AEROACOUSTIC DESIGN OF METAFLUID DEVICES

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The present paper examines different methods for the design of acoustic metamaterial devices required to operate in presence of an aerodynamic flow. The ultimate goal of the research is the exploitation of the acoustic properties of acoustic metafluids in the aeronautical context. Particular attention is paid to the development of innovative devices capable to modify the propagation pattern of aviation noise to mitigate its effect on the population. Possible applications are a new generation of liners to achieve the virtual scarfing of nacelle intakes or special surface treatments to enhance the shielding of engine noise by wings and fuselage. The paper is focused on aerodynamic flows at Mach number not higher than 0.3, which is compatible with the take-off and landing conditions of commercial aircraft. Different approaches are used to define appropriate corrections of static metamaterial designs to recover, partially or completely, the efficiency of the meta-response lost as a consequence of the aerodynamic convection. All the approaches are based on classic aeroacoustic coordinate transformations, suitably revisited and adapted to specific application at hand. All the corrections presented are independent on the technique used in the static design of the device. Preliminary numerical results are obtained in the scattering abatement (cloaking) of obstacle with simple geometries. The numerical simulations are obtained using an original, general integral formulation of the problem, solved using an extended boundary element method.

Keywords: metamaterials, cloaking, transformation acoustics, Boundary Element Method.

1. Introduction

Starting from the observation by Cummer and Schurig [1] about the formal analogy between mass and momentum equations for an inviscid fluid at rest under small pressure perturbation and the single polarization Maxwell's equations, the concepts and methods used by Pendry *et al.* [2] and Leonhardt [3] to achieve electromagnetic invisibility of objects could be directly ported into the acoustics domain. That was the birth of the so–called Standard Transformation Acoustics (STA) approach.

This created a major interest by the research community during the last decade on the development of highly innovative acoustic devices that exploited the principles of acoustic cloaking, i.e. the total abatement of the scattering effects induced by an obstacle, hyper-focusing or arbitrary reshaping of wavefronts. The approach, imported from electromagnetism, involves the exploitation of governing equations formal invariance under coordinate transformations and the reinterpretation of the arising coefficients as mechanical properties of an ideal material suitable to obtain the desired modification of the scattered field. A detailed review of the enormous literature produced on the topic is far beyond the scope of this paper, nevertheless it worth mentioning the key work by Norris [4], who developed the details of the acoustic cloaking theory based on coordinate transformations and the papers by Cai and Sanchez-Dehesa [6] and Torrent and Sanchez-Dehesa [7] who illustrated how to practically realize such cloaking devices through homogenization of layered materials to achieve the required anisotropic behaviour of the ideal material.

One of the main limitations of the Standard Transformation Acoustic approach lies in its non applicability in case of acoustic propagation in a moving medium, or in presence of moving obstacles in a fluid at rest, as the formal invariance of the equations under coordinate transformation is lost in presence of a background aerodynamic flow. Recently the issue has been faced, for example, in Garcia-Meca *et al.* [8], where the authors introduced a novel approach called Analogue Transformation Acoustic. However, despite the claimed capability to extend the range of applications to the aeroacoustic domain, the described method requires a transformed background velocity inside the cloak, which implies a not easy to achieve aerodynamic permeability of such device. Furthermore, in Huang [11] the author derives a correction factor for the metamaterial parameters to obtain cloaking capability in presence of a moving medium surrounding an object at rest, but the assumptions of planar wave fronts impinging the obstacle and the low speed of the medium represent a strong limitation that prevents the application of the proposed approach in aeronautics.

Trying to overcome the above limitations, in this paper the Taylor's coordinate transformation is used to obtain different corrections to the metafluid design, aimed at recovering, at least partially, the meta-device efficiency in presence of aerodynamic flows at M ≤ 0.3 and sound sources at a close distance to the obstacle. In particular, preliminary numerical results are obtained for the classic cloaking problem of a circular obstacle using an original integral formulation of the problem, firstly presented in Iemma and Burghignoli [12] and then developed in Iemma [13], solved with a zeroth–order extended Boundary Element Method. It is important to point out that all the proposed corrections are independent on the specific technique used in the metamaterial static design.

The paper is structured as follows: in Section 2 the theory of acoustic metafluid and cloaking is briefly recalled and the integral formulation valid in presence of a background flow is then outlined. Taylor's transform is presented in Section 3 and its applications in the present context are shown. Results of the performed numerical simulations can be found in Section 4.

2. Moving metafluid devices

A basic assumption made in this paper is that the domain occupied by the metamaterial is aerodynamically not permeable, *i.e.* no flow is present inside the meta–device.

According to Norris [5], the most general form of the scalar equation governing the propagation of an acoustic perturbation within an acoustic metamaterial (or *metafluid*) has the form

$$\mathcal{K}\,\mathbf{Q}\colon \nabla\left(\boldsymbol{\varrho}^{-1}\,\mathbf{Q}\,\nabla p\right) - \frac{\partial^2 p}{\partial t^2} = 0 \tag{1}$$

The above equation encompasses all the characteristics of a metamaterial capable of an arbitrary pseudo–acoustic behavior. Both density and stiffness are assumed to be anisotropic, represented by the tensors $\boldsymbol{\varrho}$ and $\mathcal{K}\mathbf{Q}$, respectively, where $\mathbf{Q}$ can be any tensor such that $\nabla \cdot \mathbf{Q} = 0$. The Cauchy stress tensor for such a material is given by $\boldsymbol{\sigma} = -p\,\mathbf{Q}$, where the pseudo–pressure is related to the strain tensor $\boldsymbol{\epsilon}$ by $p = -\mathcal{K}\,\mathbf{Q}\colon \boldsymbol{\epsilon}$. With the underlying assumption that the acoustic meta–behaviour can be obtained introducing fictitious sources, related to the metafluid properties, that distort the free propagation within a reference medium of density ϱ_0 and bulk modulus $\mathcal{K}_0$, Eq. 1 can be rewritten, extending the acoustic analogy introduced in Iemma and Burghignoli [12], as

$$\nabla^2 p - \frac{\varrho_0}{\mathcal{K}_0}\frac{\partial^2 p}{\partial t^2} = \mathbf{I}\colon \nabla\left(\nabla p\right) - \mathbf{Q}\colon \nabla\left(\hat{\boldsymbol{\varrho}}^{-1}\mathbf{Q}\nabla p\right) + \varrho_0\left(\frac{1}{\mathcal{K}_c} - \frac{1}{\mathcal{K}_0}\right)\frac{\partial^2 p}{\partial t^2} \tag{2}$$

and, recalling that $\mathbf{Q}$ is by definition a symmetric, divergence–free tensor, we can write

$$\nabla^2 p - \frac{\varrho_0}{\mathcal{K}_0}\frac{\partial^2 p}{\partial t^2} = \nabla \cdot \mathbf{q}\left(\mathbf{x}, t\right) + \sigma\left(\mathbf{x}, t\right) \tag{3}$$

where

$$\mathbf{q}\,(\mathbf{x},t) = \nabla p - \mathbf{Q}\hat{\boldsymbol{\varrho}}^{-1}\mathbf{Q}\nabla p, \qquad \sigma\,(\mathbf{x},t) = \varrho_0\left(\frac{1}{\mathcal{K}_c} - \frac{1}{\mathcal{K}_0}\right)\frac{\partial^2 p}{\partial t^2}, \qquad \hat{\boldsymbol{\varrho}} = \boldsymbol{\varrho}/\varrho_0 \tag{4}$$

Fourier transforming Eq. 3 we can obtain the generalized non homogeneous Helmholtz equation governing the propagation within the reference medium of the perturbation induced by the sources in Eq. 4.

$$\begin{aligned} E_c(\mathbf{y})\,\tilde{p}(\mathbf{y},k) = \oint_{\Gamma_c}\left[G(\mathbf{x},\mathbf{y},k)\,\left(\frac{\partial\tilde{p}(\mathbf{x},k)}{\partial n} - \tilde{\mathbf{q}}(\mathbf{x},k)\cdot\mathbf{n}\right) - \tilde{p}(\mathbf{x},k)\,\frac{\partial G(\mathbf{x},\mathbf{y},k)}{\partial n}\right]\mathrm{d}\Gamma(\mathbf{x}) \\ - \int_{\Omega_c}\tilde{\mathbf{q}}(\mathbf{x},k)\cdot\nabla G(\mathbf{x},\mathbf{y},k)\,\mathrm{d}\Omega(\mathbf{x}) + \int_{\Omega_c}G(\mathbf{x},\mathbf{y},k)\,\tilde{\sigma}(\mathbf{x},k)\,\mathrm{d}\Omega(\mathbf{x}), \qquad \mathbf{y}\in\Omega_c \end{aligned} \tag{5}$$

where $\tilde{}$ indicates the Fourier transform, $k = \omega/c_0$, G is the free-space Green function for the reference medium, Ω_c is the domain occupied by the metafluid and Γ_c is its boundary. For devices impinged by a moving medium at undisturbed uniform velocity $\mathbf{v}_0$, Eq. 5 can be coupled with the convective wave equation for the velocity potential $\varphi(\mathbf{x},t)$, that governs the propagation of an acoustic perturbation in the hosting domain Ω_h

$$\nabla^2\varphi - \frac{1}{c_0^2}\left(\frac{\partial}{\partial t} + \mathbf{v}_0\cdot\nabla\right)^2\varphi = 0 \qquad \mathbf{x}\in\Omega_h \tag{6}$$

that can be reformulated into a boundary integral equation in frequency domain as

$$\begin{aligned} E_c(\mathbf{y})\,\tilde{\varphi}(\mathbf{y},k) = \oint_{\Gamma_h}\left[\hat{G}(\mathbf{x},\mathbf{y},k)\,\left(\frac{\partial\tilde{\varphi}(\mathbf{x},k)}{\partial n} - M_0^n\boldsymbol{M}_0\cdot\nabla\varphi(\mathbf{x},k)\right)\right]\mathrm{d}\Gamma(\mathbf{x}) \\ + \oint_{\Gamma_h}\left[\tilde{\varphi}(\mathbf{x},k)\left(2ik_0M_0^n\hat{G}(\mathbf{x},\mathbf{y},k) + M_0^n\boldsymbol{M}_0\cdot\nabla\hat{G}(\mathbf{x},\mathbf{y},k) - \frac{\partial\hat{G}(\mathbf{x},\mathbf{y},k)}{\partial n}\right)\right]\mathrm{d}\Gamma(\mathbf{x}) \quad \mathbf{x}\in\Omega_h \end{aligned} \tag{7}$$

Equations 5 and 7 are coupled by imposing the continuity of particles' acceleration at the interface between the host and cloak media, noting that the unit normals to the boundaries are counter–signed

$$\left.\frac{\partial\tilde{p}}{\partial n}\right|_{\mathbf{x}\in\Gamma_h} = -\,\varrho_0\left(\boldsymbol{\varrho}^{-1}\,\nabla\tilde{p}\right)\cdot\mathbf{n}\big|_{\mathbf{x}\in\Gamma_c} \tag{8}$$

3. Taylor's coordinate transformation

Taylor's coordinate transformation was conceived in [14] to take into account effects of aerodynamic convection on acoustic propagation and to correct acoustic measurements in presence of a low–speed flow (motion). With the assumption of a steady homentropic potential background aerodynamic flow, the $\mathcal{O}(M)$ approximation of the governing equation for an acoustic disturbance is

$$c_0^2\nabla^2\varphi - 2\,\frac{\partial}{\partial t}\left(\nabla\Phi\cdot\nabla\varphi\right) - \frac{\partial^2\varphi}{\partial t^2} = s(\mathbf{x},t) \tag{9}$$

where Φ is the aerodynamic potential and φ is the acoustic velocity potential, related to the acoustic pressure perturbation through the linearized Bernoulli theorem $p = -\varrho_0\left(\dot{\varphi} + \nabla\Phi\cdot\nabla\varphi\right)$. Taylor's transformation is defined as a spatial–dependent time shift

$$(\bar{\mathbf{x}},\bar{t}\,) = T(\mathbf{x},t) = \left(\mathbf{x}, t + \frac{\Phi(\mathbf{x})}{c_0^2}\right) \tag{10}$$

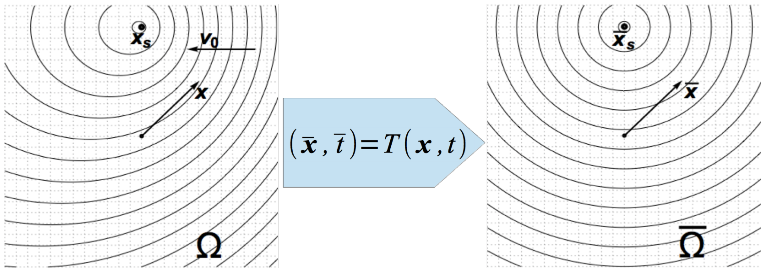


Figure 1: Effect of the Taylor's coordinate transformation in presence of point source in $\mathbf{x}_s \in \Omega$ surrounded by a uniform stream at speed $\mathbf{v}_0$. In the virtual space $\bar{\Omega}$ the wave fronts (black curves) recover the isotropic propagation pattern of the stationary case.

and, applied to 9, recasts it in form of the classic wave equation, when $\mathcal{O}(M^2)$ terms are neglected

$$c_0^2 \bar{\nabla}^2 \varphi - \frac{(1+M^2)}{c_0^2} \frac{\partial^2 \varphi}{\partial \bar{t}^2} = s(\mathbf{x}, t) \tag{11}$$

Under the action of the transformation 10, in fact, the wave fronts of an impinging acoustic perturbation are stretched to recover, in the transformed space, the isotropic pattern of the static case, Fig. 1

3.1 Lorentzian approach

In the present context, the transformation is used in a different fashion: Eq. 11 is interpreted as the equation governing the propagation of acoustic perturbations within a virtual reference medium with a speed of sound scaled by the (non–uniform) factor $c_v = c_0/\sqrt{1+M^2}$. This virtual medium is assumed to be the reference medium that would make the statically designed metamaterial have the expected target response also in presence of a background potential flow with Mach number M (with local Mach number $M(\boldsymbol{\xi})$). The mechanical properties of the virtual reference medium are obtained by imposing the matching of the characteristic impedance $z_v = \varrho_v \, c_v$ with that of the real reference medium $z_0 = \varrho_0 \, c_0$, to yield

$$\varrho_v(\boldsymbol{\xi}) = \varrho_0 \sqrt{1 + M(\boldsymbol{\xi})^2} \qquad \mathcal{K}_v(\boldsymbol{\xi}) = \frac{\mathcal{K}_0}{\sqrt{1 + M(\boldsymbol{\xi})^2}} \tag{12}$$

This approach leads to scaling factors, depending on the local Mach number, for the static metamaterial design, independently on the method used to obtain it.

3.2 ATA approach

In this section we use the Taylor's coordinate transformation to restore in the transformed space the formal invariance of the governing equations under the action of conformal transformations, recovering the usability of methods like the STA in presence of a background aerodynamic flow. As it allows to use transformations that mix space and time, the ATA approach proposed by Garcia-Meca *et al.* in [8] and [10] is adopted.

We start from the $\mathcal{O}(M)$ approximation of the convective wave equation for the velocity potential and, following the ATA approach, define a virtual and a real medium. Then we derive the relative analogue model: introducing the four–dimensional coordinates $\xi^\mu = (t, x^i)$ (Latin superscripts ranging from 1 to 3 and Greek superscripts from 0 to 3), the $\mathcal{O}(M)$ approximation of the convective wave equation in Cartesian coordinates can be written as (Einstein's notation on repeated indices is used)

$$\frac{\partial}{\partial \xi^\mu}\left(W^{\mu\nu}\frac{\partial \varphi}{\partial \xi^\nu}\right) = 0, \quad \text{with} \quad W^{\mu\nu} = \frac{\rho_0}{c_0^2}\begin{pmatrix} -1 & \vdots & -v^j \\ \cdots & \cdot & \cdots \\ -v^i & \vdots & c_0^2\delta^{ij} \end{pmatrix}. \tag{13}$$

The form of D'Alembert operator on a curved Lorentzian (or *pseudo–Riemannian*) manifold is

$$\frac{1}{\sqrt{-\gamma}}\frac{\partial}{\partial \xi^\mu}\left(\sqrt{-\gamma}\gamma^{\mu\nu}\frac{\partial \varphi}{\partial \xi^\nu}\right) = 0, \tag{14}$$

where $\gamma^{\mu\nu}$ is the contravariant metric tensor and $\gamma = \det(\gamma_{\mu\nu})$. The metric tensor of the $\mathcal{O}(M)$ approximation of the convective wave equation in a curved space–time can be obtained by comparing Equations 6 and 14 and imposing $W^{\mu\nu} = \sqrt{-\gamma}\gamma^{\mu\nu}$. This yields $\det(W^{\mu\nu}) = \gamma$ (which becomes $\det(W^{\mu\nu}) = -\sqrt{-\gamma}$ in the three–dimensional space–time used for the numerical simulations of Section 4)

$$\gamma^{\mu\nu} = \frac{1}{\rho_0 c_0\sqrt{1+M^2}}\begin{pmatrix} -1 & \vdots & -v^j \\ \cdots & \cdot & \cdots \\ -v^i & \vdots & c_0^2\delta^{ij} \end{pmatrix}. \tag{15}$$

Then we perform the Taylor's coordinate transformation on the virtual medium to obtain the new metric in the transformed space-time using the standard tensorial transformation rule

$$\bar{\gamma}^{\alpha\beta} = \bar{\Lambda}^\alpha_\mu\,\bar{\Lambda}^\beta_\nu\,\gamma^{\mu\nu} \tag{16}$$

indicating with $\boldsymbol{\Lambda} = \nabla T(\boldsymbol{\xi})$ the Jacobian matrix of transformation 10 (in components form $\Lambda^\alpha_\mu = \partial\bar{\xi^\alpha}/\partial\xi^\mu$).

Finally, imposing $[\sqrt{-\gamma}\gamma^{\mu\nu}]_V = [\sqrt{-\gamma}\gamma^{\mu\nu}]_R$, we obtain the relations between the real and virtual medium parameter

$$\rho_R = \frac{\rho_V}{\sqrt{1+M^2}}, \qquad c_R = \frac{c_V}{\sqrt{1+M^2}}, \qquad v_R = 0. \tag{17}$$

Starting from the $\mathcal{O}(M)$ approximation of the convective wave equation we obtain that, in the real medium, the formal invariance of the equation under coordinate transformations is recovered at the only cost of scaling ρ and c of our medium, allowing the standard transformation acoustic approach to be used again effectively, at least in low Mach number flow situations.

The above procedure can be repeated for the convective wave equation to obtain

$$W^{\mu\nu} = \frac{\rho_0}{c_0^2}\begin{pmatrix} -1 & \vdots & -v^j \\ \cdots & \cdot & \cdots \\ -v^i & \vdots & c_0^2\delta^{ij} - v^i v^j \end{pmatrix}, \qquad \gamma^{\mu\nu} = \frac{1}{\rho_0 c_0}\begin{pmatrix} -1 & \vdots & -v^j \\ \cdots & \cdot & \cdots \\ -v^i & \vdots & c_0^2\delta^{ij} - v^i v^j \end{pmatrix} \tag{18}$$

For the sake of simplicity, we consider the case of simple convection and following the step above-written, the relations between virtual and real medium parameters are obtained

$$\rho_R = \rho_V(1-M^2)^{(3/2)}\sqrt{1+M^2+M^4}, \qquad c_R = \frac{c_V\sqrt{1-M^2}}{\sqrt{1+M^2+M^4}}, \qquad v_R = \frac{v_V\,M^2}{1+M^2+M^4}. \tag{19}$$

Equation 19 shows how, starting from the convective wave equation, one should in principle take in account a velocity in the real medium. To reach our goal of recovering the formal invariance of the governing equations under conformal coordinate transformations, this term is neglected; noting also that it related to the convection velocity by a term of order smaller than $\mathcal{O}(M^2)$, we expect this assumption to be acceptable only for low Mach number flows.

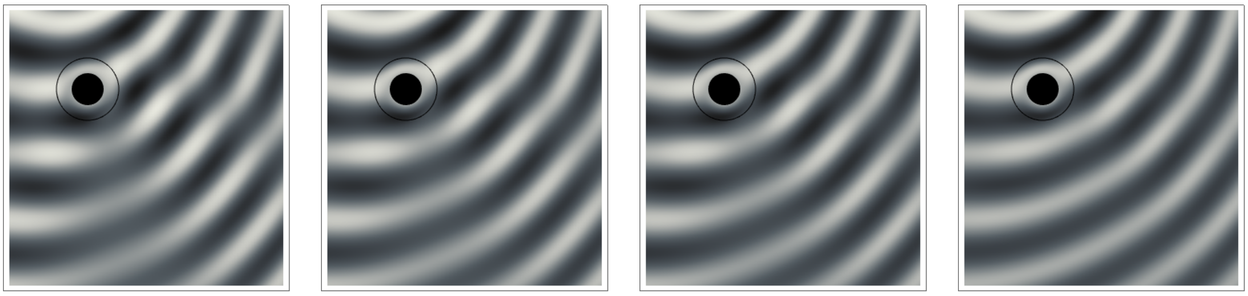
4. Numerical results

Numerical results presented in this paper have been obtained by applying a zero-th order boundary element method for the discretization of the boundary integral formulation previously presented in Section 2. Specifically, the formulation has been applied to an inertial–cloaking device for a circular obstacle of radius r_1, designed with the coordinate transformation approach proposed by Pendry *et al.*[2]

The form of the fictitious sources in Eq. 3 reduces to

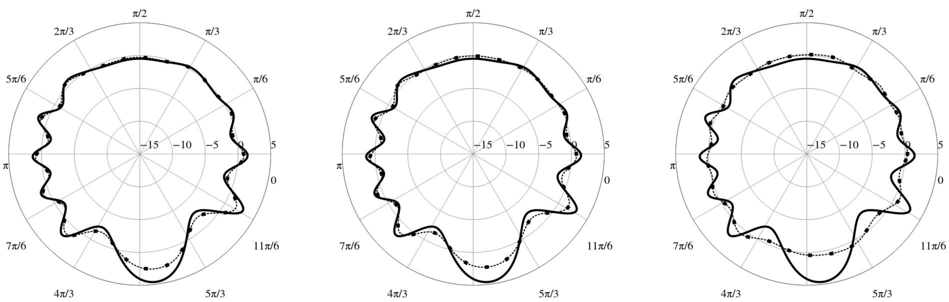
$$\mathbf{q}\,(\mathbf{x},t) = \left(\boldsymbol{I} - \hat{\boldsymbol{\varrho}}^{-1}\right)\nabla p \qquad \sigma\,(\mathbf{x},t) = \varrho_0 \left(\frac{1}{\mathcal{K}_c} - \frac{1}{\mathcal{K}_0}\right)\frac{\partial^2 p}{\partial t^2} \tag{20}$$

where $\mathcal{K}_0 = J$, $\boldsymbol{\varrho} = J\ \left(\mathbf{D}\mathbf{D}^T\right)^{-1}$ with $\mathbf{D} = \bar{\nabla} f(\bar{\mathbf{x}})$ (the bar indicates differentiation w.r.t. $\bar{\mathbf{x}}$). For simplicity, in the BEM solver, the background velocity field is considered to be uniform, i.e. $\boldsymbol{M} = \mathrm{M}\hat{\boldsymbol{\imath}}$ in Ω_h, so the perturbation induced by the presence of the cylinder on the pressure field is not taken into account. The incident acoustic field is produced by a moving mass point source, this is impinging on a co–moving rigid cylinder surrounded by a cloaking surface.



(a) Static inertial cloak (b) Correction A (c) Correction B (d) Correction C

Figure 2: Cloaking of an object in motion with $\boldsymbol{M} = 0.30\hat{\boldsymbol{\imath}}$, $kr_1 = 1.5$, impinged by the field generated by a co-moving mass point source in $\mathbf{x}_s \equiv (0, 10r_1)$. Total field in Ω_h and Ω_c.

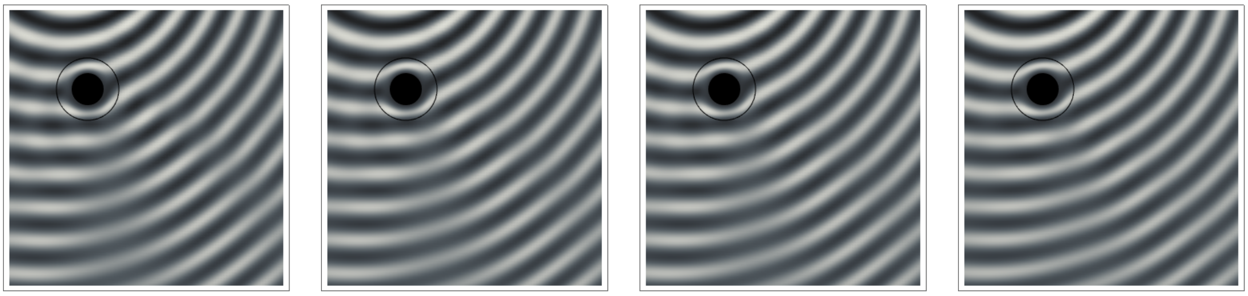


(a) Correction A vs. Static cloak (b) Correction B vs. Static cloak (c) Correction C vs. Static cloak

Figure 3: Cloaking of an object in motion with $\boldsymbol{M} = 0.30\hat{\boldsymbol{\imath}}$, $kr_1 = 1.5$, impinged by the field generated by a co-moving mass point source in $\mathbf{x}_s \equiv (0, 10r_1)$. I_L directivity pattern at $R = 10\,r_1$.

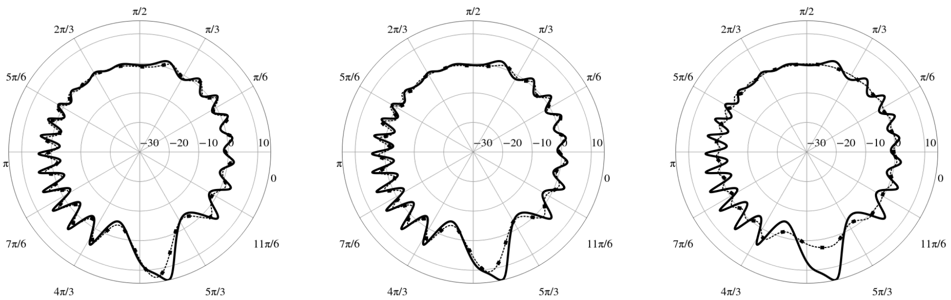
Visualizations of the total pressure field at kr_1= 1.5, 3 and 4.5 of Figs. 2 and 4 shows the scattering cancellation capabilities of the cloaks designed using the three different correction presented above (with correction A, B and C referring respectively to Equations 12, 17 and 19 where v_R is neglected).

Moreover, the scattering abatement can be observed in Figs. 3 and 5 , where the Insertion Loss, defined as $\mathrm{I}_\mathrm{L} = 20\log_{10}\left(p_{\mathrm{tot}}/p_{\mathrm{inc}}\right)$, is evaluated at a circle of microphones located at $\mathrm{r_M}$ = 25 r_2 comparing the static design with each correction at various kr_1 and $\boldsymbol{M}$.



(a) Static inertial cloak (b) Correction A (c) Correction B (d) Correction C

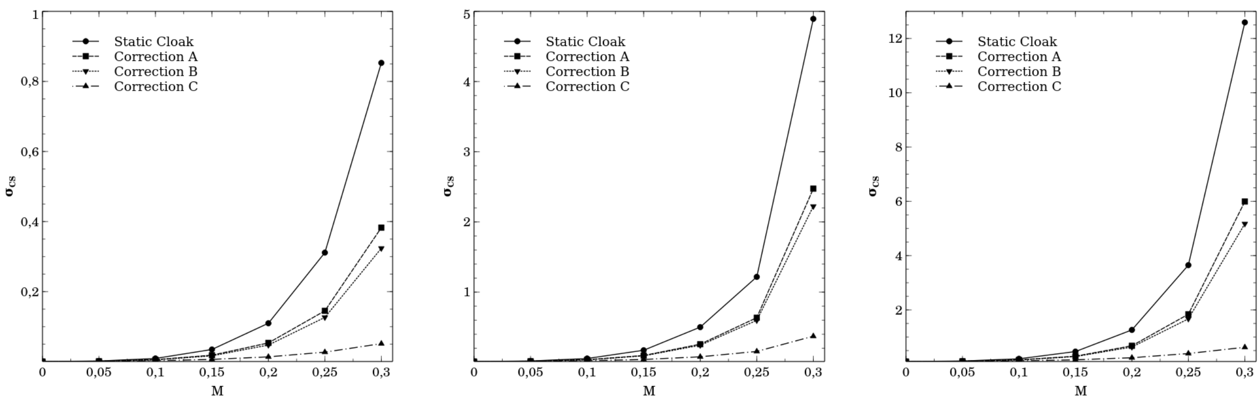
Figure 4: Cloaking of an object in motion with $\boldsymbol{M} = 0.20\hat{\boldsymbol{\imath}}$, $kr_1 = 3.0$, impinged by the field generated by a co-moving mass point source in $\mathbf{x}_s \equiv (0, 10r_1)$. Total field in Ω_h and Ω_c.



(a) Correction A vs. Static cloak (b) Correction B vs. Static cloak (c) Correction C vs. Static cloak

Figure 5: Cloaking of an object in motion with $\boldsymbol{M} = 0.30\hat{\boldsymbol{\imath}}$, $kr_1 = 3.0$, impinged by the field generated by a co-moving mass point source in $\mathbf{x}_s \equiv (0, 10r_1)$. I_L directivity pattern at $R = 10\, r_1$.

The effect of frequency and Mach number on the cloaking efficiency of tested convective designs is evaluated in terms of scattering cross section of the cloaked cylinder σ_{cs}. The trend of this efficiency estimator, illustrated in Fig. 6 in the Mach range 0–0.35 for three reduced frequencies, evidences how, for all the analysed frequencies, the corrected designs are able to recover the cloaking effect of the meta-device in presence of a background flow, improving the performance with respect to the static design.



(a) $kr_1 = 1.5$ (b) $kr_1 = 3.0$ (c) $kr_1 = 4.5$

Figure 6: Effect of Mach number and frequency on the cloaking efficiency in terms of scattering cross section σ_{cs} of the cylinder for a static (–●–) and convective cloaks (Correction A - -■- -, Correction B ···▼···, Correction C -·▲-·).

5. Conclusion

Three different design corrections based on Taylor's coordinate transformation for acoustic meta-devices in presence of relative motion respect to the surrounding fluid are derived and numerically tested on the benchmark case of a cylindrical inertial cloaking device. The modelling of the acoustic response of the cloaking device has been addressed using an original boundary integral formulation, valid for moving bodies impinged by an acoustic perturbation produced by a co-moving source. The location of the source with respect to the obstacle is such that the assumption on incoming planar wave fronts is not applicable.

The numerical results show the proposed corrections to produce a remarkable improvement of the cloaking efficiency in presence of flow, up to a Mach number $M = 0.3$, which is comparable to the take off and landing speed of a commercial aircraft.

The major limitation of the formulation presented is the assumption of a uniform stream in the hosting medium. This limitation is intended to be removed by a reformulation of the problem starting from a tailored reinterpretation of the FfowcsWilliams–Hawkings equation in Ω_h. An additional assumption on the basis of the Taylor's theory is that the background aerodynamic field is potential, thus making the transformation and the proposed methods, in principle, not applicable in case of diffused vorticity. Addressing situations in which the aerodynamic flow will depart significantly from potentiality will be object of future development of the work; the methods described in this paper will be used to provide the initial guess of a numerical optimization procedure towards the matching with a target behaviour of the meta-device in presence of a arbitrarily complex background flow.

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