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SOME ASPECTS OF THE PARAMETRIC DEPENDENCE OF THE NON-LINEAR IMPEDANCE OF PERFORATED PLATES

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1. INTRODUCTION

Non-linear acoustic behaviour in orifices (and consequently perforates-since the latter can be regarded as a set of orifices) has been appreciated for some considerable time (e.g. Reference 1) However, it is only recently, as a result of the application of resonator arrays to control rocket engine combustion instabilities and the use of perforates in aero-engine duct linings, that detailed attempts have been made at understanding the quantitative aspects of the behaviour, in particular, in identifying those parameters which are responsible in determining the non-linear behaviour (2,3,4) These approaches (3) claim to have "analysed with proper account of the primary non-linear losses" or alternatively (4) from "a fundamental fluid mechanical point of view" to describe the non-linear phenomena. The analysis of Zinn (4) and Sirgnano (3) differ in the description of end effects (attached mass) and the use of energy rather than the continuity equation to describe the flow in the resonator cavity. Zinn's analysis may be simplified on intuitive grounds if assumed that the exit losses are equal (or proportional) to the jet flow kinetic energy (reasonable assumption based on available evidence (2)) and consequently an effective force equal to the velocity head needs to be introduced into the classical linear equation for the linear damped resonator, yielding rapidly, the results obtained by Zinn.

The present approach to the problem is based on the assumption of a quasi-steady analysis, which is of general validity, as will apparent. Based on this assumption a simple model for the system may be arrived at yielding an expression for the non-linear dependence in terms of well known system parameters.

2. THEORETICAL ASPECTS

The validity of the model is dependent on the assumption of a quasi-steady flow field in the vicinity of the orifice. The flow may be considered to be quasi-steady if the time dependent term in the Euler equation for perturbed flow (acoustic field) is small in comparison with the spatially dependent terms. From order of magnitudinal consideration this condition is upheld if the ratio of the characteristic fluid displacement to the product of the period of the flow oscillation and velocity amplitude (of the perturbation), is small. Whilst for thick orifices this condition may not be upheld, in general, for typical orifices and sound pressure levels the assumption is a valid one.

Consider the case of a regularly perforated sheet mounted over a backing cavity as shown in Figure 1. To each hole, of area S_H a zone area Sc may be ascribed. For a normally incident acoustic wave $p^{(a)}(t)$, incident on the front face, two momentum equations may be written to describe the relationship between the pressure and velocity, one in terms of the velocity in the zone area (Sc) in front of the hole and one in terms of the velocity in the hole. Further, it is assumed that the cavity backing depth is a quarter wavelength this considerably simplifies the equations with out effecting the general validity of the arguement.

For the velocity in the hole (v_H)

$$p^{(a)}(t) = \rho c M_e \frac{\partial v_H(t)}{\partial t} + K |v_H(t)| v_H(t) + R_o v_H(t) \qquad (1)$$

Where: K is identified as being equivalent to the steady-flow loss factor

M_e is the mass reactance ratio of the hole.

R_o is the usual acoustic resistance term.

For the zone area of the hole (Sc) where the velocity is taken to be $v_n(t)$

$$p^{(a)}(t) = \rho c \left[R + M_e \frac{\partial}{\partial t} \right] v_n(t) \qquad (2)$$

and from continuity

$$S_c v_n(t) = S_H v_H(t) \qquad (3)$$

R in equation (2) is taken to represent the acoustic resistance term which is normally measured in impedance determination by the transmission line technique. Since the energy 'dissipated' both in the control zone and in the hole must be the same, then by taking equations (1) and (2) multiply by the appropriate velocity, integrating and equating terms it may be shown that

$$R = (8/3\pi) V_n K (S_c/S_H)^2 + R_o$$

$$= 1.055 \left[K \left(\frac{S_c}{S_H} \right)^2 \right] v_n^{rms} + R_o$$

Where v_n^{rms} is the zone area rms velocity

V_n is the zone area velocity amplitude

The velocity amplitude factor $8/3\pi$ is identical to that derived by Zinn (4)

The K factor is identified with the pressure loss which would be sustained by steady flow through an orifice; this is usually written in terms of the discharge co-efficient for the orifice whence,

$$K = \frac{1}{(C_D(R_e, N_e))^2} \cdot (1 - P^2) \rho/2$$

Where P is the effective porosity S_H/S_C

Hence, the velocity dependent impedance may be written,

$$R = \frac{1}{\rho c} 1.055 \, \rho/2 \cdot \frac{1}{C_D^2} \left(\frac{1 - P^2}{P^2} \right) u_n^{rms} \qquad (4)$$

Thus the effective resistance is directly proportional to the acoustic particle velocity and the inverse square of the discharge co-efficient.

The discharge co-efficient is dependent on the Reynolds Number (based on hole diameter) and therefore the slope of the non-linear region will be velocity dependent; only at high Reynolds Numbers will the slope become independent of the velocity ($R > 10^4$). Further, the discharge co-efficient is also sensitive to the 'effective porosity' S_H/S_C and the sharpness or shape of the orifice; in general discharge co-efficients are only quoted for sharp edge orifices. Figure 2 illustrates the general behaviour of the discharge co-efficients with orifice shape 'effective porosity' and Reynolds Number.

COMPARISON WITH EXPERIMENTAL RESULTS

Comparison between experimental work carried out on both simple orifices and perforated plates and predictions based on the above formulation show good agreement. The errors which occur are thought to result from using discharge co-efficients measured for ideal orifices; this assumption is bourne out from the comparison of the measured steady flow pressure losses and those predicted using the standard orifice plate discharge co-efficients. These differences may result from the non-ideal orifice condtion or possible interaction between adjacent orifice flows.

REFERENCES

1. L.J. Sviain — Acoustic Impedance of Small Orifices, J. Acoust. Soc. Am., 1935, 7

2. U. Ingard & S. Labate — Acoustic Circulation Effects and the Non-Linear Impedance of Orifices, J. Acoust. Soc. Am. 1950 22, 2

3. W.A. Sirignano — Non-Linear Dissipation in Acoustic Liners, Princeton University AMS Report 553 - F (in part)

4. B.T. Zinn — A Theoretical Study of Non-Linear Damping by Helmholtz Resonators, J. Sound Vib. 1970, 13, 3

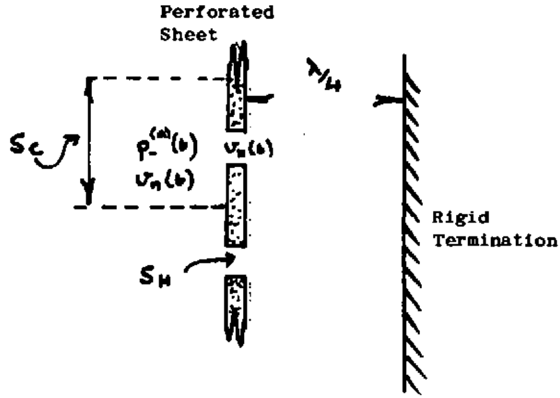

Figure 1 Model for Perforated Plate

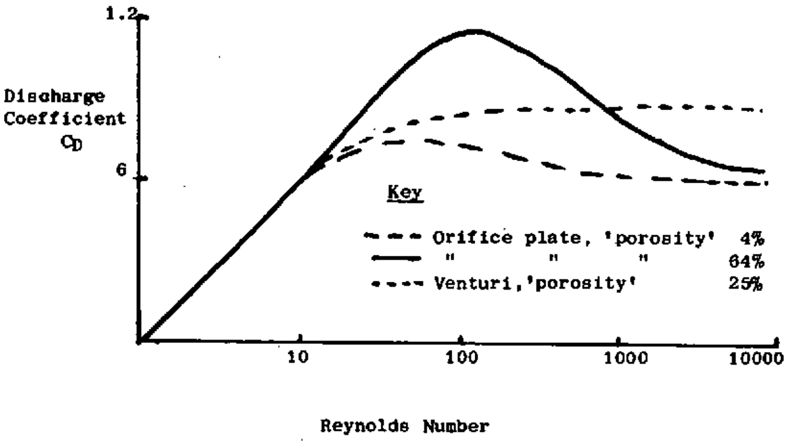

Figure 2 Behaviour of discharge coefficient as a function of Reynolds Number and effective porosity