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FREQUENCY AND TIME DOMAIN CONTROLLERS FOR THE ATTENUATION OF VIBRATION IN NONLINEAR STRUCTURAL SYSTEMS

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1 INTRODUCTION

The active control of vibrating structures is a rapidly developing field of research [1]. Hydraulic, piezoelectric, electromagnetic, magnetostrictive or other actuators provide secondary sources of vibration which are controlled to minimise structural vibration at a number of points or possibly to minimise radiated sound power [2]. In using linear feedforward control methods it is assumed that vibration amplitudes are small so that the structure (including the actuator) is linear and motion introduced by the secondary actuator is superposed with the primary disturbance. In practice, however, nonlinear distortion may arise due to non-ideal actuator characteristics (*eg* saturation at high amplitudes, hysteresis in magnetostrictive devices) or to nonlinear coupling between different parts of the structure (*eg* frictional damping at joints). This paper presents two approaches to the active control of nonlinear structural systems:

(1) for harmonic disturbances, a frequency-domain controller which synthesizes the optimum input waveform driving the secondary actuator to overcome the nonlinearity and minimise vibration at a chosen location on the structure;

(2) for random excitations, a neural network controller which approximates the inverse of the system under control.

A real-time frequency-domain controller has been constructed and applied to overcome the inherent hysteresis of a magnetostrictive actuator. Attenuation of harmonic disturbances up to 3kHz has been demonstrated.

2 FREQUENCY-DOMAIN CONTROL OF HARMONIC DISTURBANCES IN A NONLINEAR STRUCTURAL SYSTEM

2.1 Control scheme

Figure 1 shows a scheme for feedforward control of a nonlinear dynamic structure with harmonic excitation. The nonlinear structure considered is assumed to be non-chaotic and dissipative: a structure for which a periodic disturbance produces a periodic response at the same frequency (no subharmonics). It is further assumed that the structure is subject to a periodic excitation (from a rotating machine, for example) and that this excitation d(t) can be linearly superposed with the nonlinear system output y(t) to form the measured error e(t).

Assuming the disturbance is sinusoidal the output y(t) of the nonlinear system is required to be as nearly as possible a sine wave at the same frequency and opposite phase to the disturbance d(t), so that when they are superposed the error e(t) is minimised. To achieve this the input u(t) to the nonlinear plant is required to be an appropriate non-sinusoidal periodic waveform,

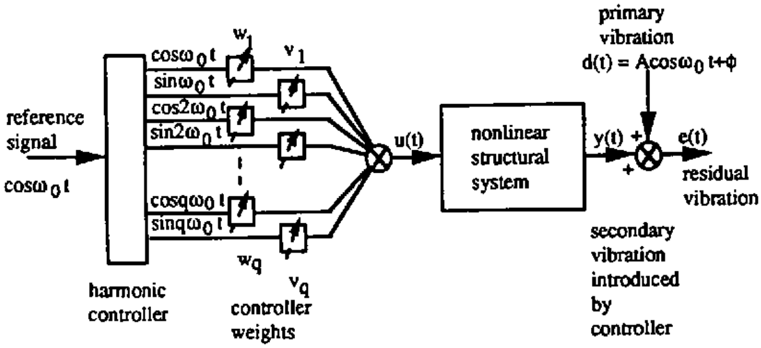

Figure 1: Scheme for control of harmonic vibration in nonlinear structure

whose form will be identified by adapting the controller coefficients. The controller synthesizes this waveform from its harmonics. Of course, there are many nonlinear systems which are incapable of generating a sinusoidal output whatever their input, but in this control scheme the aim is to obtain the closest approximation by minimising mean squared error, the chosen cost function.

The reference signal is assumed to be at the same frequency as the disturbance (*eg* engine rotation rate) and from it the controller generates a full range of harmonics. Both the in-phase and quadrature components are required. Controller coefficients w_q, v_q ($q = 0,1,2,...Q$) define the magnitude and phase of each harmonic. The coefficients are adapted by gradient descent. However, use of the gradient descent method is complicated by the fact that the controller error is not available: it is the error between the desired and actual output of the controlled system (the 'plant') which is measured. In deriving the appropriate form of the steepest descent algorithm it becomes clear that a model of the plant is required.

2.2 Coefficient adaptation by steepest-descent algorithm

The cost function J to be minimised is chosen to be mean squared error:

$$J = \frac{1}{T_p} \int_{-T_p/2}^{T_p/2} e^2(t)\, dt \qquad (2.1)$$

where T_p is the period of the fundamental disturbance d(t). The error e(t) is the sum of the disturbance and the output y(t) of the nonlinear system. Representing e(t) by its Fourier series:

$$e(t) = \frac{a_0}{2} + \sum_{p=1}^{\infty} a_p \cos(p\omega_0 t) + \sum_{p=1}^{\infty} b_p \sin(p\omega_0 t) \qquad (2.2)$$

The cost function can then be written (by Parseval's theorem) as

$$J = \frac{a_0^2}{4} + \sum_{p=1}^{\infty} \frac{a_p^2}{2} + \sum_{p=1}^{\infty} \frac{b_p^2}{2} \qquad (2.3)$$

The steepest descent algorithm to update the qth controller coefficient w_q is

$$w_q(i+1) = w_q(i) - \mu \frac{\partial J}{\partial w_q} \qquad (2.4)$$

The corresponding algorithm for $v_q(i+1)$ is given when w_q is replaced by v_q in this expression. The gradient $\partial J/\partial w_q$ follows by differentiation of (2.3):

$$\frac{\partial J}{\partial w_q} = \frac{a_0}{2}\frac{\partial a_0}{\partial w_q} + \sum_{p=1}^{\infty} a_p \frac{\partial a_p}{\partial w_q} + \sum_{p=1}^{\infty} b_p \frac{\partial b_p}{\partial w_q} \qquad (2.5)$$

In order to use (2.4) to adapt the controller coefficients it is clearly necessary to obtain information about the plant, that is, the partial derivatives $\partial a_p/\partial w_q$, $\partial b_p/\partial w_q$ and also $\partial a_p/\partial v_q$ and $\partial b_p/\partial v_q$ for all p and q. These partial derivatives describe the frequency-domain characteristics of the nonlinear system under control. For example, $\partial a_p/\partial w_q$ is the sensitivity of the pth in-phase harmonic component of the error signal to the qth in-phase harmonic component of the input. If the plant were linear, it would clearly be the case that each harmonic applied at the input (*eg* w_q) would affect only the corresponding frequency at the output (a_q and b_q). All the cross-terms would be zero, and the $\partial a_p/\partial w_q$ and other terms would form diagonal arrays. However for nonlinear systems all the partial derivatives are, in general, nonzero. These partial derivatives vary with the magnitude of each w_q and v_q (the operating point) as well as with frequency.

3 SIMULATION STUDY: CONTROL OF BACKLASH FOLLOWED BY LINEAR DYNAMICS

By way of example, the 'plant' has been taken to be a function representing a backlash (Figure 2) followed by a pure delay and short first-order lag (time constant 0.03s). The delay was chosen to introduce a phase lag of 108 degrees at the chosen fundamental frequency, sufficient to destabilise the coefficient adaptation process if no account had been taken of the plant characteristics. The fundamental disturbance frequency was arbitrarily chosen to be 1 Hz in this example.

The partial derivatives representing the plant frequency-domain characteristics were approximated by constants, obtained as best fits to the slope of the surfaces $a_p(w_1,w_2,..w_Q,v_1,v_2,..v_Q)$ *etc* over a practical range of the w_q and v_q coefficients. With these values fixed the control was allowed to run from zero initial conditions ($w_q=v_q=0$, $q=0,1,..Q$). The controller coefficients were adapted using the steepest descent algorithm Eqn (2.4) above. The transients in the controller coefficients are shown in Figure 3(a); the final waveforms for the input and output of the plant are shown in Figure 3(b). In Figure 3(a) it is seen that the

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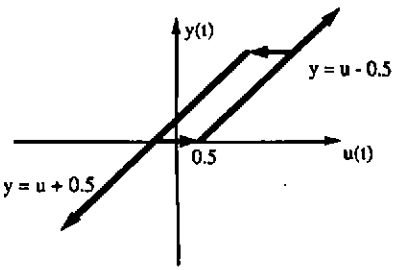

Figure 2: Characteristic of backlash nonlinearity

final controller waveforms after 300 periods contain the odd harmonic components (w_1, w_3, w_5) but not the even components. This reflects the symmetry of the backlash nonlinearity.

Figure 3(b) shows the waveform synthesized by the controller along with the sinusoidal disturbance and plant output. The plant output has been inverted to allow comparison with the disturbance. The controller output shows a sharp rise or fall at the point in the waveform when it is required to cross the backlash function rapidly and maintain a smooth output. The residual error between d(t) and y(t) reflects the fact that only 5 harmonics were used in the controller; a larger number of harmonics would give a better match and hence a lower error.

In practice the coefficient adaptation remained stable in this case even if the partial derivative arrays $[\partial a_p/\partial w_q]$ *etc* were constrained to be diagonal arrays, indicating that a linear plant model would be an adequate approximation in applications of this type; however, more severe nonlinearities (such as saturation) with stronger cross-coupling of harmonics required the full array of partial derivatives to be estimated in order to maintain stability [3].

4 APPLICATION: ACTIVE CONTROL OF HARMONIC VIBRATION USING A MAGNETOSTRICTIVE ACTUATOR

4.1 Magnetostrictive actuators

A nonlinear problem of particular interest is the active control of structural vibration using secondary actuators made from Terfenol-D. Terfenol-D is a highly magnetostrictive alloy of iron, terbium and dysprosium developed at the US Naval Surface Warfare Centre [4]. The material increases in length by up to about 1500 ppm in a magnetic field. Substantial forces are developed if the material is constrained. Actuators using this effect show promise for active control of vibration in structures. However the length changes in Terfenol-D are not linearly related to the applied magnetic field; there is hysteresis due to the magnetic character of the material. Thus sinusoidal input variations produce periodic but nonsinusoidal strain changes.

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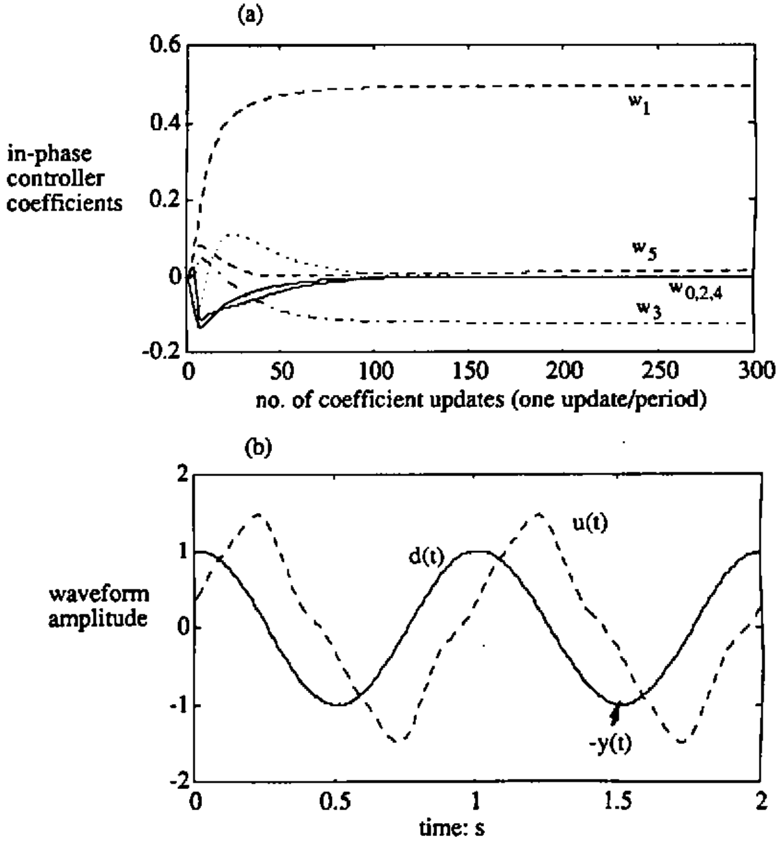

Figure 3: Attenuation of 1 Hz sine wave of unit amplitude using frequency-domain controller acting through backlash with dynamics (Figure 2)
(a) movement of in-phase controller coefficients from zero initial condition;
(b) waveforms after adaptation: disturbance d(t) (solid line), controller output u(t) (dashed) and inverted backlash output -y(t) (dotted)

4.2 Scheme for real-time control

A set of routines has been written in the 'C' language to implement real-time control of a harmonic disturbance using a magnetostrictive actuator. The routines are written for an IBM-compatible PC with a Loughborough extension board carrying a Texas Instruments

TMS320/C30 digital signal processor. The control strategy used is that outlined in Section 2. A frequency-domain model of the plant (*ie* the actuator, structure under control and attached error accelerometer) is needed for the algorithm and is identified in a separate learning phase.

The experimental results are similar to those obtained in the simulations outlined above. Because the excitation is periodic, the waveform need not be updated every cycle and the fundamental frequency can be as high as 3 kHz with dc and 7 harmonics under control.

5 CONTROL OF NONLINEAR STRUCTURAL SYSTEMS USING NEURAL NETWORKS

5.1 Control strategy

In active vibration control it is sometimes possible for nonlinear actuators to be linearised by local feedback. However in cases where this is not possible (*eg* the nonlinearity is distributed round the structure) it may still be possible to implement feedforward active vibration control by passing the reference signal through a nonlinear adaptive controller which models the inverse nonlinear response of the plant. This Section considers the possible use of neural networks for this purpose.

The use of neural networks for feedforward control of nonlinear systems has been considered by a number of writers [5,6]. A central difficulty with such an approach, as illustrated in Figure 4, is that the error at the output of the neural controller is not available for weight adjustment using in the backpropagation algorithm because of the presence of the nonlinear plant. Nguyen and Widrow [5] proposed that a separate neural model of the nonlinear plant be identified first and that this could be used as a channel for the backpropagation of errors to the controller. When trained, the controller approximates to an inverse model of the nonlinear plant.

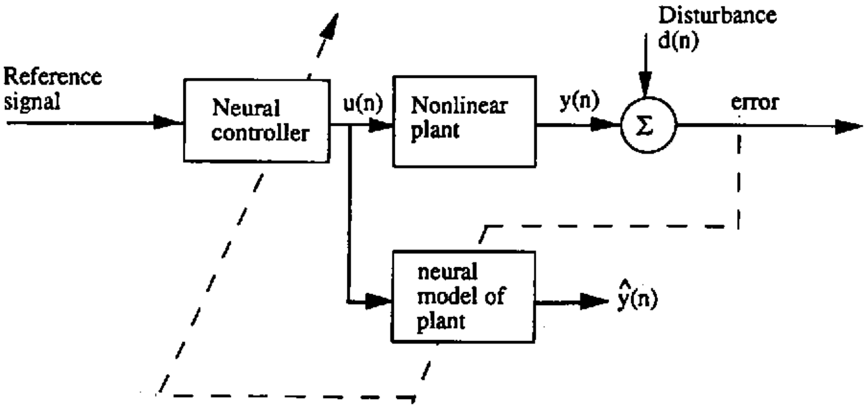

Figure 4: Coefficient adaptation for a neural network controller

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Several different forms of neural network have been devised [7]: they include the multilayer perceptron (MLP) network, the radial basis function network and the B-spline network [8]. It is necessary to choose an architecture which is appropriate for the system under control. The backlash function discussed earlier lends itself to description by a recurrent MLP network: it can be well-represented by a two-layer MLP network with just two inputs and two hidden units, one of the inputs being the network output at the previous time-step. (Recurrent networks are those in which signals are fed back from higher layers to lower layers in the network in addition to the normal feedforward connections.)

The neural controller and model are also required to be capable of representing the *dynamics* (or the inverse dynamics) of the system to be controlled. The basic feedforward MLP network is simply a static mapping from inputs to outputs, but a number of ideas have been advanced for representing dynamic systems. These include the time delay neural network (TDNN) [9] in which an input time series is passed through a tapped delay line from which past values can be obtained and presented as separate inputs to the network. Other writers have suggested incorporating dynamic characteristics into each node of the network [10,11]. Recurrent networks also have dynamic properties if the output is delayed by one or more time-steps before being fed back to the input. For modelling problems in structural dynamics recurrent networks have significant advantages because they require relatively few weights. On the other hand, a TDNN model of a structural system having a few lightly-damped modes (and thus a long ringing response) is likely to need a long tapped delay line with a correspondingly large complement of weights to be identified. Recurrent MLP networks have the disadvantage that they are difficult to train because the backpropagation algorithm cannot be used directly. A possible option is the RTRL (real-time recurrent learning) algorithm of Williams and Zipser [12].

Fortunately it is not necessary for the system model to be strictly recurrent when it is only used to update weights in a control scheme. It is possible to use past values of the actual plant output as inputs to the neural model instead of past outputs of the model itself, as shown in Figure 5. This form of model is discussed by Hunt and Sbarbaro [13] and is similar to the 'series-parallel' model of Narendra and Parthasarathy [6]. It is similar to the equation-error identification method for linear systems discussed, for example, by Norton [14]. With this modification the model can be trained by the backpropagation technique.

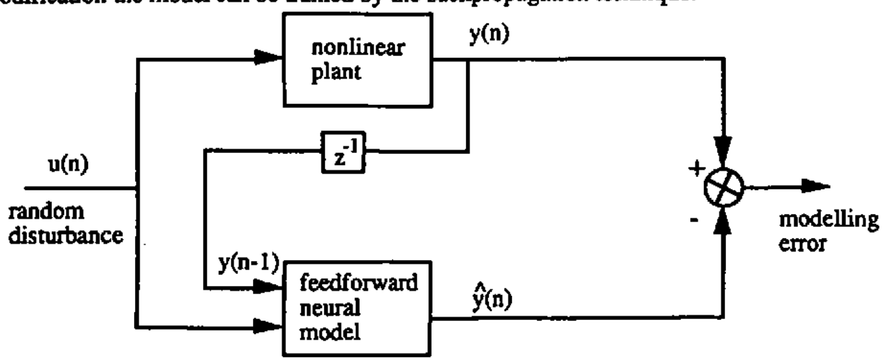

Figure 5: Neural model of nonlinear plant using the plant output as an input to the neural model

5.2 Control example

By way of illustration, this approach has been applied to the case of a simple backlash function with no dynamics. The disturbance to be attenuated at the backlash output was a narrow-band random noise signal. The disturbance was also presented to the neural controller as a reference signal. Figure 6(a) shows the output of a feedforward neural model with two hidden units and two inputs: u(n) and y(n-1). Its output is compared with the output of the backlash itself, which it models reasonably well.

After the weights in the neural model had been adapted using the backpropagation algorithm they were fixed while a neural controller was trained. The controller was a feedforward net with two inputs, d(n) and d(n-1), and two hidden units whose weights were also adapted using the backpropagation algorithm. Figure 6(b) shows the controller output and also compares the controlled backlash output with the disturbance (inverted to allow comparison). The weights of both networks were held constant at their fully-trained values for this run. When the disturbance signal changes direction, a jump in controller output is required to maintain a smooth output from the backlash function. Such a signal appears to be efficiently generated by the simple feedforward controller used here. Further work is in hand to develop time-domain neural controllers for dynamic nonlinear systems encountered in practice.

6 CONCLUSIONS

The Paper has presented a frequency-domain approach to the active control of nonlinear systems under periodic excitation. The feedforward controller adapts to synthesize the appropriate periodic waveform required at the nonlinear system input in order to cancel a sinusoidal disturbance. Adaptation of the controller coefficients requires an array of partial derivatives which express the sensitivity of each harmonic of the nonlinear system output to each harmonic of the input. This array is obtained as a best fit in a chosen operating range. Real-time control of a sinusoidal disturbance using a nonlinear magnetostrictive actuator has been demonstrated using this approach. The fundamental frequency could be varied up to 3 kHz with dc and 7 harmonics under control. The strength of this frequency-domain technique is that no extra complexity is introduced by the dynamics of the system under control. The plant dynamics are implicitly incorporated in the frequency-domain description which is expressed by the four arrays of sensitivity derivatives.

Time-domain neural networks also show promise for control of nonlinear structural systems but an appropriate neural structure must be chosen to represent the structural dynamics without allowing the network size to become excessive. Recurrent networks have been found to be very efficient models of backlash behaviour, and simple feedforward networks with memory have been found to be able to effectively compensate for backlash behaviour when used as feedforward controllers.

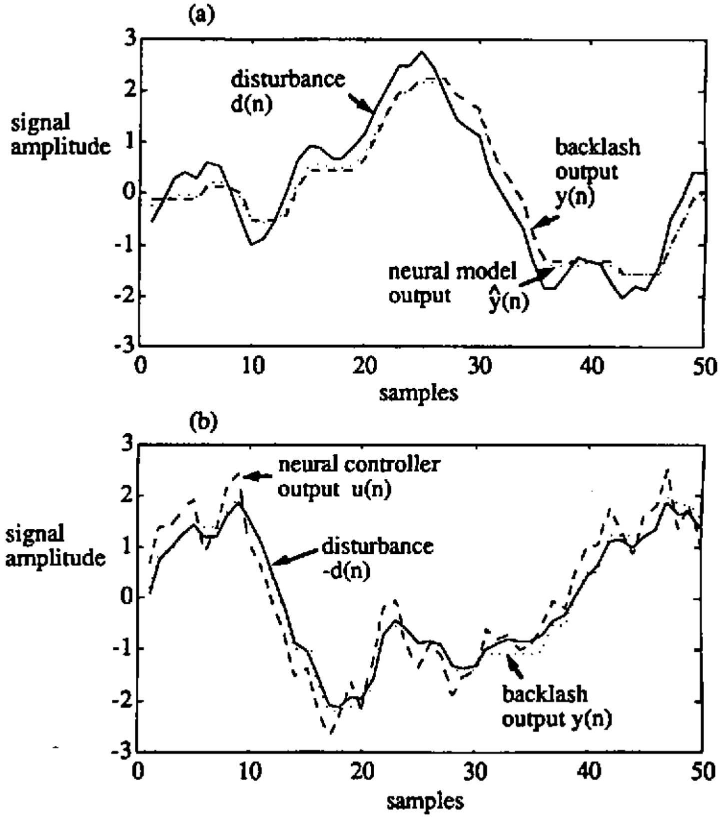

Figure 6: (a) Neural model of backlash function. Narrowband random noise d(n) (solid line) is applied to both a backlash function of unit width (output y(n)) and a feedforward neural model with fixed weights (output $\hat{y}(n)$)

(b) Attenuation of narrowband noise d(n) (solid line; shown inverted) using a feedforward neural net acting through a backlash function of unit width.

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