HIGH RESOLUTION SONAR DF SYSTEM

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ABSTRACT: *This paper presents the results of some experiments carried out to implement certain high resolution DF algorithms on a transputer-based sonar system. These experiments were made in a large water tank at Loughborough University of Technology using a 10 element array working at a frequency of 40 kHz. Four main algorithms were tested. These were the Capon Method or the so called Maximum Likelihood Method (MLM), the Maximum Entropy Method (MEM), the Multiple Signal Classification method (MUSIC) and the Minimum Norm Method (MNM). A comparison between the performance of these methods is also presented.*

1. INTRODUCTION

During the past few years many publications on algorithms for resolution enhancements beyond that of conventional beamforming have appeared. However, most of these publications have been based on computer simulations and very little has been published on the practical implementation of these algorithms. One of the main reasons for this lack of practical applications of the algorithms is the extensive computation load involved, but there have been great advances in semiconductors and computer technologies in the last few years which have led to the availabilty of more powerful computational and storage devices. An example of such devices is the transputer, a programmable VLSI component that can be used as a conventional microcomputer as well as a building block for a high performance concurrent system. These devices have opened the doors for the possibility of implementing high resolution DF algorithms in real time. A study of the feasibility of using transputers to implement these algorithms has shown that it is quite possible to do so for some applications such as in sonar where the movement of the possible targets is relatively slow, allowing time for computation to be carried out[1&2].
Following these studies, a transputer-based sonar DF system has been designed[3] and has now been built. Some preliminary experiments have been carried out to test several high resolution DF algorithms and the purpose of this paper is to report the results of these experiments.

The paper contains a brief review of the main high resolution DF algorithms, a description of the system hardware, a description of the experiment layout and a presentation of the results. It also contains a comparison between the performance of the different DF algorithms. The practical problems involved in these experiments are also discussed.

2. BRIEF REVIEW OF THE MAIN HIGH RESOLUTION ALGORITHMS

Sabbar[4] and Jeffries[5] have presented in their theses detailed reviews of almost all the high resolution techniques. However, for convenience a very brief review of the best known algorithms is presented in this section.

2.1. Capon-Type Methods :

Perhaps the most well known high resolution array processing algorithm is the so called Maximum Likelihood Method (MLM) of Capon. The MLM seeks to obtain an optimum solution by constraining the output response of a filter. If R represents the spatial correlation matrix of the sensors outputs, then the MLM solution is derived by finding a weighting vector W which yields the minimum beam energy $W^H R W$ subject to the constraint that $W^H C = 1$, where C represents a plane wave corresponding to the look direction. The purpose of the constraint is to fix the processing gain in each look direction to unity. Minimizing the resulting beam energy reduces the contributions to this energy from sources and/or noise not arriving from the look direction. The solution to this constrained optimization problem is:

$$W = \frac{R^{-1} C^H}{C^T R^{-1} C^H}$$

and the power in the direction θ (the angular spectrum) is:

$$P_{MLM}(\theta) = (C^T R^{-1} C^H)^{-1}$$

which means that an estimation and inversion of the covariance matrix are needed to generate the scan pattern.

2.2. Linear Prediction Methods:

These methods assume that the signal of interest is a stationary spatial stochastic process which is spatially sampled by the array. The task is to estimate the corresponding (angular) spectrum. The most popular method is the Maximum Entropy Method (MEM) which computes the angular spectrum as:

$$P(\theta)_{MEM} = \frac{e_1^H R^{-1} e_1}{| e_1^H R^{-1} C |^2}$$

where e_1 is the first unit vector, i.e $e_1=(1,0,...,0)$.

These methods are restricted by their construction to linear equispaced arrays and the performance depends heavily on the number of samples used and the chosen filter length. A filter which is too short yields a smooth spectrum with bad resolution, whereas long prediction filters give a very fluctuating spectrum with good resolution but random, unpredictably high sidelobes. Sometimes spectral peak splitting also occurs.

2.3. The Projection Methods:
These methods are based on the orthogonality between the signal direction vectors and the noise eigenvectors of the sample covariance matrix obtained from the measured data.

The signal subspace is defined by a set of orthogonal vectors $(x_1, x_2, \ldots, x_M) = X$, if M targets are given. If C points in a target direction then $C^H XX^H C$ will have a maximum equal to $C^H C$, and $C^H(I - XX^H)C$ will be zero. $P_s = XX^H$ is a projection on to the space spanned by the columns of X, called the 'signal subspace'. $P_n = I - XX^H$ is a projection orthogonal to this space, called the 'noise subspace'.

In general, the angular spectrum is:

$$P(\theta)_{projection} = \frac{1}{C^H(\theta) P_n C(\theta)}$$

If the columns of X are the eigenvectors corresponding to the largest eigenvalues of R, then P is called the MUSIC algorithm (Multiple Signal Classification). MUSIC produces very good resolution, especially for a small number of data vectors (e.g. of the order of the numbers of array elements). The scan patterns are very stable with low sidelobes.

All the projection type methods require a test procedure to determine the dimension of the signal subspace. For MUSIC this test gives the number of the targets. There are different techniques available for this, most of which require a subjective threshold to be set.

Important modifications to the MUSIC algorithm have been suggested by Kumareson and Tufts. Their method, known as the Minimum Norm Method (MNM), is similar to MUSIC except that only one vector is used to represent the noise subspace. This vector is a linear combination of all the eigenvectors of the noise subspace. One attractive point about this method is that the computation load is much less than that of MUSIC.

3. DESCRIPTION OF THE SYSTEM

Fig.(1) shows the block diagram of the high resolution sonar DF system. The system is based on a network of two transputers which can be expanded later to any number of transputers. The first transputer, the T414, is responsible for integer data management. The second transputer, the T800, takes care of the floating point calculations (e.g. eigenvalues and eigenvectors, matrix inversion and calculation of the DF functions). The T800 transputer can do floating point calculations much faster than the T414 because it has a built-in floating point processor. The two transputers are linked together in a pipeline.

The system uses an array of 10 elements each of which is connected to an A/D board. The elements are spaced by 37.5mm and their resonance frequencies are around 40kHz. The function of the A/D boards is to extract the signal's I&Q components and convert them into a 12 bit digital form.

The system is controlled by a BBC microcomputer which generates all the timing and triggering signals through its user port and the 1MHz bus. This BBC computer is interfaced to the T414 transputer by a specially designed board based on an Inmos link adapter chip. The BBC initiates the system by sending a START signal from its user port. This signal will initiate the generation of the transmitted pulse from the Signal and Reference Generation Board. The length of the transmitted pulse in selected by setting 8 DIP-switches either to 0 or 1. This will determine the number of cycles per pulse.

After finishing the generation of the transmitted pulse, a data collection phase starts. In this phase the Signal and References Generator board starts generating the SINE and COSINE reference signals required by the receiver boards in order to extract the In-phase and the Quadrature components (I&Q) of the received signal. It also generates a Sample & Hold signal and a multiplexing signal which multiplexes the I & Q channels to the 12 bit analogue to digital convertor (A/D). The converted signals from the receiver boards are stored temporarily in a memory buffer. This memory is shared between the receiver boards and the transputer. In the data collection phase, the receiver boards access this memory while the transputer is isolated from it by a tri-state buffer. After collecting a certain number of samples, the BBC will isolate the receiver boards from this memory and signal the transputer to start reading the data from it. When the transputer finishes reading the data, it sends an 'End of Reading' signal to the BBC to start a new cycle.

The T414 will then start processing the data batch while the BBC is collecting a new set of data i.e they are working concurrently. The T414 will search through the data to find out if there is an echo. If an echo is detected, it will extract the I & Q components of that echo. After firing a certain number of snapshots, the T414 will form the covariance matrix and pass it to the T800 for further processing. At present, a second BBC is used to display the angular spectrum but this will eventually be replaced later by a frame store. The T800 is interfaced to this BBC by a board similar to the one used to interface the controlling BBC to the T414.

4. EXPERIMENTAL LAYOUT AND THE PRACTICAL PROBLEMS

The system has been tested in a large water tank at Loughborough University of Technology. The dimensions of the tank are 9x6x2 meters. The receiving array was mounted on a pan-and-tilt unit and deployed in the water at a depth of about one meter. Two sources driven by two separate transmit pulses were fixed on a mobile trolley at a distance of about 4.5 meters from the array. The frequencies of the two transmit pulses were centred about 40 kHz, but were deliberately chosen to be different from each other so as to avoid full correlation.

A pulse width of 0.75 msec. was chosen, and the received signal was sampled at a point about 0.2 msec from the start of the pulse. This allowed the received pulse to reach its full amplitude, but the sampling was at a point prior to that where interference from multipath signals became a problem. The sampling frequency was about 10 kHz and fifteen snapshots were used in these experiments at a rate of two snapshots per second.
The practical problems can be summarized as:

a. The relatively small size of the water tank caused many reflections from the surrounding walls, the bottom of the tank and the surface of the water which restricts the maximum distance possible between the targets and the receiving array. Although the targets were approximately 4.5 meters from the the receiving array, the near field effect was still apparent as will be seen later.

b. The phase error of the receiving array caused by the mechanical mismatch of the receiving elements. This ranges from -4 to +7 degrees.

c. The electronic phase error of the receiving system caused by components mismatch of the ten receiver boards. This was around ±5 degrees.

5. RESULTS

The results of implementing certain high resolution DF algorithms are shown in fig.(2). Two targets located at positions -14 and -8 degrees have to be resolved by these algorithms.

Fig.(2.a.) shows the position of the two targets and Fig.(2.b.) shows how the conventional beamforming failed to resolve these targets.

Fig.(2.c.) shows the results of implementing Capon's method (or the MLM). The two targets are resolved and the background is well behaved although sometimes a small mirror image of the real targets appears in the spectrum. This is caused by the the effect of the near field. The performance of this algorithm degraded as the correlation between the two sources increased.

Fig.(2.d.) shows the spectrum of the Maximum Entropy Method. Although the two targets are resolved, the spectrum is too spiky and many small peaks appeared in the spectrum. This method is also very sensitive to the error caused by the near field effect. The computation load of this method is much less than that of the MLM.

Fig.(2.e.) shows the spectrum of the MUSIC method. The resolution is very good and the background is very stable. This method is less sensitive to the phase errors.

Finally, the spectrum of the MNM method is shown in fig.(2.f.). The resolution is very good but the background is a little spiky. The computation load of this method is much less than that of the eigenvector or MUSIC method.

6. CONCLUSIONS

The following conclusions have been made:

a. A practical high resolution sonar system has been designed and build using a two-transputer network.

b. Certain high resolution algorithms have been tested on this system. These were the Maximum Likelihood Method, the Maximum Entropy Method, the MUSIC algorithm and the Minimum Norm Method.

c. Results showed that the practical performances of these algorithms agree essentially with the results of the simulation studies.

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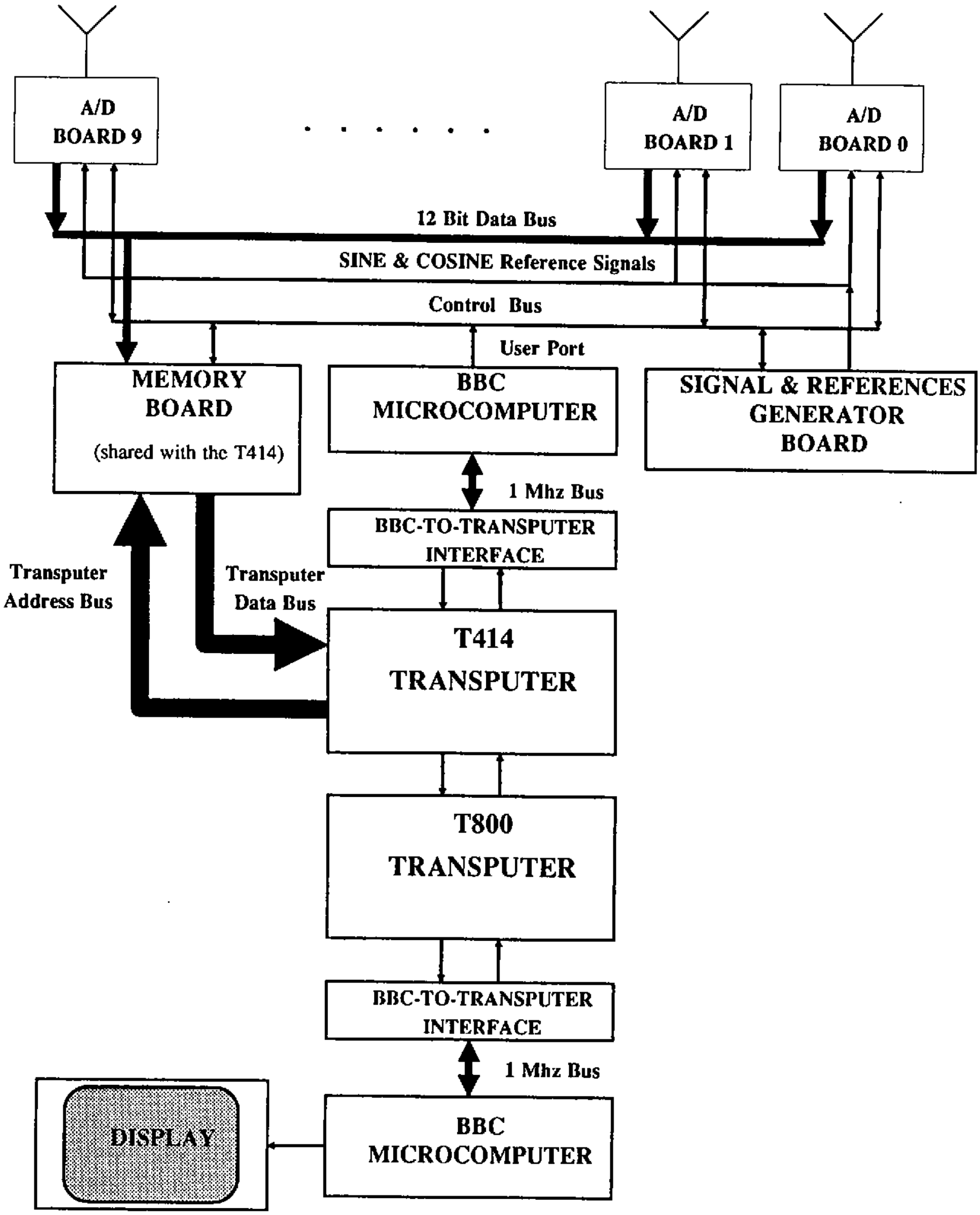


FIG.(1): System Layout.

FIG.2.a

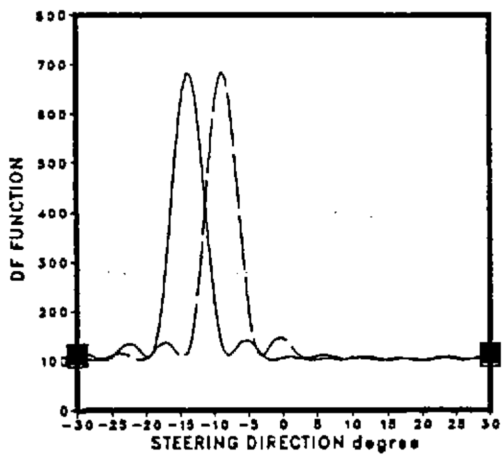


FIG.2.b

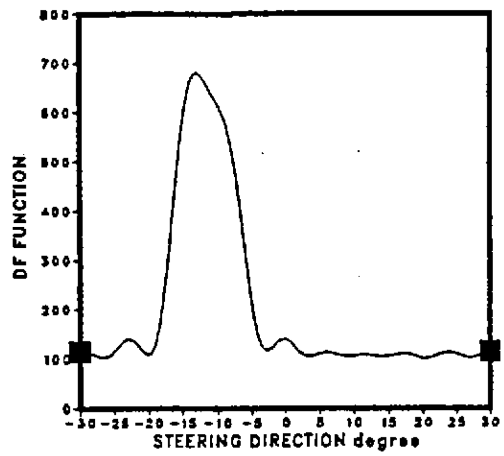


FIG.2.c.

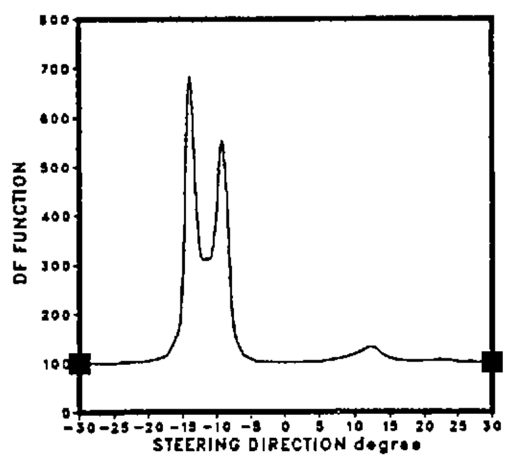


FIG.2.d.

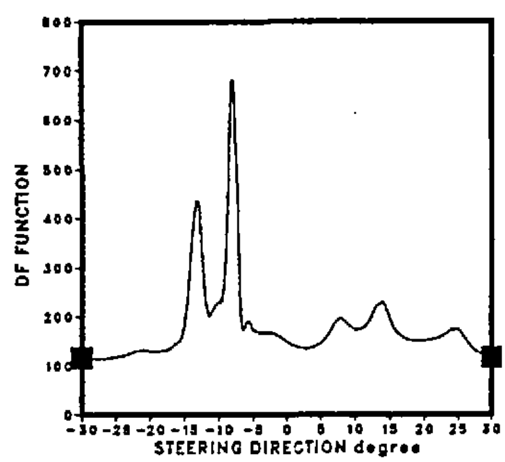


FIG.2.e.

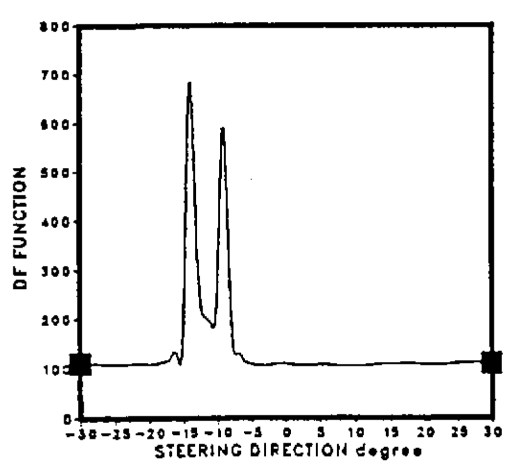
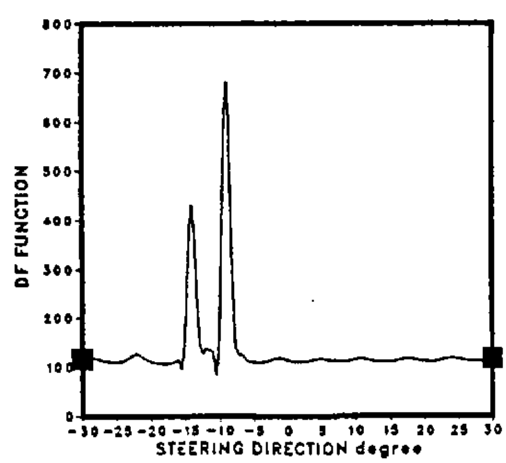


FIG.2.f



SIGNAL PROCESSING ASPECTS FOR TOWED ARRAYS

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Abstract

The performance of direction of arrival estimators is reduced by differences between the true and assumed sensor locations. Such differences are common in towed arrays. This paper presents an iterative algorithm to estimate both source directions and sensor locations from the acoustic data gathered by the array. No knowledge of the source directions or number of sources is required, however it is necessary to have obtained a good prior estimate of the sensor locations.

1. Introduction

It has been shown that imprecise knowledge of the locations of sensors used in direction-finding algorithms may severely reduce their performance[1]. This is particularly true of the so called high-resolution methods such as MUSIC[2]. A situation in which such sensor errors could occur is that of a towed array of hydrophones. These may be perturbed by, among other things, the motion of the tow ship.

Most direction-finding algorithms do not require a particular array geometry, only that the array geometry is known. Thus, if the locations of the perturbed sensors can be estimated, an increase in direction-finding performance can be expected. In particular, wave-number peaks should increase in height and become less broad, thus increasing the system resolution.

Several approaches have been proposed to estimate the shape of the array. For example, a large number of positional sensors could be placed at regular spacing along the array[3]. At another extreme a single position sensor could be placed at the head of the array and a hydrodynamic model used to predict the location of the hydrophones[4]. Both these methods would still result in positional errors however, due, in the first case, to sensor bias and drift, and in the second case to modelling errors and unknown forcing (such as turbulence).

In this paper we propose an algorithm to estimate the residual errors left after using one, or both, of the above methods from acoustic data gathered by the array. It does not require knowledge of the direction of any sources, nor that any sources present are spectrally or temporally disjoint. These assumptions have been made in several previous solutions to the problem[5][6][7]. Recent work without these assumptions has been presented on the estimation of two-dimensional errors in sonobuoy fields[8].

2. Problem formulation

An acoustic field is sampled with a towed array consisting of M hydrophones. The separation between these the sensors is known and assumed constant. The array lies in the horizontal plane. It may not be perfectly rectilinear as this would result in an ambiguity in the source direction estimates derived later.

The array is thus modelled as a set of rigid rods connected together at the hydrophones by a hinge allowing movement in the horizontal plane.

The array position is described by the position of the first (reference) sensor and the angles of the rods relative to the x axis (East). These angles are donated θ_m where m=1 to M-1. The angle θ_m denotes the direction of the rod between sensors m and m+1 relative to the axis and the separation between these two sensors is r_m. The coordinate origin is coincident with the reference sensor. The position vector $\underline{p}_m$ of a sensor is thus given by the equation

$$\underline{p}_m = \sum_{i=1}^{m-1} r_i \left[\cos\theta_i \quad \sin\theta_i \right] = \left[x_m \quad y_m \right] \tag{1}$$

The acoustic field is generated by N (<M) uncorrelated, narrow-band far-field sources with wavelength λ and angular frequency ω. Their directions are measured relative to the y axis (North) and are donated ϕ_n where n=1 to N. A positive ϕ_n representing a source to the right of the axis. Each has a complex strength s_n as measured at the reference sensor. Each sensors output is subject to additive independent identically-distributed Gaussian white noise of variance σ. It is denoted v_m.

The acoustic field is sampled by the sensor array mentioned above. The array is assumed stationary while Q snapshots (taken at time interval δ and indexed q, q=1 to Q) of the acoustic field are taken to yield data vectors $\underline{d}(q)$

$$\underline{d}(q) = A\, \underline{s}(q) + \underline{v}(q) \tag{2}$$

Here $\underline{v}(q)$ is a column vector of v_m and $\underline{s}(q)$ are the source strengths given by

$$\underline{s}(q) = \underline{s}\, e^{j\,\omega\, q\, \delta} \tag{3}$$

A is the array manifold, which has elements

$$a_{mn} = e^{j \frac{2\pi}{\lambda} \left[x_m \sin\phi_n + y_m \cos\phi_n \right]} \tag{4}$$

In order to estimate the position of the array in space it is necessary to have a positional and directional reference. These are provided by a positional sensor on the first hydrophone and a heading sensor between the first and second hydrophones. If these are not used then there is a translational and rotational ambiguity in the array position estimates (see[9][10]).

In the next section an algorithm is presented that will estimate both the source directions $\hat{\underline{\phi}}$ and the rod directions $\hat{\underline{\theta}}$. It relies on having a reasonably good initial estimate of $\hat{\underline{\theta}}$. Such an estimate could be obtained from heading sensors in the array or a hydrodynamic model.

3. The algorithm

The algorithm proposed to estimate the sensor and source locations has an iterative form. It has 3 distinct phases; initialisation, source estimation and sensor estimation.

In the initialisation stage a sample covariance matrix is formed from the available snapshots. It is then decomposed into its eigenvalues and vectors. From this the number of sources present and the noise eigenvectors of the covariance matrix are calculated. A relatively noise free data vector is estimated by averaging the data vectors from all the snapshots.

The source estimation stage estimates the source directions using a previous (or initial) estimate of the sensor locations. This is the classic direction finding problem. Here the MUSIC algorithm is used. Source strengths must also be estimated.

In the third stage the errors in the sensor locations are estimated, based on the previously calculated source directions and strengths. The sensor locations can then be updated. If the errors in the source locations are large, then stage two is re-entered.

The three stages are now considered in more detail.

3.1 Stage 1 (Initialisation)
Data from the Q snapshots is formed into a covariance matrix and an averaged data vector by applying the equations

$$C = \sum_{q=1}^{Q} \underline{d}(q)\, \underline{d}(q)^{H} \qquad \text{and} \qquad \underline{d} = \sum_{q=1}^{Q} \underline{d}(q)\, e^{-j \omega q \delta} \qquad (5,6)$$

The number of sources N, and a matrix U, whose columns are the noise eigenvectors, are then calculated (see[2]).

3.2 Stage 2 (Source Estimation)
The MUSIC[2] method is used to estimate the directions of the incident plane waves. The directions corresponding to the N highest peaks of the MUSIC "spectrum"

$$P_{music} = \frac{1}{b(\phi)\ U\ U^{H}\, b(\phi)^{H}} \qquad \text{where} \qquad b_m(\phi) = e^{j\frac{2\pi}{\lambda}(\hat{x}_m \sin\phi\ +\ \hat{y}_m \cos\phi)} \qquad (7)$$

are taken as the new estimate of the source directions $\hat{\underline{\phi}}$. A least-squares estimate of the source strengths is calculated from the estimated array manifold $\hat{A}$ and the data vector $\underline{d}$ (see equation 2). It is given by

$$\hat{\underline{s}} = (\ \hat{A}^{H}\, \hat{A}\ )^{-1}\, \hat{A}^{H}\, \underline{d} \qquad (8)$$

3.3 Stage 3 (Sensor Estimation)
The sensor locations are estimated by finding the intersection between two loci along which the sensor must lie. The first locus is determined by the difference between the actual and estimated measurement at each sensor. The second locus is given by the known sensor separation. Both these loci are highly nonlinear, but for small errors in the sensor positions they are approximately linear.

To determine the first locus consider the actual data measured at sensor m, given by

$$d_m = \sum_{n=1}^{N} s_n\, e^{j\frac{2\pi}{\lambda}(x_m \sin\phi_n\ +\ y_m \cos\phi_n)} \qquad (9)$$

If Δx and Δy are the errors between the true and assumed positions in the x and y directions where

$$\Delta x_m = \hat{x}_m - x_m \qquad \text{and} \qquad \Delta y_m = \hat{y}_m - y_m \tag{10,11}$$

then
$$d_m = \sum_{n=1}^{N} s_n\, e^{j\frac{2\pi}{\lambda}(\hat{x}_m \sin\phi_n + \hat{y}_m\cos\phi_n)}\, e^{j\frac{2\pi}{\lambda}(\Delta x_m \sin\phi_n + \Delta y_m \cos\phi_n)} \tag{12}$$

If Δx and Δy are small and $\underline{\phi} \simeq \hat{\underline{\phi}}$ this may be approximated by

$$d_m \simeq \sum_{n=1}^{N} s_n\, e^{j\frac{2\pi}{\lambda}(\hat{x}_m \sin\hat{\phi}_n + \hat{y}_m\cos\hat{\phi}_n)}\left[1 + j\frac{2\pi}{\lambda}(\Delta x_m \sin\hat{\phi}_n + \Delta y_m \cos\hat{\phi}_n)\right] \tag{13}$$

The estimated measurement at sensor m is

$$\hat{d}_m = \sum_{n=1}^{N} \hat{d}_{mn} = \sum_{n=1}^{N} s_n\, e^{j\frac{2\pi}{\lambda}(\hat{x}_m \sin\hat{\phi}_n + \hat{y}_m\cos\hat{\phi}_n)} \tag{14}$$

Taking this from the measured data d_m gives

$$j\frac{\lambda}{2\pi}(\hat{d}_m - d_m) = \sum_{n=1}^{N} \hat{d}_{mn}(\Delta x_m \sin\hat{\phi}_n + \Delta y_m \cos\hat{\phi}_n) \tag{15}$$

The second locus is a circle of radius r_{m-1} about the previous sensor location i.e. $|\underline{p}_m - \underline{p}_{m-1}| = r_{m-1}$. For small sensor deviations it may be approximated by the tangent to the circle at $\hat{\underline{p}}_m$

$$\Delta x_m = -\Delta y_m \tan\hat{\theta}_{m-1} \tag{16}$$

Substituting for Δx_m in equation (15) gives

$$\Delta y_m = \mathrm{Re}\left[\frac{j\frac{\lambda}{2\pi}\left(\sum_{n=1}^{N}\hat{d}_{mn} - d_m\right)}{\sum_{n=1}^{N}\hat{d}_{mn}(\cos\hat{\phi}_n - \tan\hat{\theta}_{m-1}\sin\hat{\phi}_n)}\right] \tag{17}$$

The error in $\hat{\theta}_{m-1}$ is denoted $\Delta\hat{\theta}_{m-1}$ and is given approximately by

$$\Delta\hat{\theta}_{m-1} \simeq \frac{\sqrt{\Delta x_m^2 + \Delta y_m^2}}{r_{m-1}} = \frac{\Delta y_m\sqrt{1+\tan^2\theta_{m-1}}}{r_{m-1}} \tag{18}$$

Finally we update $\hat{\theta}_{m-1}$ and $\hat{\underline{p}}_m$. The position is updated using equation (1) and the angle by

$$\hat{\theta}'_{m-1} = \hat{\theta}_{m-1} + \Delta\hat{\theta}_{m-1} \qquad (19)$$

Note: if the angle θ_1 is known (by measuring with a heading sensor) then $\Delta\hat{\theta}_1$ may be used to determine the rotational bias in the source locations estimates $\hat{\underline{\phi}}$.

4. Numerical Example

To demonstrate the effectiveness of the algorithm the following example is presented;

A uniformly spaced curvilinear array of 8 sensors (spacing $\lambda/2$) samples an acoustic wavefield generated by three equal power sources. These sources fulfil the conditions in Section 2 and are in the directions ϕ=10°, -25° and -40°. Additive uncorrelated sensor noise is injected at a SNR of 10dB referenced to the signal sources. Two hundred and fifty snapshots of data are used.

The array is assumed to be curved with $\hat{\theta}_m = (m-1)*5°$. The true array angles are given by $\theta_m = \hat{\theta}_m + w$, where w is a Gaussian random variable of standard deviation 2°.

Figure 1 shows the MUSIC spectral estimate before applying the algorithm and after 5 iterations of the algorithm. A spectral estimate with no sensor errors is also plotted.

5. Conclusions

In this paper an algorithm has been derived to improve an initial estimate of the shape of a curvilinear array of sensors with known spacing. No prior knowledge of the direction and number of observed sources is required.

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Figure 1. MUSIC Spectrum
(showing 2 of the 3 peaks)