APPLICATION OF SYSTEM MODELLING TO THE DESIGN OF DIGITAL SYSTEMS FOR ACTIVE CONTROL OF ACOUSTIC NOISE IN A DUCT

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INTRODUCTION

This paper contains a description of a series of experiments and calculations concerned with the design of a single channel active noise control system for an experimental duct. It is shown how the characteristics for the digital filters used to implement the ANC system were obtained by injecting various finite duration signals into the duct and capturing the responses for subsequent off-line processing. The filters themselves were implemented as a pair of oppositely directed FIR filters configured such that one acted as a feedback canceller while the other provided the characteristic required to cancel the propagating noise.

PRINCIPLES OF THE SYSTEM

Figure 1 shows a diagram of the experimental duct. Near the left hand end is a loudspeaker (PS) used as the primary noise source, then moving from left to right there is a (omnidirectional) microphone (D) to detect the incident sound, another loudspeaker (SS) to act as the secondary source, and another microphone (M) to measure the residual field. Though only the detector and the secondary source are strictly part of the ANC system the primary source loudspeaker and the monitor microphone, M, play a vital part in the system design.

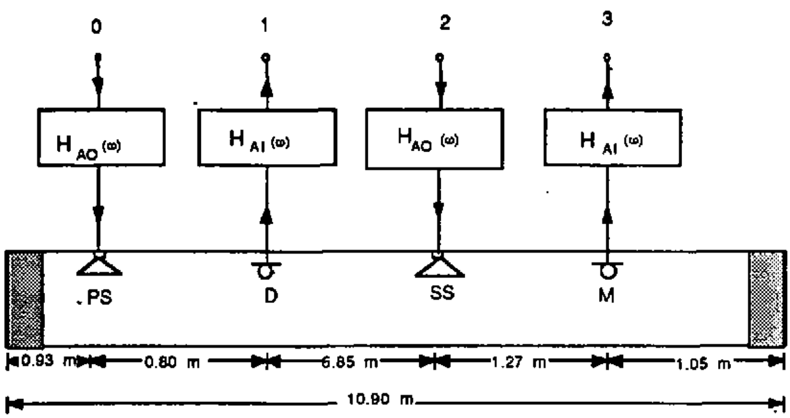

Figure 1. Diagram showing the arrangement of the transducers in the experimental duct.

APPLICATION OF SYSTEM MODELLING TO THE DESIGN OF DIGITAL SYSTEMS FOR ACTIVE CONTROL OF ACOUSTIC NOISE IN A DUCT

The ANC system consisted of two FIR filters connected in the arrangement shown in Figure 2, previously suggested by, amongst others, Poole et al [1]. This was connected between points 1 and 2 in Figure 1. The filter D_{FB} is intended to cancel the feedback from the secondary source to the detec-

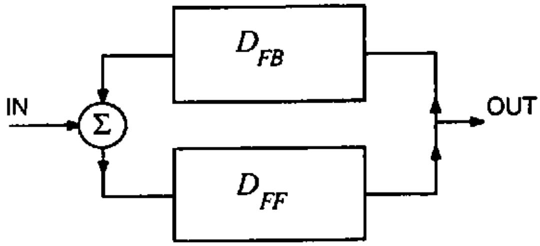

Figure 2. Configuration of the filters in the ANC system

tor while the filter D_{FF} is then chosen to be such that atteuation is achieved at the monitor microphone position.

The advantage of this approach over some others, such as using just a single feedforward filter or using a recursive filter of the conventional type where the feedforward part of the filter is cascaded with the recursive part, is twofold. The first advantage is that the role of each filter is clearly defined and directly related to a physical process; this also makes it relatively easy to deduce a series of measurements to be made on the real system from which the precise characteristics for each filter may be obtained. The second advantage is that this arrangement decouples the two important aspects of system stability and system attenuation performance. Provided care is taken in the design of D_{FB} to make sure that it is a good match for the feedback path, then the system eventually produced is bound to have a high degree of stability regardless of the match of the feedforward filter to that actually required to produce attenuation. Both the other approaches involve the coupling of stability and attenuation performance. In addition to this undesirable coupling a feedforward-only filter has the further disadvantage that it is not efficient in terms of the number of coefficients needed in the filter. For example to produce the correct characteristic using one FIR filter would require a very long filter indeed unless there is very substantial attenuation of sound travelling along the duct between the detector and the secondary source.

It is in principle possible to produce an ANC system, using the filter configuration described here, that will need only short responses in each filter even when the duct in which the system is to be installed is very reverberant.This, however, means that the feedback filter is *not* designed to cancel the feedback; once again, therfore, such an approach involves the stability of the system being crucially dependent on the characteristics of *both* constituent filters, and it also removes the easy physical interpretation of the role of each filter.

APPLICATION OF SYSTEM MODELLING TO THE DESIGN OF DIGITAL SYSTEMS FOR ACTIVE CONTROL OF ACOUSTIC NOISE IN A DUCT

SPECIFICATION OF THE FILTER RESPONSES

Defining $H_{nm}(z)$ as the z-transfer function between points n and m in Figure 1 the following relationships may be deduced for the ideal $D_{FB}(z)$ and $D_{FF}(z)$:

$$H_{01}(z)\,D_{FF}(z)H_{23}(z) = -H_{03}(z) \qquad (1)$$

and

$$D_{FB}(z) = -H_{21}(z) \qquad (2)$$

Hence

$$D_{FF}(z) = -H_{03}(z)/(H_{01}(z)\;H_{23}(z)) \qquad (3)$$

The procedure adopted in order to obtain FIR estimates of the filter characteristics was as follows. A rapid swept sine wave (0-350 Hz in 0.2 s), generated by a computer program, was stored in the TMS320 system used as the signal source and capture device. This signal was then output via the analogue interface denoted by H_{AO} in Figure 1 to the primary source PS. The signal $Y_{10}(z)$ received at 1 via the detector D and the analogue interface H_{AI}, and the signal $Y_{30}(z)$ received at 3 via the monitor and its associated analogue interface were both captured and stored. Next the signal $Y_{10}(z)$ was reinjected into the system at point 2 and the resulting signal at point 3, $Y_{32}(z)$, captured. Finally the original swept sine wave was injected at point 2 and the signal received at point 1, $Y_{12}(z)$, was captured. Denoting the original swept sine wave as $Y_{SS}(z)$, the following relationships between the captured signals and the transfer functions may be written down:

$$H_{01}(z) = Y_{10}(z)/Y_{ss}(z) \qquad H_{03}(z) = Y_{30}(z)$$
$$H_{23}(z) = Y_{32}(z)/Y_{10}(z) \qquad H_{21}(z) = Y_{12}(z),$$

and substituting these into (2) and (3) leads to

$$D_{FB}(z) = \frac{-Y_{12}(z)}{Y_{SS}(z)} \qquad (4)$$

$$D_{FF}(z) = \frac{-Y_{30}(z)\,Y_{SS}(z)\,Y_{10}(z)}{Y_{SS}(z)\,Y_{10}(z)\,Y_{32}(z)} = \frac{-Y_{30}(z)}{Y_{32}(z)} \qquad (5)$$

Note how, owing to the careful choice of signals, the z-tranform of each filter response is given by the ratio of the z-tranforms of just two signals. The importance of this is that it means that in the time domain the impulse response of each of the digital filters is equal to just the deconvolution of two of the captured signals.

An algorithm to do this convolution is required, and the one that was adopted for the work described here is that due to Marple [2], which involves a recursive least squares fit in the time domain to obtain an estimate of the deconvolution. The advantage of this approach over, for example, simply taking discrete Fourier transforms of the signals, dividing these as appropriate, and then taking the

APPLICATION OF SYSTEM MODELLING TO THE DESIGN OF DIGITAL SYSTEMS FOR ACTIVE CONTROL OF ACOUSTIC NOISE IN A DUCT

inverse transform, is that it provides a best fit in a least squares sense for a filter of any desired length and the residual error provides a direct indication of the goodness of fit obtained. Early tests using a program based on that in [2] showed some discrepancies until the program was modified to use double precision variables throughout.

RESULTS

Examples of some of the captured signals and their frequency spectra are shown in Figure 3.

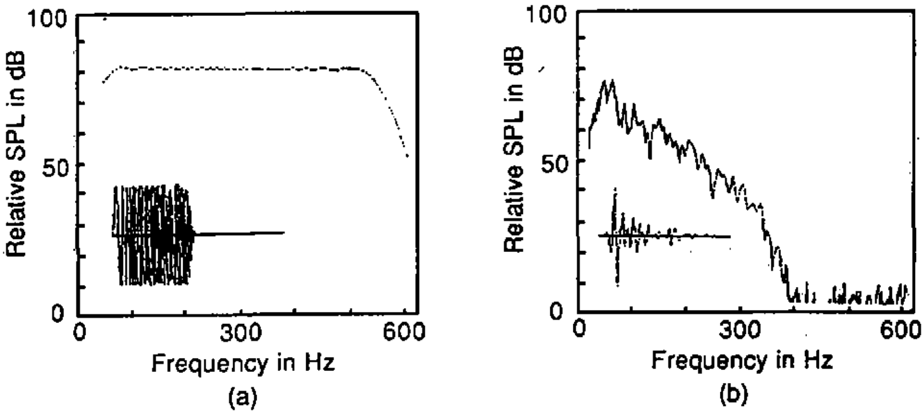

Figure 3. Time series and frequency spectrum of
(a) swept sine wave
(b) captured signal Y_{32}

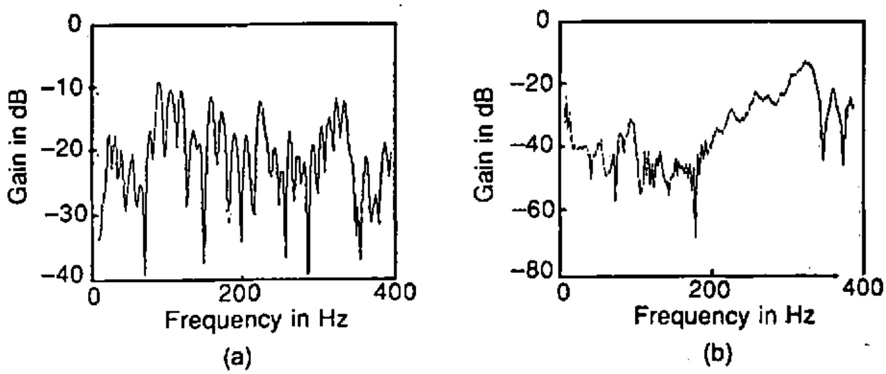

Figure 4. Open loop gain of the ANC system for filter lengths of:
(a) 64 and 64
(b) 256 and 256

APPLICATION OF SYSTEM MODELLING TO THE DESIGN OF DIGITAL SYSTEMS FOR ACTIVE CONTROL OF ACOUSTIC NOISE IN A DUCT

The stability of a system using filters of given lengths was investigated by predicting, on the basis of the previously-captured signals, the open loop gain of the ANC system when installed in the duct. Examples of some results so obtained are shown in Figure 4 for two different pairs of filter lengths, (64 and 64) and (256 and 256). It can be seen that even with the shorter filter pair the open loop gain was around –10 dB over most of the frequency range, with the longer pair the gain was mostly around –40 dB, though at the top end of the system passband (the system was designed to work up to a frequency of around 350 Hz) the gain rises to about –15dB. The fact that these figures were substantially less than 0 dB implied that, regardless of phase of the transfer functions, the overall system would be comfortably stable.

Finally Figure 5 shows the extent to which the predicted attenuation depended on the filter length.

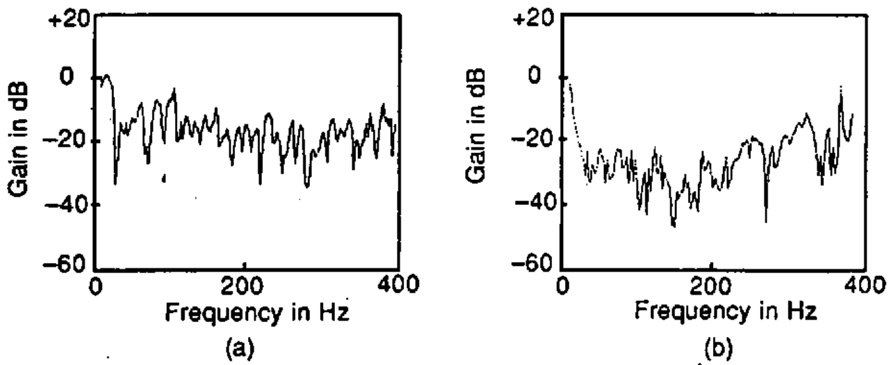

Figure 5. Graphs showing the attenuation performance of the ANC system for filter lengths of: (a) 64 and 64 (b) 256 and 256

The results for the shorter filter pair show rather poor performance, giving typically only around 10 dB attenuation, but for the (256 and 256) filters the attenuation was about 25 dB over most of the system bandwidth.

Results were also obtained for various different end terminations on the duct, altering the amount of reverberation within it and hence the effective lengths of the various impulse responses, and showing the extent to which the achievable performance depended on the filter lengths.

CONCLUSIONS

A method of designing an ANC system has been described that has the major advantage that it may be used to assess the feasibility of using an ANC system to control a real noise problem. All that is needed is to make a series of measurements on site analogous to those described above: these may then be analysed in the laboratory firstly to discover if it is even possible to obtain the required per-

APPLICATION OF SYSTEM MODELLING TO THE DESIGN OF DIGITAL SYSTEMS FOR ACTIVE CONTROL OF ACOUSTIC NOISE IN A DUCT

formance, and secondly to specify the complexity of digital filters that will be required. Such information is extremely useful even when it is intended to use adaptive filters to realise the system, as it is necessary to know in advance the filter sizes that will be needed in order to specify the hardware for the filter implementation.

ACKNOWLEDGMENTS

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SIMULATION OF THE BEHAVIOUR OF THE ERIKSSON ADAPTIVE NOISE CONTROL SYSTEM IN A REVERBERANT DUCT

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INTRODUCTION

This paper describes the results of some computer simulations carried out to investigate the behaviour of the active noise control system described by Eriksson et al [1]. This system has the advantage over previously described adaptive schemes that, although two filters are being adapted, since only one error signal is used as input to the adaptive algorithm, both adaptations can take place simultaneously. It is based on the use of an adaptive recursive filter and it is shown here that the values to which the coefficients of this filter converge depend upon the number of coefficients permitted to each of the two constituent FIR filters making up the recursive filter, and also upon the amount of reverberation present in the initial sound field. However a study of the pole-zero plots corresponding to these various different filters show that they are in fact all equivalent to each other.

THE ERIKSSON ANC SYSTEM

A block diagram of the component parts of a single channel active noise control system of the type described in [1] is shown in Figure 1.

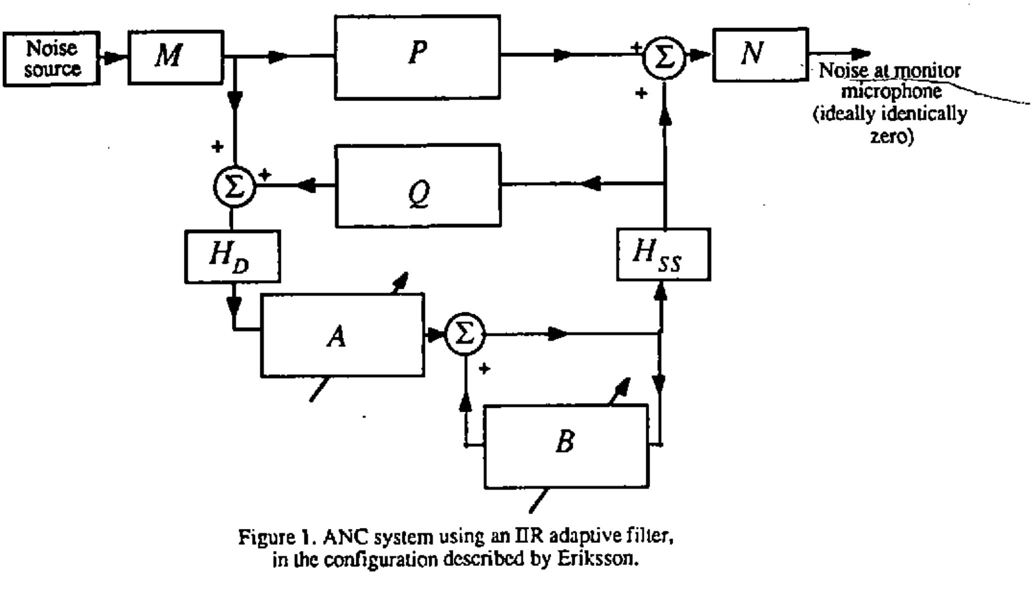

Figure 1. ANC system using an IIR adaptive filter, in the configuration described by Eriksson.

SIMULATION OF THE BEHAVIOUR OF THE ERIKSSON ADAPTIVE NOISE CONTROL SYSTEM IN A REVERBERANT DUCT

Noise from the primary source propagates via the transfer function M to the detector microphone. The noise continuing down the duct then travels via the transfer functions P and N to the monitoring microphone. Meanwhile the noise at the detector is converted to an electrical signal, via the transfer function of the microphone H_D, before being fed into the ANC system. The ANC system itself is realised as an adaptive recursive filter made up from two constituent FIR filters A and B. The filter output then travels via the secondary source transfer function H_{SS} back into the duct. It then travels two ways: downstream via the duct transfer function N to the monitor where it is seen superposed on the direct sound that has travelled via the duct only; and upstream in the direction of the primary source via the transfer function Q back to the detector.

The two adaptive filters making up the recursive filter are both adapted simultaneously, using the Widrow-Hoff LMS algorithm, using as inputs to the algorithm the error signal received at the monitor together with the respective inputs to the filters. This method of adapting an IIR filter was suggested over 10 years ago by Feintuch [2] but never seems to have come into widespread use. This may well be on account of the correspondence [3,4] that immediately followed its publication, showing that it could not be guaranteed to find the global minimum of the error function owing to the potentially multimodal error surface possessed by a recursive filter. The cloud that therefore hangs over the algorithm, leading one to question its usefulness, was one of the main reasons for embarking on the simulations described in this paper. It was hoped to discover whether it was indeed capable of forming the basis of a reliable ANC system.

TRANSFER FUNCTIONS OF DUCT ANC SYSTEMS

The condition for perfect operation of the ANC system is that, regardless of the output from the primary noise source, the signal received at the monitor should be identically zero. The overall transfer function of the ANC system in figure 1 is equal to $H_D H_{SS} A/(1-B)$ and hence that of the combination of this with Q is equal to $[H_D H_{SS} A/(1-B)]/\{1 - Q[H_D H_{SS} A/(1-B)]\}$. Thus the condition for perfect cancellation is that

$$P = \frac{-H_D H_{SS} A/(1-B)}{1 - Q[H_D H_{SS} A/(1-B)]}$$

which after rearrangement to separate the adjustable from the fixed parts gives the following equation specifying A and B:

$$\frac{A}{1-B} = \frac{1}{H_D H_{SS}} \frac{-P}{1-PQ} \qquad (1)$$

For the rest of this paper it will be assumed, in order to make the equations shorter, that H_D and H_{SS} are both equal to unity; if it is desired to keep these quantities explicit it is an elementary task to go through the equations inserting them in the appropriate places.

SIMULATION OF THE BEHAVIOUR OF THE ERIKSSON ADAPTIVE NOISE CONTROL SYSTEM IN A REVERBERANT DUCT

In order to explore the implications of equation (1) further it is necessary to ask about the nature of the transfer functions P and Q. In an anechoic duct this is easy as both of them are (neglecting propagation losses along the duct) just pure delays of a time equal to the propagation time between the detector and the secondary source, i.e. $P = Q = e^{-j\omega l/c}$ where ω is the angular frequency, l is the distance between the detector and the secondary source, and c is the speed of sound in the the duct.

This in turn means that $A/(1 - B)$ must be equal to $\dfrac{-e^{-j\omega l/c}}{1 - e^{-2j\omega l/c}}$ for perfect cancellation to occur.

A plot of the poles of this function in the z-plane is shown in Figure 2, assuming the delay time to be equal to 3 sample delays. In addition to the poles shown (the six sixth roots of 1) the function also has three zeros; these are not shown on the plot as they are the three cube roots of infinity!

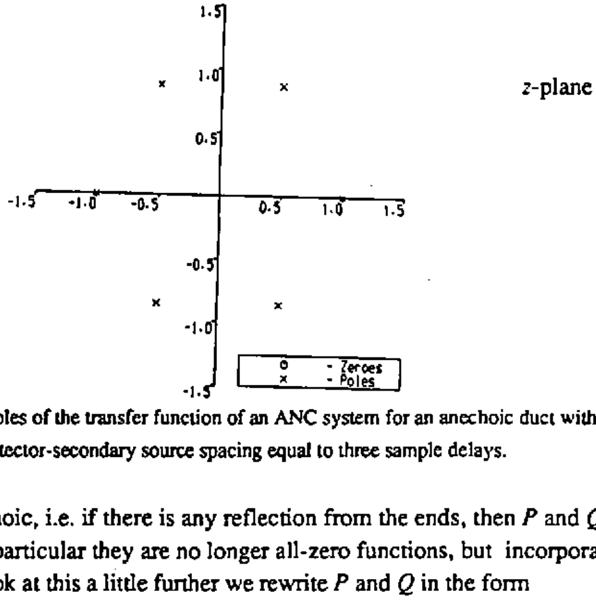

Figure 2. Poles of the transfer function of an ANC system for an anechoic duct with detector-secondary source spacing equal to three sample delays.

If the duct is not anechoic, i.e. if there is any reflection from the ends, then P and Q become much more complicated. In particular they are no longer all-zero functions, but incorporate poles as well as zeros. In order to look at this a little further we rewrite P and Q in the form

$$P = \frac{P_z}{1 - P_p} \quad \text{and} \quad Q = \frac{Q_z}{1 - Q_p} \qquad (2)$$

i.e. we look upon both of them as IIR systems of the type shown in Figure 3. Hence

$$\frac{P}{1 - PQ} = \frac{P_z(1 - Q_p)}{(1 - P_p)(1 - Q_p) - P_z Q_z} \qquad (3)$$

SIMULATION OF THE BEHAVIOUR OF THE ERIKSSON ADAPTIVE NOISE CONTROL SYSTEM IN A REVERBERANT DUCT

and this can be rearranged into the form

$$\frac{P}{1-PQ} = \frac{P_z(1-Q_p)}{1-(P_zQ_z+P_p+Q_p-P_pQ_p)} \qquad (4)$$

It follows therefore that one possible pair of choices for A and B that will satisfy equation (1) is

$$A = -P_z(1-Q_p) \qquad (5)$$

and $$B = (P_zQ_z + P_p + Q_p - P_pQ_p). \qquad (6)$$

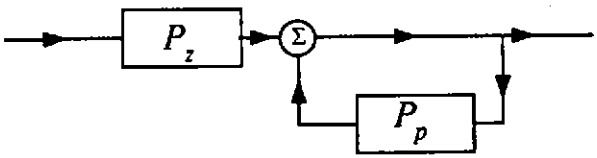

Figure 3. Form of the acoustic transfer functions when assumed to have poles as well as zeros.

Expressions for P_z, Q_z, P_p and Q_p in terms of the duct geometry and end reflection coefficients may be obtained from [5]. Q has zeros, in addition to those for the anechoic case, that arise due to the first reflection from each end and also poles that arise from the reverberation in the duct, while P has zeros that exactly correspond to the poles of Q as well as two sets of poles associated with the first reflection from each of the two ends. This is because (see, for example, equation (3) of reference [6]) P is the ratio of one in-duct transfer function divided by the product of two more such functions, and all of these have the same form as Q; hence the general pole-zero nature of P is the opposite of that of Q. Hence it is possible to calculate finite length z transfer functions for each of P_z, Q_z, P_p and Q_p, given values for the duct lengths and reflection coefficients, and hence, using (5) and (6) to obtain values for A and B which will satisfy the requirement to provide perfect cancellation.

However it is also known that the transfer function for an ANC system in a reverberant duct is the *same* as for an anechoic duct, so the question arises as to how these two apparently conflicting results for A and B can be reconciled, as (5) and (6) do not evaluate to simply $-e^{-j\omega l/c}$ and $e^{-2j\omega l/c}$ respectively. The answer to this question may be discerned by looking at Figure 4 which is a pole-zero plot of $A/(1-B)$ where A and B have been evaluated using (5) and (6) for a duct with the following specification:

time delay from end of duct to primary source	1 sample period
time delay from primary source to detector	1 sample period
time delay from detector to secondary source	3 sample periods
time delay from secondary source to monitor	2 sample periods
time delay from monitor to end of duct	2 sample periods
reflection coefficients of the ends of the duct	0.2.

SIMULATION OF THE BEHAVIOUR OF THE ERIKSSON ADAPTIVE NOISE CONTROL SYSTEM IN A REVERBERANT DUCT

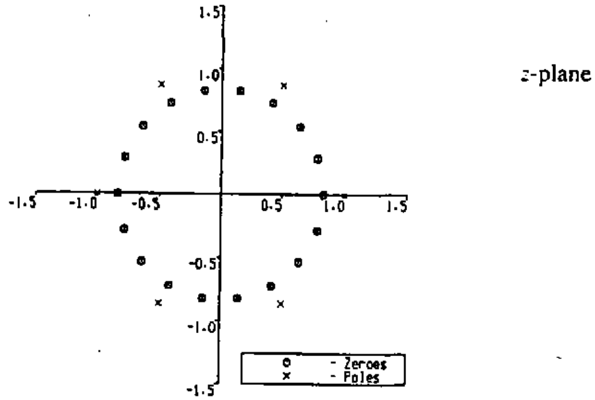

Figure 4. Poles and zeros of $A/(1 - B)$ for the duct specified above, calculated from equations (5) and (6).

It can be seen from this figure that, despite the apparently different nature of the transfer function from the simple one for the anechoic duct, the pole-zero plots are very closely related. All of the six poles shown in Figure 2 are here, as also are the (necessarily unplotted) three zeros at the cube roots of infinity, but there are also apparently eighteen more poles and eighteen more zeros. In fact, however, there are thirty six extra poles and thirty six extra zeros as all of these poles and zeros are double, but as the extra poles and zeros are exactly paired with each other they do not have any effect on the overall transfer function; it, therefore, is exactly the same as that represented by Figure 2.

PERFORMANCE OF THE ADAPTIVE ALGORITHM

Having now gained some knowledge about the ideal solution, we are in a position to study intelligently the behaviour of the adaptive algorithm when faced with the ANC problem. Figure 5 shows the convergence behaviour of the algorithm for the anechoic case, with a three sample period delay beween detector and secondary source, using four forward and seven feedback coefficients (the minimum possible) in the adaptive filter. It can be seen, like the essentially similar Figure 2(b) in [1], that there is steady convergence. The floor of around −100 dB is set by the numerical accuracy of the computer used for the simulation. When the poles and zeros of the adapted filter were inspected the poles proved to be exactly where they would be expected, namely at the six sixth roots of unity. The three zeros which should be at infinity were approximately (but not exactly) equally spaced near a circle in the z-plane, as compared with the exactly equal spacing of cube roots round such a circle. The distance of these zeros from the origin was approximately 219, which, relative to unity, is a fair approximation to infinity.

SIMULATION OF THE BEHAVIOUR OF THE ERIKSSON ADAPTIVE NOISE CONTROL SYSTEM IN A REVERBERANT DUCT

Moving on to look at the behaviour of the algorithm when reverberation is present in the duct, Figure 5 also shows the convergence behaviour when adapting to the reverberant duct previously described. The result of two different length models are shown, the four-forward/seven-feedback that is known to be the minimum necessary and also a forty-feedforward/forty three-feedback that corre-

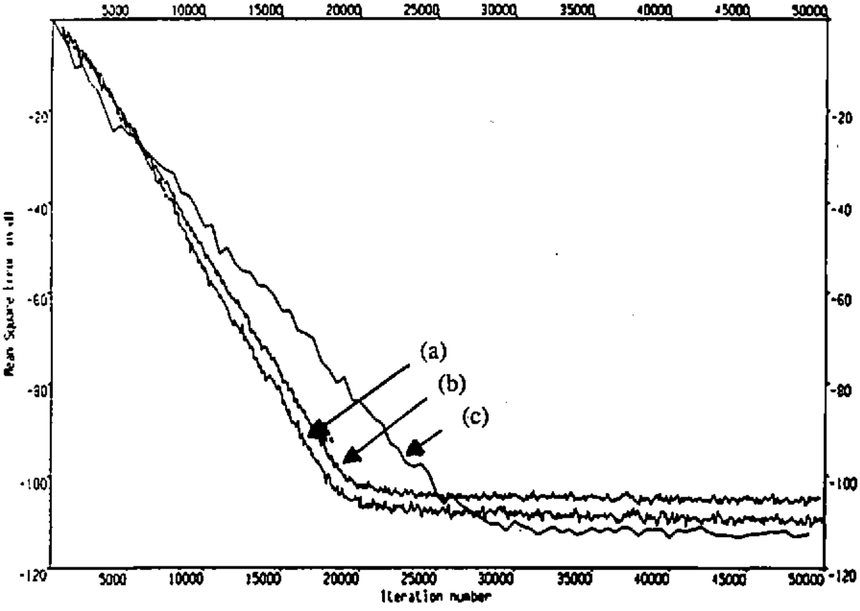

Figure 5. Plot of the mean square error v. iteration number for a recursive LMS filter based ANC system
(a) anechoic duct, filter length 4 by 7,
(b) reflective duct, filter length 4 by 7,
(c) reflective duct, filter length 40 by 43.

sponds to the lengths of A and B when calculated using equations (5) and (6). As can be seen the overall convergence behaviour is similar in both cases, and is also similar to that for the anechoic case. The longer filter seems to converge marginally faster at the start, but then slows down relative to the shorter one. Even so the difference in the time to reach maximum convergence is only about 30%.

The pole-zero plots corresponding to the adapted filter coefficients in each case were inspected. For the short filter they were, as expected, essentially the same as those for the same length filter modelling the anechoic duct. The pole-zero plot for the 40 by 43 filter is shown in Figure 6. It is interesting to compare this with Figure 4, which corresponds to the same number of potentially non-zero

SIMULATION OF THE BEHAVIOUR OF THE ERIKSSON ADAPTIVE NOISE CONTROL SYSTEM IN A REVERBERANT DUCT

coefficients in the filter. Once again there are the six poles at the sixth roots of unity as well as three zeros (not shown on the plot) well outside the unit circle. There are also thirty six paired poles and zeros, but this time they are not double poles and double zeros and they do not all lie on a circle in the z-plane, though they are spread around (in conjugate pairs) something approaching such a circle.

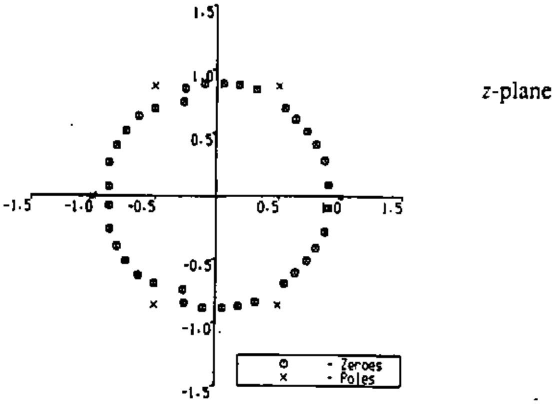

Figure 6. Poles and zeros of the transfer function of the ANC system after adaption to the reverberant duct described above.

CONCLUSIONS

The convergence behaviour of the adaptive ANC system described by Eriksson has been investigated for some fairly simple duct situations. The algorithm, the performance of which seems from the literature to be open to doubt, appears to cope well with the presence of slight reverberation, resulting in adapted transfer functions that are very close to the ideal, whether the adaptive filters used are short or long. Initial indications of the performance in the presence of considerable reverberation, however, are less promising, and this is currently being looked at in more detail.

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