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PROPAGATION OF PARAMETRIC WAVES IN SHALLOW WATER

T. G. Muir and J. R. Clynch

Applied Research Laboratories
The University of Texas at Austin
Austin, Texas 78712 U.S.A.

Introduction

In recent years the parametric end-fire array[1] has been the subject of many experimental and theoretical investigations. Parametric arrays have themselves been researched and more recently they have been applied to research on other problems. The present paper continues an examination[2-4] of parametric arrays in the context of shallow water acoustics.

Sound propagation in a shallow water waveguide is illustrated in Fig. 1, where we see the situation for conventional low frequency transmission sketched in the upper portion. Most conventional projectors

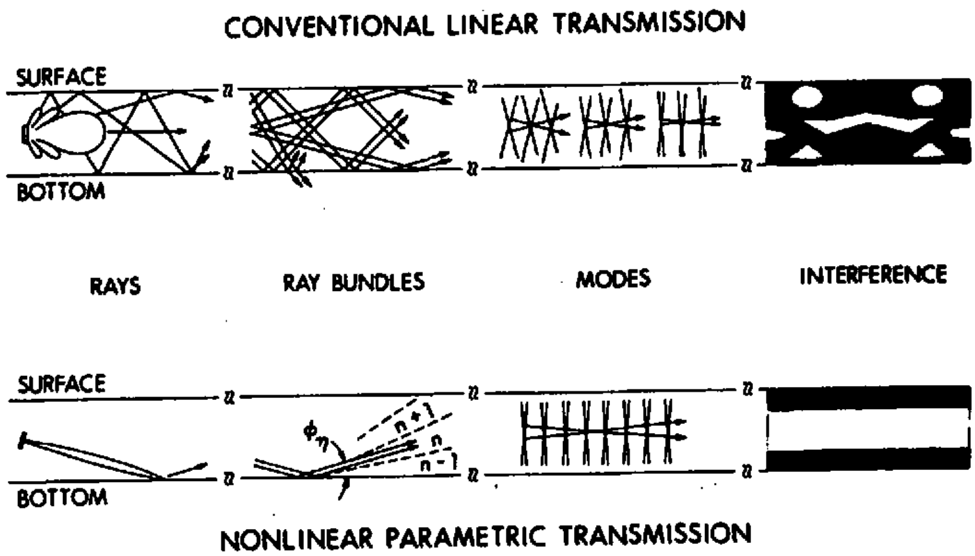

FIGURE I
MODE FORMATION IN ISOTHERMAL SHALLOW WATER

radiate a continuum of rays in a broad beam with, at best, a shaded distribution of minor lobes. The reflection of these rays from the boundaries leads to considerable multipath interference which destroys the coherence of the radiated beam. The medium eventually selects those rays grouped about preferred ray bundles, in accordance with the spatial resonance phenomena of waveguide propagation. Allowed ray bundles then develop into a series of up going and down going wave sets, at discrete grazing angles. The self-interference of each wave set characterizes a particular mode of propagation. Since the modes follow different angular paths in the vertical plane, their horizontal velocities down the waveguide are different. This difference leads to quasiperiodic interference between the modes that further destroys the coherence of the transmission, giving rise to "hot spots" and "dead spots" in the insonification of the water column. The interference diminishes with increase in range, as the amplitude of the higher order modes become damped, while the lowest order mode eventually diminishes into the noise.

The transmission of sound with a narrowbeam parametric array, on the other hand, enables one to ask the water column where it would prefer to be insonified. By directing the sound beam to some parametric eigenray angle (dictated by both waveguide and parametric considerations), it is possible to maximize the energy coupled, say to mode n, at the expense of that coupled to other modes. This reduces the intermodal interference, leaving a more uniform band of insonification down the waveguide.

The parametric radiation has other interesting features pertinent to acoustic work in shallow water. Sonar targets become more recognizable because the reverberation is reduced with respect to the echo when the narrow beam is utilized. Parametric frequency agility may also be used to advantage in wideband signal processing. Doppler techniques are similarly well suited to the narrow, monotonic parametric beam, thereby minimizing frequency spreading that comes from the reverberation and not from the target.

The present analysis begins with a theoretical description of the sound field, and examines the interesting possibility of selective modal excitation. Experiments are then discussed in support of the theory. Several sonar aspects of parametric arrays in shallow water are also pointed out and illustrated experimentally.

Theory of Parametric Excitation

Previous parametric array theory assumes the absence of boundaries in the volume of interaction where the secondary frequency sound is created. We have considered the case of a parametric array in isovelocity shallow water where boundaries play an important role in determining the farfield pressure. The analysis begins by considering the standard form of the velocity potential, which is proportional to a source strength Q times a sum over modes,

$$\Phi(r,z) \propto Q\sum_n \frac{U_n(z_t)U_n(z)}{\sqrt{k_n r}}\, e^{ik_n r} \quad , \qquad (1)$$

where $U_n(z)$ is the depth function of the nth mode, z is the depth coordinate, z_t is the transducer depth (here an omnisource), $k_n = k \cos \varphi_n$, φ_n is the usual eigenray angle of the nth mode (independent of parametric considerations) and r is range.

In the parametric case, the velocity potential becomes a sum over modes with each term containing an integral over the volume of interaction. The integrand consists of a parametric source strength density,[1] $q(\cdot)$, times the mode function. The result is

$$\Phi(r,z) \propto \sum_n \frac{U_n(z)e^{ik_n r}}{\sqrt{k_n r}} \int_{V'} q(\vec{r}\,')U_n(z')dV' \quad , \qquad (2)$$

where $\vec{r}\,'$ is a vector from the origin to the elemental volume of parametric interaction, z' is the depth of the elemental volume, and V' denotes integration over the interaction volume. Performing the integration over range r' in Eq. (2) yields the two-dimensional Green's functions for parametric excitation of the individual normal modes in shallow water.

Examination of this Green's function sheds considerable light on the parametric excitation of given modes. Consider, for example, the three-dimensional plots of this Green's function in Fig. 2, for the case of a parametric array operating in a water column 15 wavelengths in depth over a hard bottom. The angles φ and ψ on the plot axes correspond to the transducer elevation angle in the vertical plane and to the azimuth angle in the horizontal plane, respectively. Amplitude is indicated by height above the φ-ψ plane.

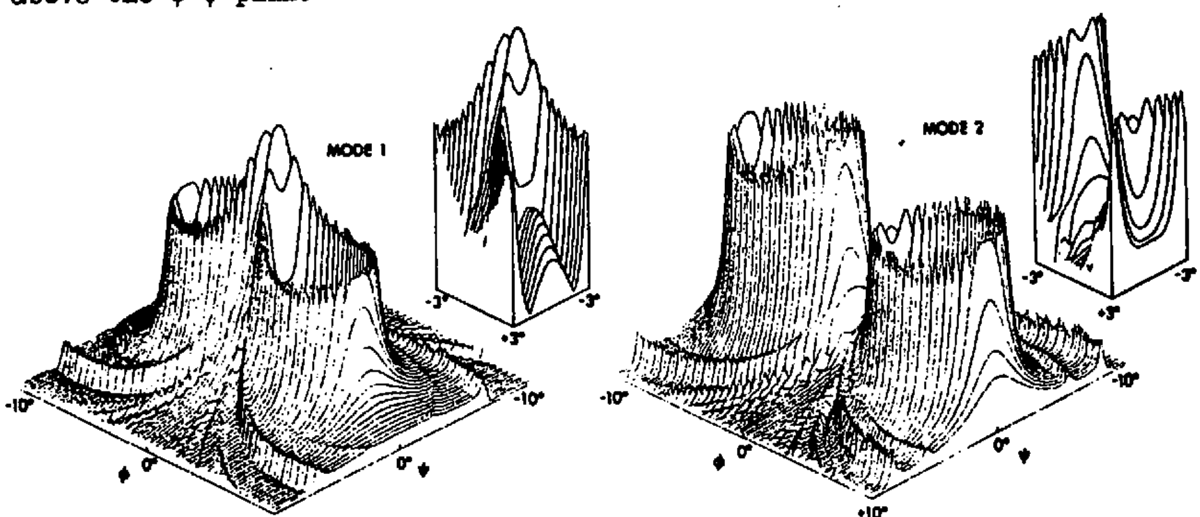

FIGURE 2
NORMAL MODE PARAMETRIC ARRAY
GREEN'S FUNCTION FOR MODES 1 AND 2

In order to preferentially excite a given mode, one should concentrate the radiated energy in regions defined by the Green's function plot. In practice, one can preferentially excite the first mode by simply aiming the parametric beam in the direction of the highest Green's function lobe (see cut-away plot). The lobal structure for the second mode follows a curious structure, much like a pair of volcanic craters. Concentrating the energy along the lobal peaks is therefore more difficult as they turn out to be ridges with rather sharp slopes.

Model Tank Experiments

In a previous model tank study[3,4] (along the lines of A. B. Wood's work in the 1950's at the Admiralty Research Laboratory) the feasibility of parametric propagation in shallow water was examined. An aluminum plank was suspended 11.4 cm below the surface of a fresh water tank. At one end of the plank, a 1.5 in. piston projected collinear 1.53 MHz and 1.73 MHz sound beams, creating a 200 kHz difference frequency. The attenuation at the primary frequencies was 1 dB/m, and the water was approximately 15 difference frequency wavelengths in depth. A spherical hydrophone probe was used at middepth for reception. A continuous propagation curve, shown in Fig. 3, was taken when the 200 kHz signal was produced both linearly and nonlinearly. The linear curve shows the usual effects of multimode interference in its rapid variations. By contrast the parametric curve is smooth, indicating the reduction of multimode interference. In both cases, the projector was oriented horizontally.

For the linear case, the 15° half-power beamwidth insonified several modes while the 2° parametric beam insonified predominately the first parametric eigenray, which lies very near the horizontal axis. A favorable comparison

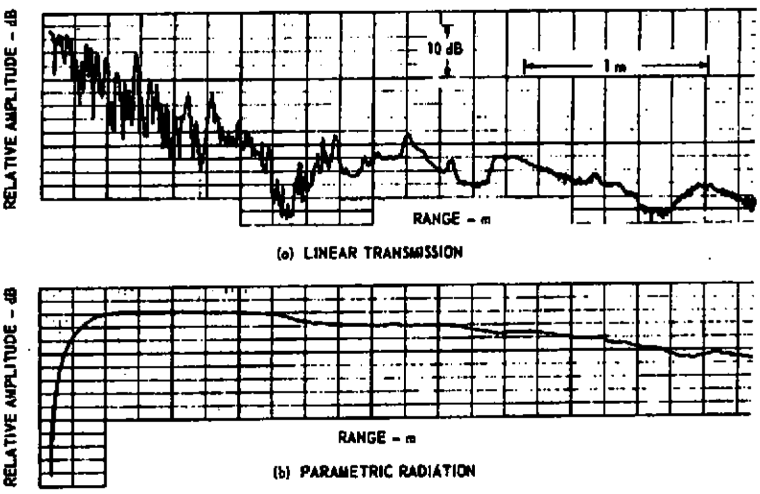

FIGURE 3
MODEL TANK PROPAGATION CURVES

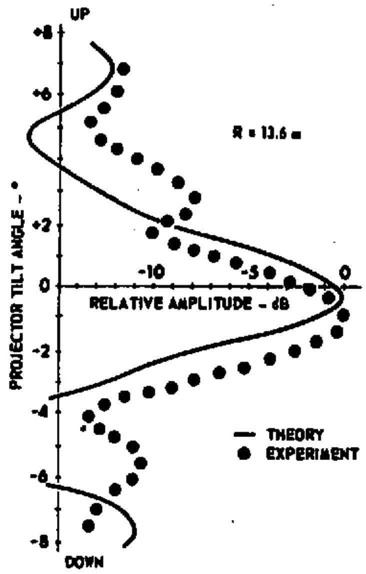

FIGURE 4
DEPENDENCE OF RECEIVED AMPLITUDE AT MIDDEPTH ON PROJECTOR TILT ANGLE

of theory and experiment, shown in Fig. 4, further illustrates this point by exploring the dependence of middepth sound pressure level as a function of the angular position of the projector in the vertical plane. The theory curve was computed for the first mode component of Eq. (2). The measurement was made at a fairly short range of 13.6 m, which corresponds to about one cycle or skip distance for the first mode. It can be seen that the first mode prefers to be excited at a parametric eigenray angle lying just below the horizontal. This slight asymmetry is due to a slight difference in the phase of reflection between the aluminum bottom and the air/water surface at low grazing angles. The effects of a real bottom can be accounted for in Eq. (2) by the appropriate designation of boundary values.

Depth function and other measurements made in the model tank further confirmed the enhancement of the first mode; however, these observations are best illustrated in presentation of results from a field experiment that was conducted after the model tank work.

Field Tests

An experimental parametric sonar system has recently been deployed in the Laguna Madre, a south Texas lagoon encompassed by the continental mainland and Padre Island, on the Gulf of Mexico.

A parametric projector, 11 in. in diameter, was mounted horizontally at middepth in a water column whose depth varied from 4 to 5 ft, depending on the tide. This projector operated at primary frequencies centered at 220 kHz, with a difference frequency of 15 kHz. The water column was about 15 difference frequency wavelengths deep and was used over test ranges out to 3/4 of a mile. The primaries were transmitted in 1 msec pulses at power levels of 1 kW each. During these tests, the water was isothermal and isohaline, and the projector was placed at middepth and oriented in the horizontal direction.

This orientation enabled the projector to insonify the "bull's eye" of the integrated Green's function for the first mode, as is given by the integral of Eq. (2). This important parameter was computed for the experimental conditions and is shown in Fig. 5 as a function of transducer azimuth and elevation angle. Like Fig. 2, this plot has some complicated low level features, but it is not necessary to mimic the whole contour plot to preferentially excite the first mode. It is sufficient to insonify the center region with the narrow parametric beam.

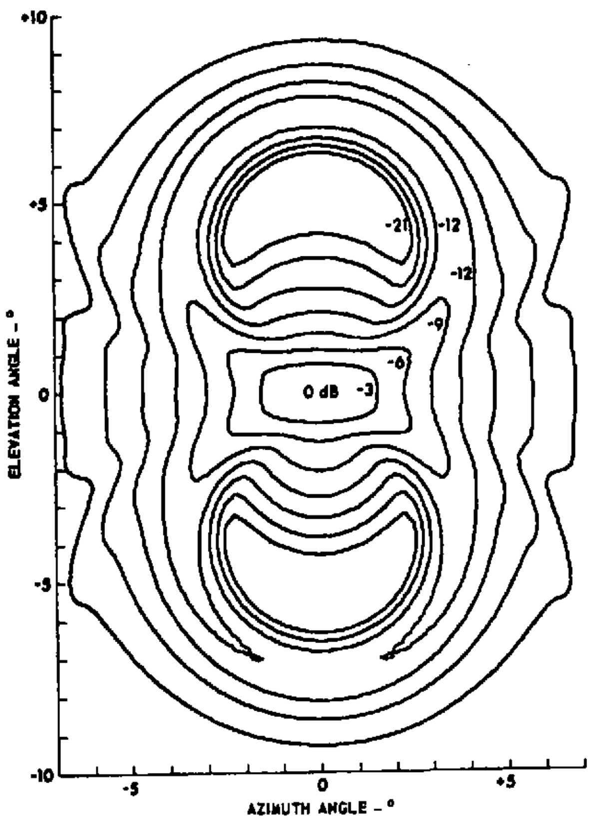

FIGURE 5
INTEGRATED GREEN'S FUNCTION

The very first measurements, made with an omnidirectional hydrophone, showed the parametric transmission to be highly coupled to the first mode of propagation. The 3° half-power beamwidth of the parametric radiation was well matched to the parametric eigenray angles of the first mode, as will be evidenced by the data to follow.

The pulse arrival data shown in Fig. 6 was taken at a range of 412 yd and typifies the pulses observed at all ranges in the experiment. A steady and clear parametric pulse having a 1 msec duration was obtained with short term fluctuations of less than 2 dB. The pulse was not elongated by multimode transmission, volume reverberation, or surface scattering.

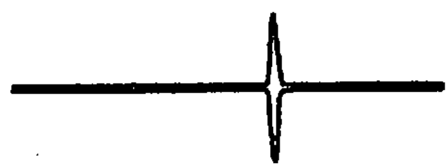

FIGURE 6
PULSE ARRIVAL
A 1 msec PARAMETRIC PULSE AT 15 kHz RECEIVED AT A RANGE OF 412 yd

The 412 yd data station corresponds to a range of about 4 1/2 cycle distances for the first mode. In order to determine whether or not the water column was really being resonated in the first mode, it was decided to make two depth function measurements--one at a short range (~1.4 cycle distances) to see what was being input to the water column, and one at a long range (~14 cycle distances) to determine the output after appreciable propagation. The results are shown in Figs. 7 and 8 in comparison to the theoretical eigenfunctions of the first mode. The agreement between theory and experiment confirms the expected selection and dominance of the first mode of propagation. It can be seen that the water column was uniquely driven in a first mode fashion and that it responded accordingly.

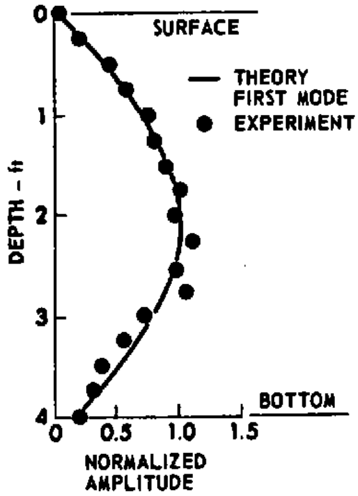

FIGURE 7
DEPTH FUNCTION
AT A RANGE OF 128 yd

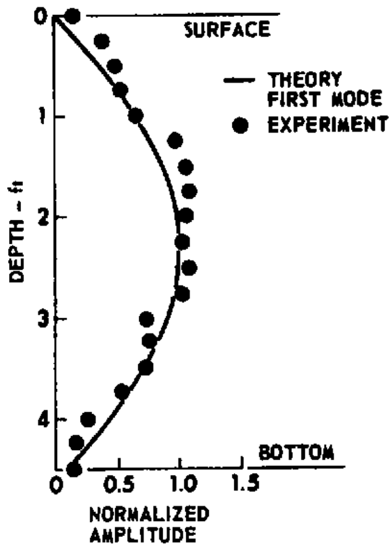

FIGURE 8
DEPTH FUNCTION
AT A RANGE OF 1,285 yd (0.73 MILE)

Applications

The question arises as to what implications parametric arrays might have for shallow water sonar work. Although the subject is quite new, we can illustrate several interesting features with some results from the field experiment at hand.

The first involves the relationship of target echoes to the reverberation. Given the fact that narrowbeam parametric arrays can operate in shallow water, we are immediately drawn to the observation that their angular discrimination (and hence their rejection of reverberation) is significantly better than that of linear systems of the same aperture, depending on the parameters. In the present experiment, for example, the ratio of linear to parametric beamwidths is $27°/2.5°=10.8$, and the parametric beam also has no side lobes. Considering surface backscattering and comparing the two areas of insonification, we should observe a 10 log (Area Ratio) improvement with the parametric system, which would amount to about 10 dB for the present case. This expectation is borne out by the A-scan echo/reverberation data of Fig. 9, which was taken against a brass reflector of the tip-to-tip biconic design that was placed at middepth at a range of 43 yd. The improved suppression of reverberation with the parametric system is obtained, even though the parametric data was taken in rougher water than was the linear data. Statistical data on the echo and echo plus reverberation were also acquired and are being analyzed with respect to the surface wave height statistics.

The suitability of parametric arrays to wideband signal generation[4] raises the next question concerning the potential use of wideband parametric signals in shallow water. This issue depends on whether or not the medium will permit the coherent transmission of such a signal. This question was subjected to a test and appears to be answerable in the affirmative. For this measurement, one of the primaries was projected in a 1 msec pulse at 230 kHz while the other primary pulse was swept from 215 to 210 kHz in an FM slide. This produced the difference frequency pulse shown in Fig. 10. This data was acquired at middepth at a range of 500 yd which corresponds to 5.4 cycle distances for the first mode. A measurement

LINEAR ECHO AND REVERBERATION

PARAMETRIC ECHO AND REVERBERATION

FIGURE 9
COMPARISON OF ECHOES

FIGURE 10
A PARAMETRIC FM PULSE

of zero crossings shows the difference frequency pulse to be swept from 16 to 24 kHz. The fact that the difference frequency sweep is somewhat higher and broader than the difference of the two primaries is probably due to several frequency weighting factors arising from both parametric and waveguide effects. The main observation, however, should be that wideband operation was in fact achieved and this remains to be explained. Since we usually consider the eigenray angles in the shallow water waveguide as being highly dependent on frequency, would not their change with frequency lead to multimode generation and interference? This did not occur and we surmise that the first mode eigenrays, or more generally the Green's function integral of Eq. (2), may have changed shape but not location with change in frequency, thereby leaving the first mode in a good position to be excited. Examination of the Green's function plots (like that of Fig. 2) does support this argument in that the major lobes get steeper with increase in frequency, but they remain in the near vicinity of the origin. Irrespective of the reason for the achievement of parametric wideband transmission, we are left with important implications for the use of wideband signal processing enhancement in underwater communications and quite possibly in sonar systems as well. This may significantly offset the inefficiency of the parametric array and lead to new concepts in system design.[4]

In the other extreme, that of narrowband applications, the parametric sonar has potential shallow water applications of comparative interest. Here, doppler measurements come to mind in the light of the increased directivity of the parametric array. Consider, for example, the comparison of Fig. 11 which shows the 15 kHz beam patterns of the present experiment cast in the role of an active doppler system. For these applications, it is usually desired to measure the frequency shift created by the relative motion of objects in some target area B with respect to own ship's velocity, v_A. There may be no movement in area B if the system is used for topographic navigation. In either case, there will be an own doppler noise induced by movement of the sonar platform in the presence of reverberation that masks the doppler signals from the desired target. This increases with beam size and becomes more complex with amplitude changes across the minor lobes. For shallow water work, the problem is often two-dimensional, with the bottom and surface playing dominant roles in doppler backscattering. It can be shown that the frequency of doppler noise induced by own ship movement is given by

$$f = \frac{2v_A f_o}{c} \cos\theta \quad , \quad (3)$$

and that the spectrum level of own doppler noise is proportional to

$$N(f) \propto 20 \log b(f(\theta)), \quad (4)$$

where b(·) is the directivity function of the system beam expressed as a function of doppler frequency, f_o is the radiated frequency, and the other symbols are defined in Fig. 11. The beam pattern data shown in this figure has been employed with Eq. (4) to compare the linear and

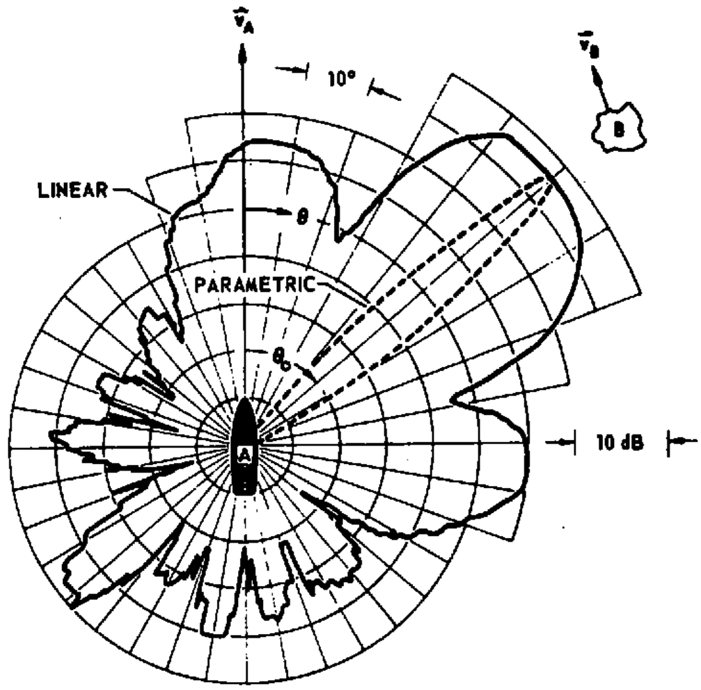

FIGURE 11
BEAM CONSIDERATIONS FOR DOPPLER MEASUREMENTS

parametric noise spectrums of the present 15 kHz experiment. The results are given in Fig. 12 for the up-doppler quadrant. The main ordinate is calibrated in terms of the $2v_A f_o/c$ parameter, while the insert ordinate provides a numerical example for an own ship's velocity of 15 kt. It can be seen that the parametric system is capable of vastly improved suppression of induced doppler noise, except at the one doppler frequency component corresponding to equal insonification by each beam. It can also be seen that the doppler noise present with the parametric system should decrease sharply with decrease in doppler frequency. This implies a significant improvement in doppler sonar performance against low velocity targets.

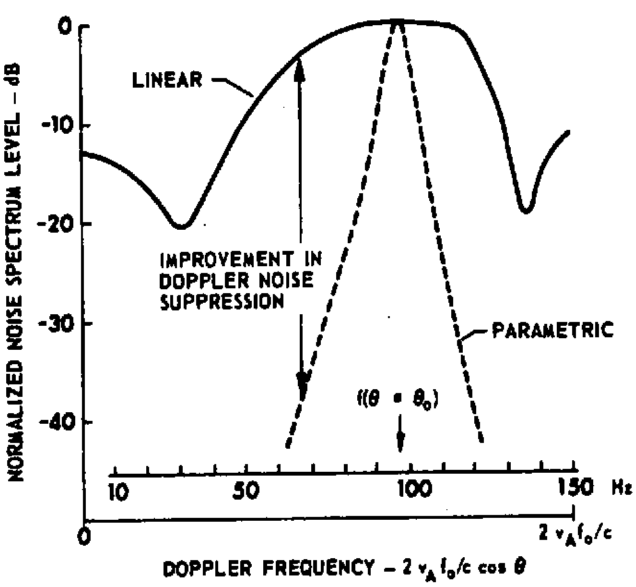

FIGURE 12
COMPARISON OF UP-DOPPLER NOISE

Summary

The application of parametric arrays to shallow water acoustics has been outlined with a Green's function formulation appropriate for waveguide propagation. Experimental results from both a model tank and a natural lagoon support the theory, primarily in the validation of certain possibilities it predicts. These include the selective excitation of the first mode of propagation. Applications to the suppression of sonar reverberation, to the suitability of wideband propagation and its compatibility for signal processing enhancement, and to the reduction of own ship's doppler noise were discussed and demonstrated experimentally.

Acknowledgements

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