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THE USE OF THE DISCRETE FOURIER TRANSFORM TO CALCULATE THE SPATIAL RESPONSE OF DAMPED PANELS

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1. INTRODUCTION

In this paper we present a technique for calculating the vibration response of damped panels when forced by a line input. The damping that will be considered here is provided either by radiation into an adjacent acoustic medium (including porous media) or by a damping treatment applied to the panel, or both. It is well-known that formal solutions to problems of this sort are easily obtained using wavenumber transform techniques as outlined in recent books by Junger and Feit [1] and Fahy [2]. However, the wavenumber domain solution must then be inverse transformed to obtain the spatial response. This inversion may be performed analytically in only the simplest of circumstances. In this paper it will be shown that for damped panels, the inversion integral may be evaluated efficiently and "exactly" by using the Fast Fourier Transform (FFT) algorithm to evaluate the Inverse Discrete Fourier Transform (IDFT). The use of this technique will be demonstrated by calculating the damping effect of finite depth layers of porous media when they are confined between a panel and a hard surface.

Line-excited panels loaded by an acoustic half-space have been studied extensively: see, for example, Morse and Ingard [3], Nayak [4], Keltie and Peng [5], Feit and Liu [6], Innes and Crighton [7] and Seren and Hayek [8]. In each case, the differential equation representing the single frequency, transverse displacement of the panel is Fourier transformed in space to yield an algebraic relation between the input force and the panel displacement in the wavenumber domain. To obtain the spatial response, e.g., the magnitude of the transverse displacement *versus* distance from the input force, it is necessary to inverse transform the wavenumber solution. The inverse transform is evaluated, in principle, by integrating from $-\infty$ to ∞ along the real wavenumber axis. Since the required inversion integral cannot be evaluated along this path in closed form when the panel is fluid-loaded, alternative approaches must be adopted. Most often, the techniques of contour integration are used; by an appropriate choice of integration contour and asymptotic forms of the integrand, approximate values for the panel displacement or radiated sound pressure may be obtained that are valid in particular domains [3,6,7,8]. Alternatively, the contour integral may be evaluated "exactly" by numerically integrating along the appropriate contours and branch cuts [4,8]. The simplest approach to inverting the integral, however, is to integrate numerically, directly along the real axis [5,9]. Unfortunately, the latter approach cannot be used in the absence of damping since the integrand then possesses poles on the real axis that make direct numerical integration impossible. The poles can, however, be shifted off the real axis, if desired, by adding a small amount of damping to the panel (by making its Young's modulus complex, for example) or by adding damping to the fluid medium (by making the fluid wavenumber complex) [5]. Thus, what might be called the *direct* approach, i.e., integration directly along the real axis, can usually be made to work in practical circumstances since any realistic structure will exhibit finite damping. The potential benefit of the direct approach is that the inversion integral may be evaluated efficiently by using an FFT algorithm to evaluate the IDFT. To-date, this approach has not been used to calculate panel responses, although the FFT has been used to calculate the sound radiation from vibrating planar surfaces: see, for example, Stepanishen and Benjamin [10] and Williams and Maynard [11].

In the work described here, we are concerned explicitly with structures other than panels loaded by a fluid half-space: i.e., panels that are relatively heavily damped, whether by addition of a damping treatment to the panel or by the action of porous materials adjacent to the panel. Our intention is to develop a design tool that can be used to investigate multi-layer damping teatments and to establish their relative merit. The mathematical structure of the wavenumber solution that results in these cases is sufficiently complex that the effort required to approximate the inversion integral by using the techniques of contour integration may outweigh the benefits of the approach. If, instead, it were decided to perform the contour integration numerically, the structure of the integrand may be complicated enough that it is difficult to identify an appropriate path over which to integrate. This observation suggests that, when considering arbitrarily complex structures, it is only feasible to evaluate the integral directly along the real axis. Fortunately, as noted above, when attention is directed specifically towards damped structures, singularities do not occur on the real axis, thus making the direct approach straightforward. In addition, as will be seen in the following section, the effect of any acoustic medium (including many types of porous media) can be represented by a plane wave surface normal impedance. The latter quantity is easily obtained for even arbitrarily complicated layered structures [12,13]. Thus, the final benefit of the direct approach is the ability to accommodate realistic structures of the type used in noise control practice. For all these reasons, it was decided to pursue the use of the FFT to calculate the response of damped panels.

The theory behind the approach and its implementation is outlined in the next two sections. The use of the technique will then be illustrated by reference to the response of a plane panel separated by a space from a hard backing. It will be seen that the insertion of a porous material into the space has a dramatic effect on the panel response, while, in contrast, the addition of damping to the panel itself has a negligible effect. This conclusion could not have been arrived at by using conventional damping theories.

2. THE RESPONSE OF LINE-EXCITED PANELS

The panel configurations considered here are shown in Figure 1. We consider one side of the panel to be exposed to a vacuum and the other side to be loaded by: (i) an acoustical half-space, (ii) a finite-depth acoustical medium backed by an acoustical half-space having distinct physical properties, and (iii) a finite-depth acoustical space backed by a rigid surface. The media adjacent to the panel are referred to here as "acoustical" since only longitudinal waves are imagined to propagate in them. As a result, these spaces are considered to couple to the panel through a distributed normal stress. The acoustical spaces may therefore be considered to represent either fluids or porous materials, such as fiber glass, in which motion of the solid phase has no significant effect on the material's acoustical properties [14,15]. The loading provided by elastic solids or elastic porous media will be considered elsewhere [16].

The response of an Euler-Bernoulli panel to a harmonic line input applied at x=0 is [1]

$$D\partial^4 w(x,t)/\partial x^4 + m_s \partial^2 w(x,t)/\partial t^2 = -p(x,0,t) + f(t)\delta(x) \tag{1}$$

where: w is the transverse panel displacement, D is the flexural bending stiffness per unit width, m_s is the mass per unit area, $p(x,z,t)$ is the acoustic pressure coupling the acoustical space to the panel, $f(t) = Fe^{-i\omega t}$ where F is the force magnitude per unit length applied to the panel, and $\delta(x)$ is the Dirac delta function. Equation (1) may be Fourier transformed in time to give

$$D\partial^4 W(x,\omega)/\partial x^4 - \omega^2 m_s W(x,\omega) = -P(x,0,\omega) + F\delta(x) \tag{2}$$

where: ω is the circular frequency, and $W(x,\omega)$ and $P(x,0,\omega)$ are the temporal Fourier transforms of $w(x,t)$ and $p(x,0,t)$, respectively. Equation (2) may then be Fourier transformed in space, i.e.,

$$(D\gamma^4 - \omega^2 m_s)\tilde{W}(\gamma,\omega) = -\tilde{P}(\gamma,0,\omega) + F \tag{3}$$

where: γ is the spatial circular frequency (i.e., the wavenumber), and $\tilde{W}(\gamma,\omega)$ and $\tilde{P}(\gamma,0,\omega)$ are the spatial Fourier transforms of $W(\gamma,\omega)$ and $P(x,0,\omega)$, respectively. By introducing the mechanical impedance of the panel, $Z_p = i[(D/\omega)\gamma^4 - \omega m_s]$, and making use of the fact that $-i\omega\tilde{W} = \tilde{V}$ where $\tilde{V}$ is the time and space transformed transverse panel velocity, equation (3) may be written more compactly as

$$Z_p\tilde{V}(\gamma,\omega) = -\tilde{P}(\gamma,0,\omega) + F. \tag{4}$$

Finally, the acoustic pressure may be eliminated by use of the normal acoustic impedance of the acoustic medium: i.e., $Z_a = \tilde{P}(\gamma,0,\omega)/\tilde{V}(\gamma,\omega)$. Note that the impedance may be expressed in terms of the panel normal velocity owing to the continuity of normal velocity at the panel surface [12]. Therefore, the transformed panel velocity (normalized with respect to the input force) may be written as

$$\tilde{V}_f(\gamma,\omega) = 1/(Z_a+Z_p) \tag{5}$$

where: $\tilde{V}_f(\gamma,\omega) = \tilde{V}(\gamma,\omega)/F$. The spatial response of the panel is found by inverse transforming this result to give $V_f(x,\omega) = V(x,\omega)/F$. If desired, the line input impedance of the panel may then be obtained from the latter as $Z_i = 1/V_f(0,\omega)$.

The acoustic impedance Z_a, provided by each configuration shown in Figure 1 is easily calculated [12]. For the acoustical half-space, $Z_a = \omega\rho/k_z$ where ρ is the density of the acoustical medium, $k_z = (k^2 - \gamma^2)^{1/2}$ and k is the acoustical wavenumber. Note that if the acoustic medium is a porous material, both k and ρ may be complex [14]. Further, when using the sign convention we have adopted here, it is necessary that $\mathrm{Re}[k_z]$, $\mathrm{Im}[k_z] \geq 0$. In the case when the loading is provided by a finite-depth acoustical layer backed by an acoustical half-space, the acoustical impedance is

$$Z_a = Z_1(\zeta_2 - i\tan k_z l)/(1 - i\zeta_2 \tan k_z l)$$

where: $Z_1 = \omega\rho_1/k_{1z}$, $\zeta_2 = Z_2/Z_1$, $Z_2 = \omega\rho_2/k_{2z}$, $k_{1z} = (k_1^2 - \gamma^2)^{1/2}$, $k_{2z} = (k_2^2 - \gamma^2)^{1/2}$, the subscripts 1 and 2 denote properties of the layer and backing space, respectively, and l is the layer depth. Note as before that $\mathrm{Re}[k_{1z}]$, $\mathrm{Im}[k_{1z}] \geq 0$ and $\mathrm{Re}[k_{2z}]$, $\mathrm{Im}[k_{2z}] \geq 0$. Finally, the acoustic impedance of a layer confined between a panel and a hard backing is $Z_a = iZ_c \cot k_z l$ where $Z_c = \omega\rho/k_z$, and all other parameters are as defined in connection with the acoustical half-space.

3. DISCRETE IMPLEMENTATION

The spatial response of the panel is found by taking an inverse transform of equation (5):

$$V_f(x,\omega) = (1/2\pi)\int_{-\infty}^{\infty} \tilde{V}_f(\gamma,\omega)\, e^{i\gamma x} d\gamma \;. \tag{6}$$

When approximated as a finite sum, equation (6) can be written as

$$V_f(k\Delta x,\omega) = (1/L\Delta x)\sum_{l=-L/2+1}^{L/2} \tilde{V}_f(l\,\Delta\gamma,\omega)\, e^{i2\pi lk/L} \tag{7}$$

where: Δx is the spatial sampling interval (equal to $2\pi/\gamma_s$ where γ_s is the spatial sampling frequency), $\Delta\gamma$

is the spatial frequency resolution (equal to γ_s/L), and L is the transform length. By making use of the implied periodicity of a sampled signal of finite duration [17], equation (7) can be rewritten as:

$$V_f(k\Delta x, \omega) = (1/L\Delta x) \sum_{l=0}^{L-1} \tilde{V}_f(l\,\Delta\gamma, \omega)\, e^{i2\pi lk/L} \quad . \tag{8}$$

Equation (8) is in the form of an IDFT, i.e.,

$$V_f(k\Delta x, \omega) = (1/\Delta x)\, IDFT\,[\tilde{V}_f(l\Delta\gamma, \omega)], \tag{9}$$

and can be evaluated using an FFT algorithm. We employ the convention that the IDFT is defined as [17]:

$$g(k) = IDFT\,[G(n)] = (1/K) \sum_{k=0}^{K-1} G(n)\, e^{i2\pi nk/K} \quad . \tag{10}$$

Note that some care must be exercised when using equation (8). The wavenumber spectrum level at the Nyquist frequency (i.e., one-half the sampling wavenumber) must be low enough to avoid any significant spectral truncation that can otherwise result in "ringing" in the space domain. If necessary, high wavenumber components may be windowed out of the wavenumber spectrum (an operation equivalent to lowpass filtering the spatial data). The latter approach produces an effect similar to an excitation of the structure by a force distributed over a finite width in the x-direction. In addition, the wavenumber spectrum must be sampled sufficiently often that all significant features are adequately resolved. This is especially true of lightly damped structures, in which case the use of double precision calculations is recommended. Lack of adequate sampling of the wavenumber spectrum is usually indicted by a spurious oscillation of the spatial response.

4. RESPONSE OF A PANEL BACKED BY A FINITE-DEPTH SPACE

To illustrate the usefulness of the approach described above, we will consider the response of a panel separated from a hard backing by a finite-depth space. Note that a similar configuration has been studied previously by Leung and Pinnington [18]; they considered the acoustical medium to be locally reacting rather than extended reaction as we do here.

In the present case, the appropriate acoustical impedance is $Z_a = iZ_c \cot k_z l$. The panel is imagined to be a 0.762 mm thick sheet of aluminium and the depth of the space is 6.35 mm. The Young's modulus is assumed to be complex in order to incorporate panel damping [2]: i.e., $D = D_m(1 - i\eta)$ where D_m and η are both real and η is the loss factor. The input impedance of the panel has been calculated both when the space is filled with air (at normal atmospheric conditions) and when the space is filled by a porous material whose acoustical properties can be defined by its flow resistivity [14]. In the former case, the wavenumber is made slightly complex to allow for losses in the air, i.e., $k = (\omega/c_0)(1 + i0.01)$; in the latter case, both the complex wavenumber and density have been calculated using the semi-empirical results of Delany and Bazley [14]. For the wavenumber: $k = \beta + i\alpha$, $\alpha = 0.189(\omega/c_0)f'^{-0.595}$ and $\beta = (\omega/c_0)(1 + 0.0978f'^{-0.700})$ where ρ_0 and c_0 are the ambient density and speed of sound of air, respectively, $f' = (\rho_0 f/\sigma)$, f is the frequency (in Hertz) and σ is the flow resistivity. The complex density may be calculated as $\rho = Zk/\omega$ where Z is the characteristic impedance of the porous material, and $Z = R + iX$ where $R = \rho_0 c_0(1 + 0.0571f'^{-0.754})$ and $X = 0.0870\rho_0 c_0 f'^{-0.732}$. The calculations presented below have been made using a spatial sampling frequency of 1000 rad/m and transform lengths of 4096 points.

Figure 2 shows the input impedance of the panel when air is assumed to fill the space between the panel and the hard backing; the panel loss factor was assumed to be 0.05. The major feature in this result is the mass-air-mass resonance [2] visible near 500 Hz. This effect is normally attributed to the mass of the panel resonating against the stiffness of the air trapped between the panel and the hard backing [2]. Simply increasing the panel loss factor has very little effect on this resonance, presumably since the frequency of interest is well below the panel's critical frequency (approximately 16 kHz). Figure 3 shows the input impedance when the loss factor is increased to 0.5; some effect of the damping is visible, but the net result is a very small improvement. Thus, application of a damping treatment to the panel would have little effect in this instance. However, if the space is filled with a porous material, the resonance may be effectively damped. Figure 4 shows the input impedance when the porous medium flow resistivity is 1×10^3 MKS Rayls/m while Figure 5 shows the result when the flow resistivity is raised to 10×10^3 MKS Rayls/m. Note that these flow resistivities fall towards the lower end of the range of fiber glass flow resistivities normally encountered in practice. It is evident that the addition of the fiber glass has caused the resonance to be effectively damped. The effectiveness of the fiber glass in reducing the structural response is evident from a comparison of Figures 6 and 7; both figures show the magnitude of the normalized transverse velocity, $|V_f(x,\omega)|$, plotted *versus* frequency and distance from the input force. Figure 6 shows the response when the space between the panel and the hard backing is air-filled and the panel has a loss factor of 0.5, and Figure 7 shows the response when the space is filled with a porous material having a flow resistivity of 10×10^3 MKS Rayls/m. The superiority of the latter treatment is obvious.

5. CONCLUSIONS

In this paper we have presented a method whereby the FFT algorithm may be used to predict the response of line-excited, damped panels. The technique provides a versatile and efficient alternative to classical methods of predicting the response of fluid-loaded panels. In addition, it has been shown that the panel loading may be accounted for by using an acoustical impedance. The latter quantity is easily calculated for a wide variety of layered treatments. The use of the technique has been demonstrated in a configuration in which damping provided by a layer of porous material adjacent to the panel proved much more effective at suppressing a resonance than the conventional approach of adding damping to the panel. It is expected that the present approach will prove useful in studying a variety of multi-element damping treatments that are not easily modeled using conventional damping theory. Future work will be directed at the extension of this technique to accommodate panel loadings provided by elastic solids and elastic porous materials.

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SPATIAL RESPONSE OF DAMPED PANELS

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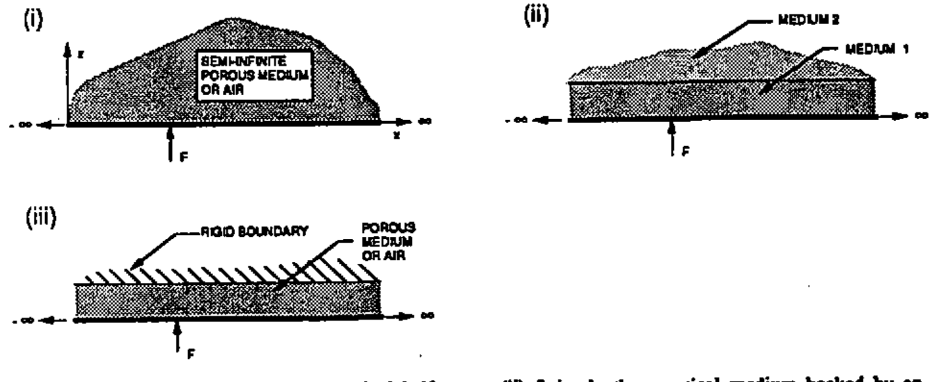

Figure 1. Panel configurations: (i) acoustical half-space, (ii) finite-depth acoustical medium backed by an acoustical half-space, (iii) finite-depth acoustical space backed by a rigid surface.

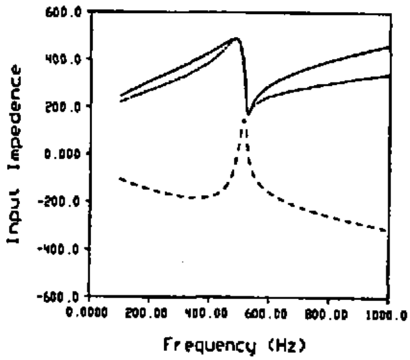

Figure 2. Input impedance of panel with air in space between the panel and the hard backing ($\eta = 0.05$). Solid line, magnitude; dotted line, real part; dashed line, imaginary part.

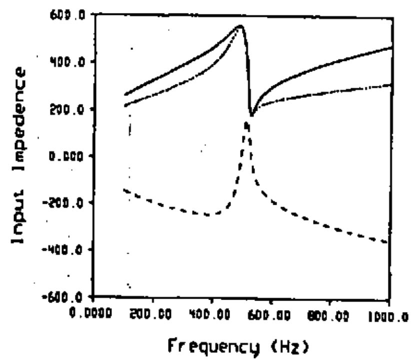

Figure 3. Input impedance of panel with air in space between the panel and the hard backing ($\eta = 0.5$). Solid line, magnitude; dotted line, real part; dashed line, imaginary part.

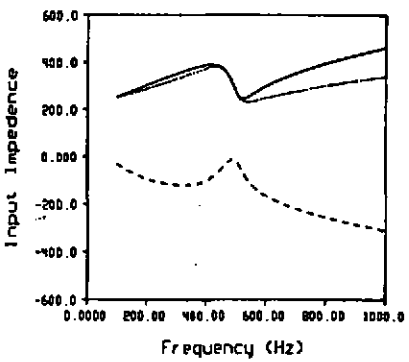

Figure 4. Input impedance of a panel with porous material in space between the panel and the hard backing ($\eta = 0.05$, $\sigma = 1x\,10^3$ *MKS Rayls/m*). Solid line, magnitude; dotted line, real part; dashed line, imaginary part.

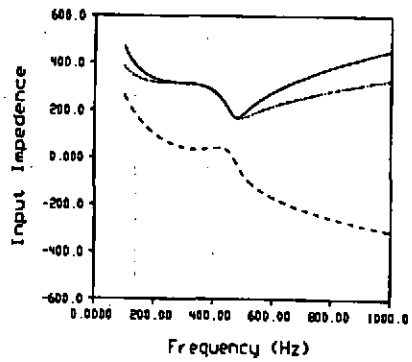

Figure 5. Input impedance of a panel with porous material in space between the panel and the hard backing. ($\eta = 0.05$, $\sigma = 10x\,10^3$ *MKS Rayls/m*). Solid line, magnitude; dotted line, real part; dashed line, imaginary part.

SPATIAL RESPONSE OF DAMPED PANELS

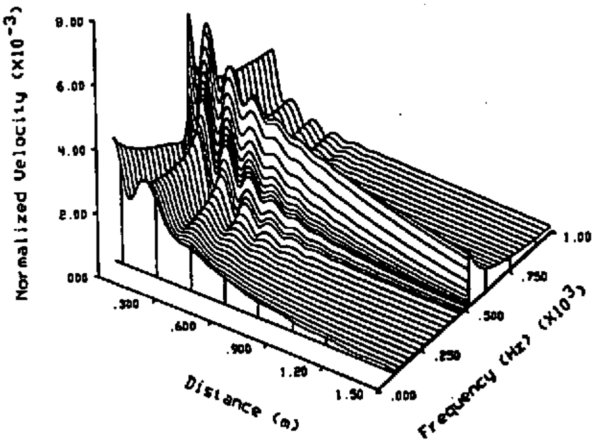

Figure 6. Normalized transverse velocity of panel: air in the space between the panel and the hard backing ($\eta = 0.5$).

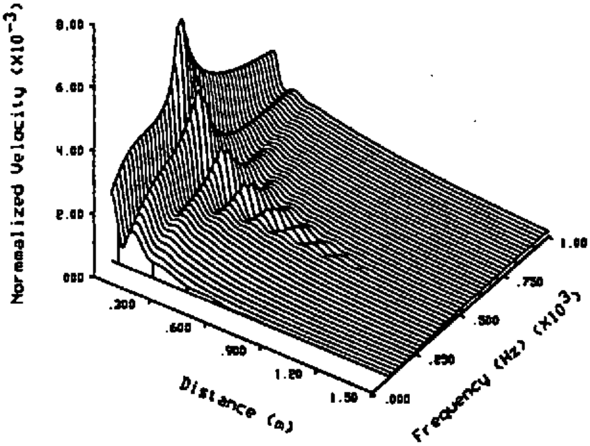

Figure 7. Normalized transverse velocity of panel: porous material in the space between the panel and the hard backing ($\eta = 0.05$, $\sigma = 10x\,10^3$ *MKS Rayls/m*).