volume of the gas (the subscript 0 denotes the initial undisturbed state). The nondimensional parameters, Ma, Re, and Pr, are defined by

$$Ma = \frac{U}{c_0}, \quad Re = \frac{\rho_0 c_0^2}{\mu\omega}, \quad Pr = \frac{\mu c_p}{\lambda}, \tag{6}$$

where U is the maximum speed of sinusoidal motion of resonator, μ is the viscosity coefficient, λ is the thermal conductivity coefficient, c_p is the specific heat at constant pressure per unit mass, and $\omega = c_0\pi/L$ is the angular frequency of the oscillation. We assume μ and λ are constants and the bulk viscosity coefficient is zero.

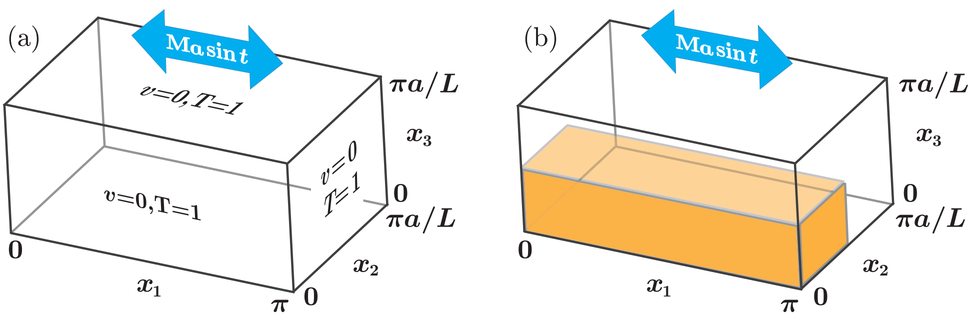

Figure 1: (a) Schematic of the model. (b) Computational region is a quarter of the resonator.

3. Result

The initial and boundary value problem presented in the previous section is solved numerically by using the second-order central-difference in space and a three-stage Runge–Kutta method for advancing in time, without introducing artificial viscosity.

Only a quarter part of resonator ($0 \leqq x_1 \leqq \pi$, $0 \leqq x_2, x_3 \leqq \pi a/2L$) is computed on the assumption of symmetry as shown in Fig. 1(b). Asymmetric streaming motions in resonators have been reported by several authors not only in experiments but simulations [12, 13]. In the present paper, however, we treat rather small Re (≈ 2000), and hence we expect the symmetry of the streaming motions with respect to the central axis of the resonator ($x_2 = x_3 = \pi a/2L$).

We here present the simulation result in the case of $Ma = 0.03$, $Re = 2000$, and $a/L = 1/6$. The computational region, a quarter of the resonator, is discretized with a uniform mesh of $1600 \times 200 \times$

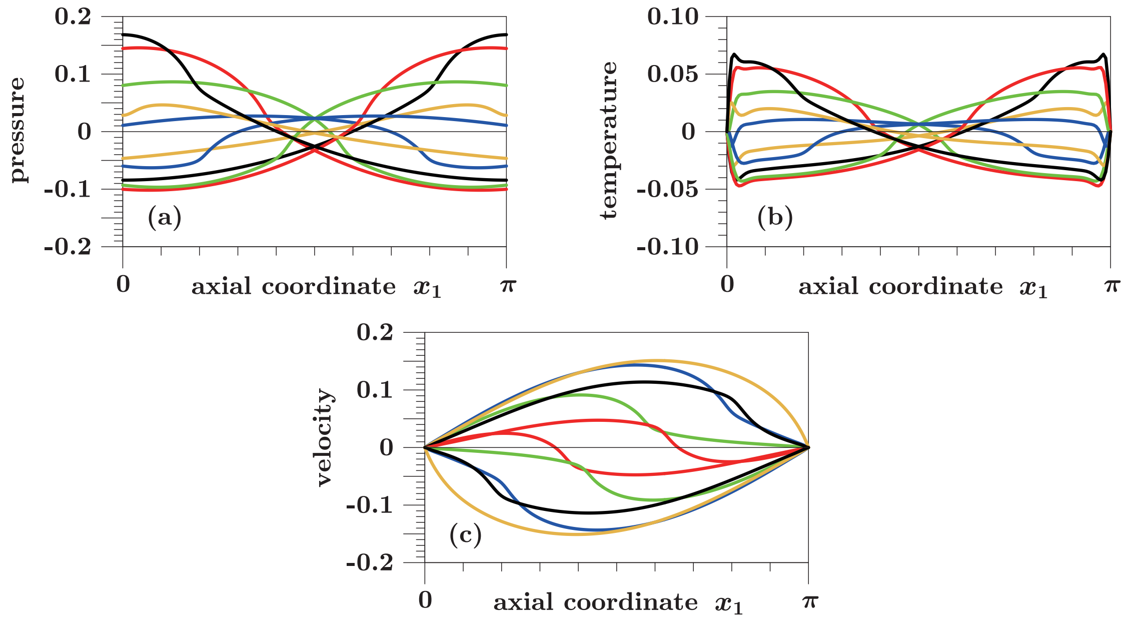

Figure 2: Wave profiles for $200\pi \leqq t \leqq 202\pi$ in the case of $Ma = 0.03$, $Re = 2000$, and $a/L = 1/6$. (a) Normalized pressure variation from the reference pressure. (b) Normalized temperature variation from the reference temperature. (c) Normalized velocity.

200; then the mesh size in the x_1-direction $\Delta x_1 \approx 0.09/\sqrt{Re}$ and that normal to the central axis is $\Delta x_2 = \Delta x_3 \approx 0.06/\sqrt{Re}$, which have been found to be sufficient for the resolution of boundary layer by our preliminary computations. The time step is $\Delta t = 0.000314$.

Figure 2 shows typical wave profiles on the central axis in one oscillation cycle $200\pi \leqq t \leqq 202\pi$, in the time range of which the wave motion is almost time periodic with period 2π. The amplitude of gas oscillation reaches almost $5 \times Ma$ and the profile is distorted by the nonlinear effect, although the shock wave is not formed. If at least one of three parameters $(Ma, Re, a/L)$ is increased, the nonlinear effect is enhanced and the shock wave will appear. The temperature profiles in Fig. 2(b) displays the steep gradient in the boundary layer. Our computation well resolves the steep gradient thanks to the fine mesh.

The velocity of acoustic streaming motion, $c_0 V_i$, is defined by the time average of mass flux density vector divided by the reference density, i.e.,

$$V_i(\boldsymbol{x}, t) = \frac{1}{t_{\text{ave}}} \int_t^{t+t_{\text{ave}}} \rho(\boldsymbol{x}, \tau) v_i(\boldsymbol{x}, \tau)\, \mathrm{d}\tau \tag{7}$$

where t_{ave} is a nondimensional arbitrary average time. In experiments, a time range sufficiently large compared with the acoustic period $2L/c_0$ is chosen for t_{ave}, while in numerical simulations $t_{\text{ave}} = 2\pi$ is often used.

In Fig. 3, we show the streaming velocity field (V_1, V_2) in the $x_1 x_2$-plane at $t = 200\pi$ and $t_{\text{ave}} = 2\pi$. The figures are enlarged in the x_2-direction so that one can distinguish the vortices inside and outside the boundary layer, which is a typical vortex structure in Rayleigh streaming [1]. In the time range $200\pi \leqq t \leqq 202\pi$, we confirmed that the time average of the first term in the left-hand side of Eq. (1) vanishes (numerically $O(10^{-6})$), which means that the streaming velocity field is divergence free. Incidentally, the time average of gas velocity v_i is not divergence free, and this is the reason why the time average of gas velocity v_i is not used for the definition of streaming velocity. The difference between Figs. 3(a) and 3(b) proves that the streaming motion is a three-dimensional flow.

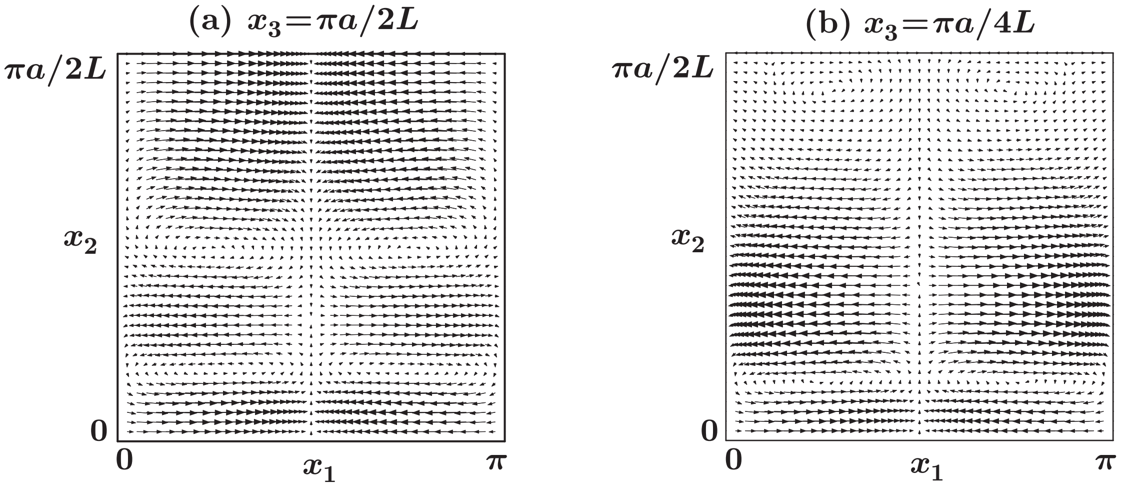

Figure 3: Streaming velocity field in the x_1x_2-plane in the case of $Ma = 0.03$, $Re = 2000$, and $a/L = 1/6$. (a) A symmetry plane at $x_3 = \pi a/2L$. (b) A plane at $x_3 = \pi a/4L$.

4. Concluding remark

The numerical solution of the nonlinear resonant gas oscillation in the resonator of square cross section is obtained by a large-scale three-dimensional computation. The three-dimensional character of streaming motion will be clarified through the detailed investigation of the numerical solution, which will be reported in a forthcoming paper.

In concluding, we make a comment on the time-periodicity of the numerical solution. We usually expect that the system of resonant oscillation will reach the time-periodic state ultimately. If the time-periodic state is realized, the time averages of the first terms in the left-hand sides of Eqs. (1)–(3) should all vanish, because the time average of derivative of periodic function is zero. In the case shown above, we confirmed that the density ρ is almost time periodic at $t = 200\pi$ and hence the streaming velocity is divergence free at that time. However, the internal energy (i.e., temperature) inside the boundary layer cannot be regarded as a time-periodic function at $t = 200\pi$. That is, the establishment of time-periodic energy flux (including heat flux) requires a longer time compared with the mass flux, i.e., acoustic streaming.

REFERENCES

1. Lord Rayleigh, *Theory of Sound*, Dover, New York (1945).

2. Westervelt, P. L., The theory of steady rotational flow generated by sound field, *J. Acoust. Soc. Am.*, **25,** 60–67, (1953).

3. Nyborg, W., Acoustic streaming due to attenuated plane wave, *J. Acoust. Soc. Am.*, **25,** 68–75, (1953).

4. Rudenko, O. V. and Soluyan, S. I., *Theoretical Foundations of Nonlinear Acoustics*, Consultant Bureau, New York (1977).

5. Lighthill, M. J., Acoustic streaming, *J. Sound Vib.*, **61,** 391–418, (1978).

6. Vainshtein, P., Rayleigh streaming at large Reynolds number and its effect on shear flow, *J. Fluid Mech.*, **285,** 249–264, (1995).

7. Yano, T., Turbulent acoustic streaming excited by resonant gas oscillation with periodic shock waves in a closed tube, *J. Acoust. Soc. Am.*, **106,** L7–L12, (1999).

8. Hamilton, M. F., Ilinskii, Y. A. and Zabolotskaya, E. A., Acoustic streaming generated by standing waves in two-dimensional channels of arbitrary width, *J. Acoust. Soc. Am.*, **113,** 153–160, (2003).

9. Thompson, W. W., Atchley, A. A. and Maccarone, M. J., Influences of a temperature gradient and fluid inertia on acoustic streaming in a standing wave, *J. Acoust. Soc. Am.* **117,** 1839–1849, (2004).

10. Marx, D. and Blanc-Benon, P., Computation of the mean velocity field above a stack plate in a thermoacoustic refrigerator, *C. R. Mecanique*, **332,** 867–874, (2004).

11. Daru, V., Baltean-Carlès, D., Weisman, C., Debesse P., and Gandikota, G., Two-dimensional numerical simulations of nonlinear acoustic streaming in standing waves, *Wave Motion*, **50,** 955–963, (2013).

12. Merkli, P. and Thomann, H., Thermoacoustic effects in a resonance tube, *J. Fluid Mech.*, **70,** 161–177, (1975).

13. Yano, T., Numerical study of high Reynolds number acoustic streaming in resonators, *INNOVATIONS IN NONLINEAR ACOUSTICS: AIP Conference Proceedings*, **838,** 379–386, (2006).