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OSCILLATORY TESTING OF SHIP MODELS

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Admiralty Experiment Works

It is customary to analyse the directional stability of ships and submarines by examining the motion following a disturbance from a straight line reference motion. The equations of motion are expressed in terms of body axes moving with the ship, and provided the hydrodynamic forces due to the disturbance can also be expressed in these terms it is a simple matter to establish stability criteria. The general force expressions are very complicated, but where small disturbances are concerned considerable simplification is possible. Taylor's Theorem is usually invoked so as to allow the sway force ΔY (for example) due to the disturbance to be written as

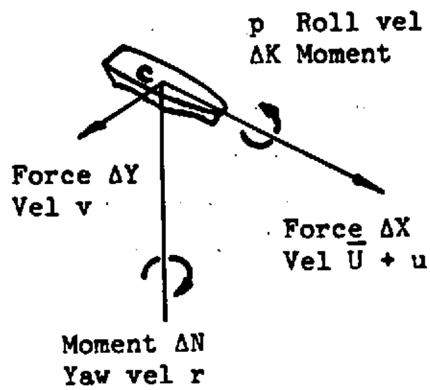
$$\Delta Y = Y\{u\} + Y\{v\} + Y\{p\} + Y\{r\} \quad .... \quad (1)$$

where

u, v, p, r, are disturbance increments

$$Y\{v\} = Y_v v + Y_{\dot v}\dot v + Y_{\ddot v}\ddot v + \text{etc} \quad .... \quad (2)$$

$$Y_v = \left(\frac{\partial Y}{\partial v}\right)_{\text{steady state}} \qquad Y_{\dot v} = \left(\frac{\partial Y}{\partial \dot v}\right)_{\text{steady state}} \qquad \text{etc} \quad (3)$$



with similar expressions for Y{u} etc.

Y_v $Y_{\dot v}$ $Y_{\ddot v}$ etc will be referred to as the 'steady state derivatives', and for a given model are dependent only on the axial speed $\bar U$.

The next step is usually to discard terms in $\ddot v$ and above on the grounds that they are insignificant if the motion is 'slow'. The resulting expression $Y\{v\} = Y_v v + Y_{\dot v}\dot v$ ... (4) is found to be a good description of the forces in many ship motions. Y_v and $Y_{\dot v}$ are commonly referred to as 'slow motion derivatives' and their measurement is the object of much tank testing. The object of this paper is to examine how Y_v and $Y_{\dot v}$ are measured by oscillatory tests.

The device used is known as the Planar Motion Mechanism (PMM for short), fully described in [1] and [2]. In essence the PMM imposes a sinusoidal oscillation on to the straight reference path of a model which is being towed. The oscillation may be one of pure sway or pure yawing. The instrumentation measures the forces applied to the model by the PMM and carries out an 'on line'

Fourier analysis to separate the force into components 'in phase' and 'quadrature' with the displacement. Since the inertia forces are known, the PMM is, in effect, a device for measuring the 'in phase' and 'quadrature' components of hydrodynamic force.

Pure Swaying　　　　　　　　　　Pure Yawing

Consider, for example, an oscillatory test in pure sway. If the displacement is $y_o \sin \omega t$ and the hydrodynamic sway force is

$A \sin \omega t + B \cos \omega t$, ... (5)

then the PMM measures A and B. Now according to equations (1) and (4) this force is $\Delta Y = Y\{v\} = Y_v v + Y_{\dot v}\dot v$ since $u = p = \dot{r} = o$.

Also $v = v_o \cos \omega t$ where $v_o = y_o\omega$, and $\dot v = v_o\omega \sin \omega t$.

Hence $\Delta Y = Y_v\, v_o \cos \omega t - Y_{\dot v}\, v_o\, \omega \sin \omega t$.

Hence $A = - Y_{\dot v}\, v_o\, \omega$ and $B = Y_v\, v_o$ .... (6)

Thus if A/v_o and B/v_o are plotted against ω, we expect the former to be a straight line through the origin of slope $- Y_{\dot v}$ and the latter a constant of value Y_v.

In practice this is not so, although it is almost true of $Y_{\dot v}$. We conclude that while equation (4) may be adequate with regard to actual ship motions, it is not adequate at the higher frequencies of a PMM experiment.

Bishop and Parkinson [3] therefore take the higher derivatives into account by the method of 'oscillatory coefficients'. Inserting a sinusoidal oscillation as before into equation (2) gives

$$\Delta Y = Y\{v\} = \tilde{Y}_v\, v_o \cos \omega t - \tilde{Y}_{\dot v}\, v_o\, \omega \sin \omega t = \tilde{Y}_v v + \tilde{Y}_{\dot v}\dot v \quad \ldots\ldots (7)$$

where

$$\left.\begin{aligned} \tilde{Y}_v &= Y_v - \omega^2 Y_{\ddot v} + \omega^4 Y_{\ddddot v} - \ldots \\ \tilde{Y}_{\dot v} &= Y_{\dot v} - \omega^2 Y_{\dddot v} + \ldots \end{aligned}\right\} \quad \ldots\ldots\ldots\ldots (8)$$

$\tilde{Y}_v$ and $\tilde{Y}_{\dot v}$ are 'oscillatory coefficients' and by comparison with equation (5) it is seen that $\tilde{Y}_v = B/v_o$ and $\tilde{Y}_{\dot v} = - A/v_o\, \omega$ ...... (9)

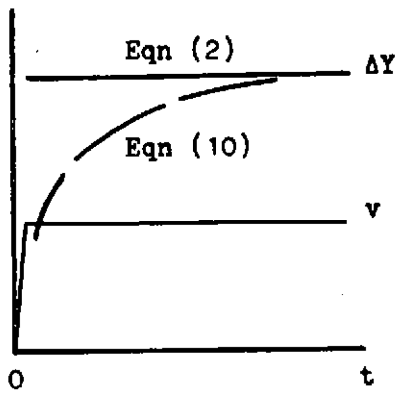
In other words, the PMM measures not Y_v and $Y_{\dot v}$, but $\tilde{Y}_v$ and $\tilde{Y}_{\dot v}$. Moreover we know the form of $\tilde{Y}_v$ and $\tilde{Y}_{\dot v}$ in terms of ω, and see that as $\omega \to o$, $\tilde{Y}_v \to Y_v$ $\tilde{Y}_{\dot v} \to Y_{\dot v}$. By plotting $\tilde{Y}_v$ and $\tilde{Y}_{\dot v}$ against ω^2 the result should be almost a straight line, whose intercept at $\omega = o$ gives the values of Y_v and $Y_{\dot v}$ precisely. This method is a considerable improvement, but the theory is open to the objection that it still does not take into account the past history of the motion - an effect which is known to exist.

Recently this limitation has been removed by an approach based on Volterra functions [4]. In this paper an alternative approach is presented based on expressing Y{v} as the solution of the differential equation

$$Y\{v\} + a_1 \frac{d}{dt} Y\{v\} + \dots a_n \frac{d^n}{dt^n} Y\{v\} = b_o v + b_1 \frac{dv}{dt} + \dots b_m \frac{d^m v}{dt^m} \quad (10)$$

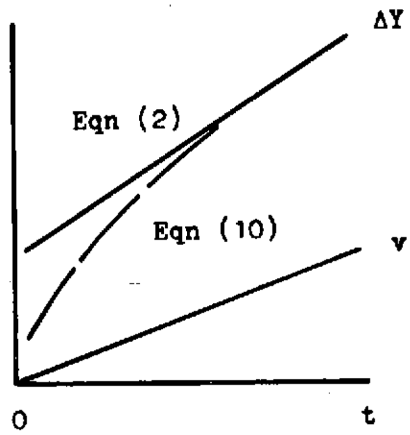
(It cannot be taken for granted that the effects of past history can be represented in this way but there is some evidence that it can.) We now seek to relate the 'a' and 'b' coefficients to those already discussed. First consider a reference motion on which is superposed a constant sway velocity v_o which develops rapidly at time t = o. Now according to equation (2) Y{v} will, after a brief transient, take the value $Y_v v_o$. According to equation (10) there will be a delay in the development of a steady state, but the force must eventually reach the same value. Since the steady state solution of equation (10) is $Y\{v\} = b_o v_o$ we have the relation

$Y_v = b_o$ .... (11)



Now consider a reference motion on which is superposed a constant acceleration in sway $\dot{v}_o$, which develops rapidly at time t = o. According to equation (2) Y{v} will, after a brief transient, take the value $Y_v \dot{v}_o t + Y_{\dot{v}} \dot{v}_o$. According to equation (10) there will be a delay in the development of the force, but it must eventually be assymptotic to the same value. The particular solution of equation (10) for $t \to \infty$ is $Y\{v\} = b_o \dot{v}_o t + (b_1 - a_1 b_o) \dot{v}_o$ and since we have established that $Y_v = b_o$, we obtain the relation

$Y_{\dot{v}} = (b_1 - a_1 b_o)$ .... (12)



Relations for the higher derivatives can be obtained in like manner and can be expressed as $Y_{v_j} = c_j$ ... (13), where the suffices v_1 v_2 etc denote $\dot{v}$ $\ddot{v}$ etc and c_j is the determinant

$$(-1)^j \begin{vmatrix} b_o & 1 & 0 & 0 & \cdot\cdot & 0 \\ b_1 & a_1 & 1 & 0 & \cdot\cdot & 0 \\ b_2 & a_2 & a_1 & 1 & \cdot\cdot & 0 \\ \cdot & \cdot & \cdot & \cdot & \cdot\cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot\cdot & 1 \\ b_j & a_j & a_{j-1} & \cdot & \cdot\cdot & a_1 \end{vmatrix}$$

Finally consider a reference motion on which is superposed a steady sinusoidal oscillation. Since equation (10) represents a linear system its asymptotic response to a sinusoidal input,

$v = v_o \cos \omega t$ is also sinusoidal, and is $A \sin \omega t + B \cos \omega t$ where

$$A = \frac{\alpha_1 \beta_o - \alpha_o \beta_1}{\alpha_o^2 + \alpha_1^2 \omega^2} \cdot \omega v_o \qquad B = \frac{\alpha_o \beta_o + \alpha_1 \beta_1}{\alpha_o^2 + \alpha_1^2 \omega^2} \cdot v_o \quad \ldots\ldots\ldots \quad (14)$$

$$\alpha_o = (1 - a_2 \omega^2 + a_4 \omega^4 - \ldots) \quad \alpha_1 = (a_1 - a_3 \omega^2 + a_5 \omega^4 - \ldots)$$

$$\beta_o = (b_o - b_2 \omega^2 + b_4 \omega^4 - \ldots) \quad \beta_1 = (b_1 - b_3 \omega^2 + b_5 \omega^4 - \ldots) \quad (15)$$

For low frequencies A and B can be expressed as polynomials in ω and the c determinants, giving the following results.

$$\left. \begin{aligned} A &= -(c_1 - c_3 \omega^2 + c_5 \omega^4 \ldots)\, \omega v_o \\ B &= (c_o - c_2 \omega^2 + c_4 \omega^4 \ldots)\, v_o \end{aligned} \right) \quad \ldots\ldots\ldots\ldots \quad (16)$$

Hence from equations (13) and (8)

$$\left. \begin{aligned} A &= -(Y_{\dot v} - \omega^2 Y_{\dddot v} + \ldots)\, \omega v_o = -\bar{Y}_{\dot v} v_o \omega \\ B &= (Y_v - \omega^2 Y_{\ddot v} + \ldots)\, v_o = \bar{Y}_v v_o \end{aligned} \right) \quad \ldots\ldots\ldots\ldots \quad (17)$$

These equations are identical with equations (9) and it follows that the procedure there described for obtaining the slow motion derivatives Y_v and $Y_{\dot v}$ is valid in this more general case also.

References

1. Gertler, M. 1959. Symposium on Tank Facilities. Zagreb.

2. Bishop, R E D and Booth T B, 1972. Monograph 'The Planar Motion Mechanism'.

3. Bishop, R E D and Parkinson, A G. Phil Trans Roy Soc (A) Vol 266 Feb 1970.

4. Bishop R E D, Burcher R K and Price W G, 1972 Papers accepted by the Royal Society.

   Functional Analysis in Ship Dynamics.

   Applications of Functional Analysis to Oscillatory Ship Model Testing.