THE LONGITUDINAL VIBRATION OF COMPOSITE DRIVE SHAFT

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The longitudinal vibration characteristics of composite drive shaft are investigated in this paper. Composite materials are sometimes considered for manufacturing long drive shafts for the sake of reducing its weight as well as vibration. The applications of composite drive shafts have already been developed in various areas such as ships, cars, helicopters, etc. As far as ship shaft vibration control is concerned, the longitudinal vibration response control is normally very difficult, which is receiving more and more concerns. An analytical method for longitudinal vibration natural frequency is developed, and then a finite element analysis is performed to explore the longitudinal vibration behaviour of drive shaft, mainly including natural frequencies, mode shapes, and longitudinal vibration reduction. Effects of thrust bearing stiffness, fibre orientation and length-diameter ratios of composite drive shaft on its longitudinal vibration behaviour were also examined. The result shows that the first order natural frequency of composite drive shaft increases with the thrust bearing stiffness while the vibration response decreases. Increasing the fibre orientation and decreasing the length-diameter ratio can both reduce the first order frequency and the vibration response.

Keywords: composite drive shaft, natural frequencies, mode shapes, longitudinal vibration

1. Introduction

Besides the steady drive force, longitudinal unsteady resonant force excited by ship propeller is also generally transferred to the hull by drive system, thrust bearing and its foundation. Due to the factors such as asymmetric mounting of ship shaft system etc., asymmetric vibration mode emerged in the hulls and radiation noise will be generated. Therefore, the longitudinal vibration of drive shaft caused by propeller excitation has a direct and big contribution to the ship's acoustic and vibration properties. Traditional drive shafts made of metal or alloy sometimes have serious propeller-shaft-ship coupling vibration problem, unsatisfactory vibration attenuation performance etc., as well as great shaft weight. Hence fibre reinforced composite material is increasingly proposed to apply on drive shaft due to high specific strength and specific modulus , high damping factor and consequently potential advantages on vibration attenuation, etc. It is thus imperative to establish a theoretical method to estimate the longitudinal dynamic response of composite shaft system in the ship and investigate the influence of structural parameters on the vibration property of shaft systems.

The research on composite drive shaft can be traced back to 1970s. Zinberg et al [1] introduced the application of composite drive shaft on helicopters at that time. Since 1990s, fibre reinforced composite drive shafts were sometimes used on ship shafting because of its light weight. Two shafts made of carbon fibre reinforced plasticity (CFRP) were developed by Singh and Gupta [2-3]. And the regulations about how coupling effects and natural frequencies change with fibre orientation were experimentally studied. Besides, it is found that the natural frequency increased with the decrease of fibre orientation [4-6]. Sino [7] found that natural frequencies and damping factors changed with fibre volume content. Chang et al. [8-9] analysed the vibration characteristics of composite shaft and the interaction between fibre reinforced plasticity elements. Boukhalfa et al. [10] analysed the vibration characteristics of composite shaft by trigonometric function model and finite element method.

In this paper, the theoretical method to natural frequencies of drive shaft was given based on a simplified model. Natural frequencies and vibration modes of the shafts with varied parameters were analysed, and the comparison of vibration characteristics of stern shaft with CFRP and steel was implemented. Finally, effects of thrust bearing, fibre orientation and length-diameter ratio on vibration response characteristics were analysed.

2. Longitudinal vibration theory calculation method of drive shaft

The ship drive shaft is usually composed of multiple sections. In this paper the three section model was considered, in which thrust, intermediate and stern sections were involved. Influences of thrust bearing, propeller and flexible coupling on the dynamic property of shaft system were also considered, as shown in Fig. 1. The spring stiffness was used to simulate the thrust bearing stiffness.

Due to the lower longitudinal stiffness of stern shaft made of CFRP material compared to the intermediate and thrust shaft sections which made of metal materials, the intermediate and thrust shaft sections and the flexible coupling can be simplified to a lumped mass when the first order longitudinal natural frequency was estimated, as shown in Fig. 2. In the figure m_1 represents the lumped mass for the total mass of thrust and intermediate shaft sections and flexible coupling, and m_2 is the mass of propeller. k is the stiffness of thrust bearing.

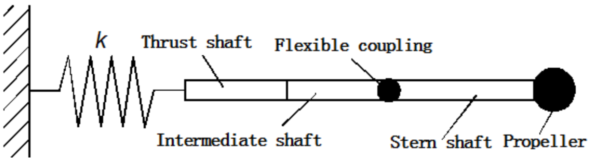

Figure 1: Model of three section shaft

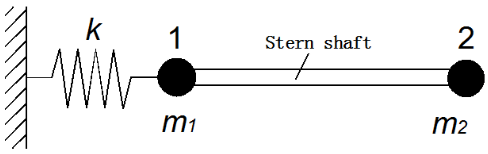

Figure 2: Simplified model of shaft

The wave motion equation of stern shaft section can be expressed as

$$U(x,t) = u(x)t(t) = (B_1 \sin\frac{\omega}{a}x + B_2 \cos\frac{\omega}{a}x)\sin(\omega t + \varphi). \tag{1}$$

So the equation in point 1 and 2 can be written as

$$\begin{cases} EAu'(0) = ku(0) - m_1\omega^2 u(0) \\ EAu'(l) = m_2\omega^2 u(l) \end{cases}. \tag{2}$$

It can be concluded that $R \cdot [B_1 \quad B_2]^T = 0$, and

$$R = \begin{bmatrix} \frac{EA}{a}\omega & -k + m_1\omega^2 \\ \alpha\cos\beta - \beta\sin\beta & -\alpha\sin\beta - \beta\cos\beta \end{bmatrix} \tag{3}$$

Where $\alpha = \rho Al / m_2$; $\beta = \omega l / A$, E、A、l、ρ represent longitudinal modulus, section area, length and density of CFRP stern shaft, respectively; $a = \sqrt{E/\rho}$; k was the stiffness of thrust bearing.

Because $R \cdot [B_1 \quad B_2]^T = 0$ have non-zero solution, the equation $|R| = 0$ can be used to calculate the natural frequency of shaft.

If the stiffness of thrust bearing was very small, meanwhile the stiffness of the whole shaft was far greater than it, the whole shaft can then be simplified as a lumped mass, thus the shaft system can be simplified as a spring vibrator.

3. Analysis of longitudinal vibration characteristic of composite shaft

3.1 Basic information of model

The aim of this paper is to investigate the vibration property of a three section drive shaft with thrust, intermediate and stern parts. The thrust and intermediate parts were made of steel, while the stern shaft was made of CFRP. The dimensions and material property of the shaft are shown in Table 1 to Table 3. In basic model the stiffness of thrust bearing was $1\times10^9 N/\mathrm{m}$. The intermediate and stern parts were connected by a flexible coupling whose mass was 45kg. A propeller with mass of 200kg was installed on the end of the stern shaft. The fibre orientation of composite stern shaft was $[\pm45]_{100}$.

Table 1: Size of shaft model

Dimension	Outer diameter (m)	Inner diameter (m)	Length (m)	Material
Stern shaft	0.16	0.06	6	GFRP
Intermediate shaft	0.11	0.06	0.5	steel
Thrust shaft	0.10	0	0.6	steel

Table 2: Material parameters of steel

$E(GPa)$	ν	$\rho(kg/m^3)$	$\eta(\%)$
210	0.3	7800	0.4

Table 3: Material properties of CFRP

$E_1(GPa)$	$E_2(GPa)$	ν_{12}	$G(GPa)$	$\rho(kg/m^3)$	$\eta_1(\%)$	$\eta_2(\%)$	$\eta_{12}(\%)$
97.4	7.27	0.32	4.89	1600	0.64	2.91	1.35

3.2 Natural frequencies and corresponding mode shapes

Based on the analytical method in the above, the longitudinal vibration natural frequency and vibration mode of composite and steel shafts (with similar torsion stiffness, the inner and outer diameters of composite shaft were 0.06m and 0.12m) were calculated respectively. The results are shown in Table 4, Fig. 3 and Fig. 4. In these two figures, thrust and intermediate shaft were in 0-1.1m part, while the rest length belong to stern shaft.

Table 4: Natural frequencies of composite and steel shaft

Order	1	2	3	4	5
Composite shaft (Hz)	64	257	413	587	831
Steel shaft (Hz)	107	339	626	975	1346

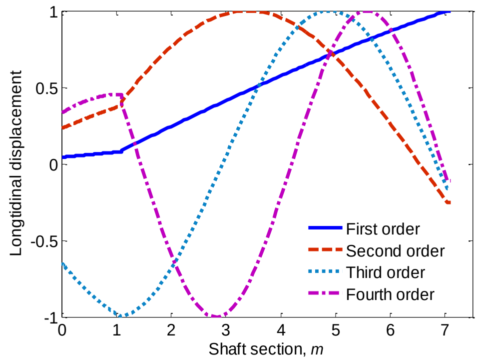

Figure 2: Mode shapes of the first four orders of composite shaft

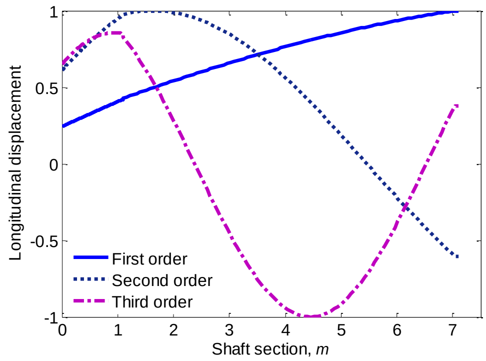

Figure 3: Mode shapes of the first three orders of steel shaft

For the composite shaft system, the first order vibration mode mainly depends on the stern shaft part. This is because the longitudinal stiffness of composite stern shaft with a [±45°] fibre orientation is much less than the other two parts of steel because its longitudinal elasticity modulus was much less but its length was much longer than the steel parts. As the vibration frequency increase, thrust and intermediate shaft begin to participate in the system vibration gradually.

3.3 Influence of model parameters on the vibration characteristics

3.3.1 Longitudinal stiffness of thrust bearing

The longitudinal vibration mode of shaft was significantly influenced by thrust bearing stiffness. First order vibration mode shapes corresponding to different thrust bearing stiffness are shown in Fig. 5. When thrust bearing stiffness was far less than the longitudinal stiffness of shaft, the longitudinal displacements of each shaft section are nearly the same value. In this case, the first vibration mode of shaft seems like a spring-mass system that thrust bearing provides the stiffness while the whole shaft provides the mass. As “spring stiffness” increases, the first order natural frequency also increased. The longitudinal displacement distribution varies along the whole shaft and the maximum value of displacement appeared on the end of CFRP stern shaft.

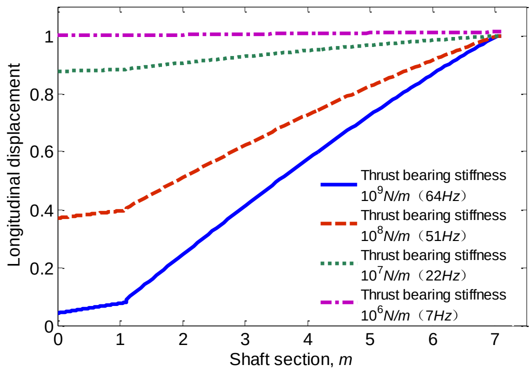

Figure 5: First order vibration mode shapes with different thrust bearing stiffness

3.3.2 Fibre orientation and length-diameter ratio

When thrust bearing stiffness was larger than that of shaft, the longitudinal vibration mode of shaft mainly depends on CFRP stern shaft. The parameters of stern shaft can be designed to change the longitudinal stiffness of shaft, which also implies that the longitudinal vibration characteristics can be optimized.

Table 5: Moduli and natural frequencies of composite shaft with different fibre orientation

Fibre orientation (°)	30	40	45	50	60
Longitudinal modulus (*GPa*)	39.4	21.6	16.6	13.3	9.7
First order natural frequency (*Hz*)	92	72	64	58	50

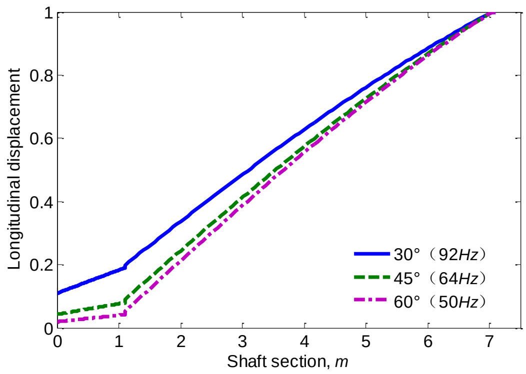

Figure 6: First order vibration mode shapes with different fibre orientations

The longitudinal moduli and first order natural frequencies at different fibre orientations were shown in Table 5, and first mode shapes of composite shaft with different fibre orientations were plotted in Fig. 6. It can be observed that the longitudinal moduli and natural frequencies decreased with the fibre orientation, and the vibration participation of stern shaft is more evident.

Table 6: Natural frequencies of composite shaft with different length-diameter ratio

Outer diameter (m)	Inner diameter (m)	Length-diameter ratio	First order natural frequency (Hz)
0.160	0.06	37.6	64
0.165	0.10	36.4	58
0.179	0.14	33.5	52
0.203	0.18	29.6	45

The inner and outer diameters of composite stern shaft were then changed to examine the influence of different length-diameter ratios on natural frequencies while the length and torsion stiffness remain constant. The results are listed in Table 6. It can be seen that the first order natural frequency increased with length-diameter ratio. The reason is supposed to be that as the length-diameter ratio increased, the longitudinal stiffness of shaft is correspondingly increased due to the relatively larger increase of the cross section of stern shaft while a relatively less increase of shaft mass.

4. Analysis of longitudinal vibration attenuation characteristics

4.1 Vibration response and attenuation analysis of typical shaft model

Vibration responses and attenuations of a steel and a composite stern shaft were individually calculated and then compared. The inner and outer diameters of steel stern shaft were 0.06m and 0.12m, respectively. The thrust bearing stiffness was $1\times10^{9}\,N/\mathrm{m}$. The mass representing propeller was excited by a unit force. In order to examine the output response, the free end of typical thrust bearing was select as the output point. The frequency band for analysis was 1-1000Hz.

Vibration mode analysis was carried out first. Damping factors of every longitudinal vibration modes were calculated in the given frequency band for both composite and steel shaft systems. Then, vibration responses were obtained by mode superposition method. The longitudinal vibration acceleration level with respect to frequency was shown in Fig. 7.

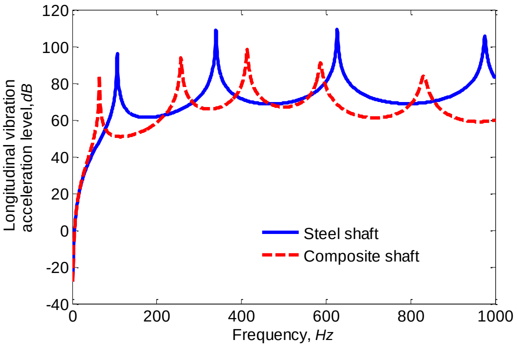

Figure 7: Longitudinal vibration acceleration of composite and steel shafts

As shown in Fig. 7, the maximum vibration acceleration responses of composite stern shaft were all smaller than that of steel stern shaft at each mode. Moreover, the total vibration acceleration level of composite stern shaft was 11.4*dB* smaller than that of steel stern shaft in whole frequency band. Thus it can be seen that composite stern shaft can significantly reduce the longitudinal vibration of drive shaft.

Considering the frequencies corresponding to the maximum responses of both shafts were different, the maximum responses of the same orders for both shafts were compared in this paper. The following equation (4) was used to calculate the “inserted loss” of composite stern shaft relative to steel stern shaft.

$$La = 20\lg \frac{a_{steel}}{a_{GFRP}} \tag{4}$$

Where a_{steel}, a_{GFRP} respectively represent the acceleration of steel shaft and composite shaft at maximum vibration response at the same order. The "inserted losses" of first four orders were shown in Table 7. It can be seen that composite stern shaft designed by parameters given in this paper was effective in vibration reduction compared with the prototype of steel stern shaft.

Table 7: First four order "inserted losses"

Order	1	2	3	4
Inserted loss (dB)	12.06	14.94	10.86	14.43

4.2 Influence of model parameters on the longitudinal vibration response

4.2.1 Longitudinal stiffness of thrust bearing

In order to investigate the effect of thrust bearing stiffness on vibration response, the shafts were analysed again but with thrust bearing stiffness changed, while other parameters of shaft structure remain constant for both composite and steel shaft. The vibration responses for steel shaft and composite shaft, and "inserted losses" of first three orders were plotted in Fig. 8, 9 and 10.

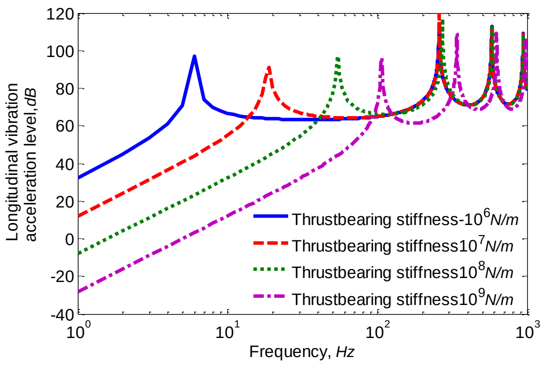

Figure 8: Acceleration level of steel shaft with different thrust bearing stiffness

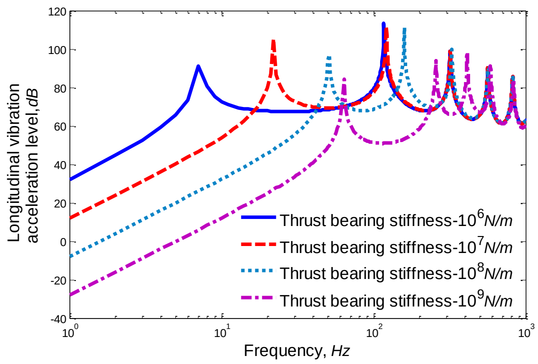

Figure 9: Acceleration level of composite shaft with different thrust bearing stiffness

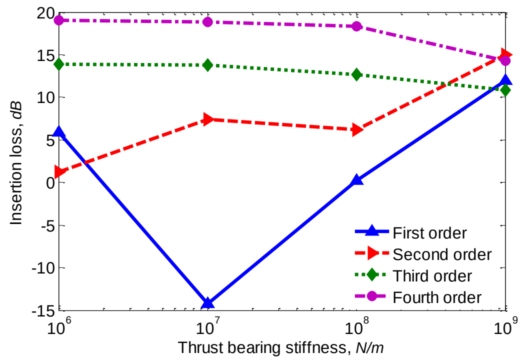

Figure 10: "Inserted losses" at first three orders vibration modes with different thrust bearing stiffness

As shown in Fig. 9, the first order vibration frequency of composite shaft increased with thrust bearing stiffness becoming big, while the first order vibration response decreased. This suggested that increasing the thrust bearing stiffness can inhibit the vibration of composite shaft, but the effect became slight at moderate or higher frequencies.

When thrust bearing stiffness was far less than longitudinal stiffness of shaft, the value of "inserted losses" of first and second order were small, which means it had little effect on vibration attenuation. When thrust bearing stiffness was about $1\times10^{7}\,N/\mathrm{m}$, the first order "inserted loss" was negative. This situation was not expected for vibration control and should be avoided in shaft design. With the thrust bearing stiffness further increasing, the "inserted losses" of first two orders were increased. Therefore the stiffness of thrust bearing should be normally larger than that of shaft in composite shaft design.

4.2.2 Fibre orientation

The fibre orientation of composite stern shaft was designed in different values, while other parameters remained constant to study the influence of fibre orientation on the vibration transfer response of shaft. The thrust bearing stiffness was still $1\times10^9 N/\mathrm{m}$ for both shafts. The vibration damping ration of composite stern shaft were calculated and shown in Table 8, the longitudinal vibration responses of shafts with different fibre orientations were plotted in Fig. 11. The "inserted losses" of first three orders for composite shaft compared with steel ones were shown in Fig. 12.

It was shown that vibration response decreased with the fibre orientations, meanwhile the damping ration of shaft increased. Thus increasing the fibre orientation can consequently control the vibration transmission effectively. However, the torsion ability of stern shaft would be reduced if the fibre orientation was too large, accordingly the fibre orientation should be designed reasonably and in balance.

Table 8: Damping ration of composite shaft with different fibre orientation

Fibre orientation (°)	20	30	40	45	50	60	70
Damping ration (%)	0.44	0.54	0.60	0.64	0.71	0.95	1.21

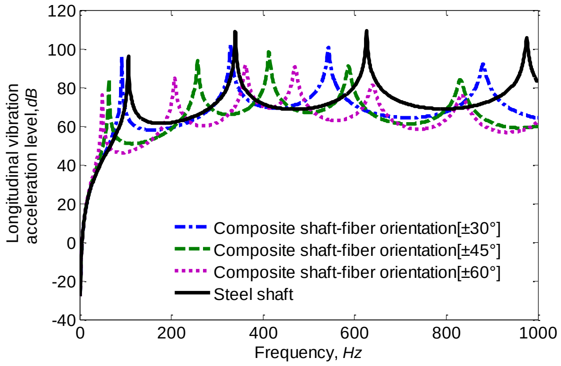

Figure 11: Acceleration level of shaft with different fibre orientation

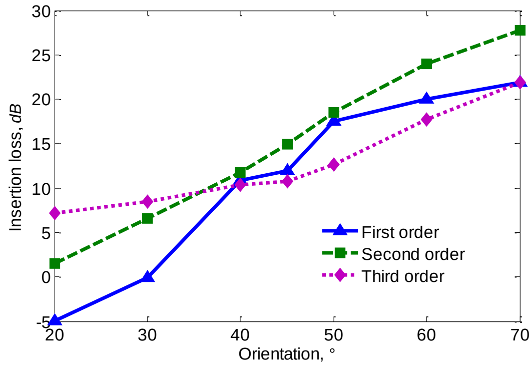

Figure 12: "Inserted losses" at the first three orders versus fibre orientation of composite shaft

4.2.3 Length-diameter ratio

The longitudinal vibration responses of composite shaft with different length-diameter ratios were studied and results were shown in Fig. 13. Here the parameters of shafts such as length, torsion stiffness keeps constant while inner and outer diameter was altered to achieve different length-diameter ratios. The thrust bearing stiffness was still $1\times10^9 N/\mathrm{m}$. The "inserted losses" were plotted in Fig. 14.

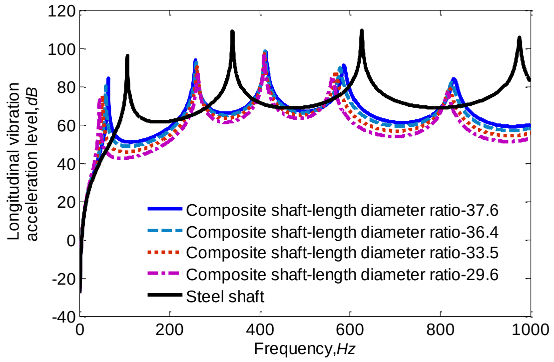

Figure 13: Acceleration level of shaft with different length-diameter ratio

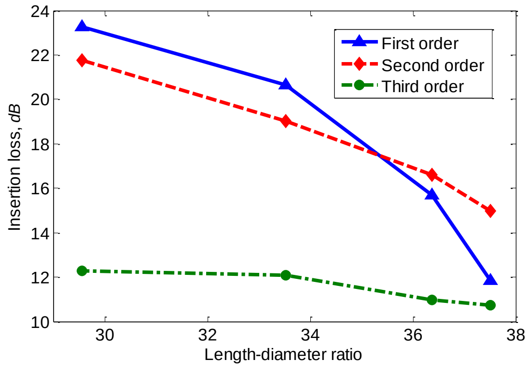

Figure 44: "Inserted losses" of the first three orders with different length-diameter ratio

From Fig. 13 and 14, it can be seen that the first order vibration frequency and vibration response increased with length-diameter ratio, but "inserted loss" decreased. It seems that increasing the length-diameter ratio goes against vibration attenuation. Hence the length-diameter ratio should be chosen reasonably.

5. Results

A theoretical model for longitudinal vibration analysis of composite propulsion shaft was developed. The frequencies and mode shapes of shaft with different parameters were analysed and longitudinal vibration characteristics of composite stern shaft was estimated and compared with that of steel counterparts. Influences of different parameters such as thrust bearing stiffness, fibre orientation and length-diameter ratio on longitudinal vibration response characteristics of composite stern shaft were studied. Conclusions of this paper were as follows:

1. The first order natural frequency of longitudinal vibration of drive shaft depends on the relative value of longitudinal stiffness of shaft over thrust bearing. When the longitudinal stiffness of thrust bearing is far less than that of the whole shaft, composite stern shaft part will take part into the first order longitudinal vibration of shaft as a rigid body. In this case, structure parameters and material properties of composite stern shaft nearly have no effects on the vibration performance of drive shaft.
2. The longitudinal stiffness of thrust bearing should be concerned when composite is used for vibration attenuation of drive shaft, for the reason that low longitudinal thrust bearing stiffness will reduce the vibration attenuation performance of composite shaft. Compared with steel stern shaft, it is effective in attenuating vibration for composite stern shaft under the circumstances where thrust bearing has enough longitudinal stiffness.
3. The vibration characteristics of composite shaft are influenced by not only thrust bearing stiffness, but also fibre orientation and length-diameter ratio. The first order natural frequency of composite drive shaft increased with bearing stiffness while the vibration response is on the contrary. Besides these, increasing the fibre orientation and decreasing the length-diameter ratio can both reduce the first order frequency and the vibration response. Therefore, these parameters should be optimized to achieve better vibration control effect in the future application.

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