24th INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
23–27 July 2017, London

NUMERICAL MODELS OF SINGLE- AND DOUBLE-NEGATIVE METAMATERIALS INCLUDING VISCOUS AND THERMAL LOSSES

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Negative index acoustic metamaterials are artificial structures made of subwavelength units arranged in a lattice, whose effective acoustic parameters, bulk modulus and mass density, can be negative. In these materials, sound waves propagate inside the periodic structure, assumed rigid, showing extraordinary properties. We are interested in two particular cases: a double-negative metamaterial, where both parameters are negative at some frequencies, and a single-negative metamaterial with negative bulk modulus within a broader frequency band.

In previous research involving the double-negative metamaterial, numerical models with viscous and thermal losses were used to explain that the extraordinary behavior, predicted by analytical models and numerical simulations with no losses, disappeared when the metamaterial was measured in physical setups. The improvement of the models is allowing now a more detailed understanding on how viscous and thermal losses affect the setups at different frequencies. The modeling of a simpler single-negative metamaterial also broadens this overview. Both setups have been modeled with quadratic BEM meshes. Each sample, scaled at two different sizes, has been represented with a detailed frequency step.

The influence of viscous and thermal losses as a function of the scale has been studied at two different scales, in both metamaerials. It is shown that the effect of losses on the scale is not the same for the different regimes of the metamaterials. Special attention is also given to the double-negative frequency band, where a fine frequency step of the simulation reveals details about the Fabry-Perot resonances in the metamaterial slab. The numerical model with losses, which is computationally very demanding, will also be commented.

Keywords: Acoustic metamaterials, boundary element method

1. Introduction

Acoustic metamaterials are acoustic setups, where specifically designed units are arranged in a periodic structure [1, 2]. The units may contain elements such as scatterers or resonators in the subwavelength scale. If the metamaterial is studied as a whole, it may show extraordinary properties that cannot be achieved with simpler structures. In particular, there are acoustic metamaterials with a negative effective bulk modulus, negative effective density, or even both at the same time. The former are called single-negative and the latter are called double-negative metamaterials in the literature.

In the past decade or so, acoustic metamaterials have become popular research topics [3, 4] due to their interesting properties and the similarities with other physical phenomena such as optics of crystalline materials. Numerical simulations where the lossless wave equation is discretized in the domain are routinely employed, together with analytical descriptions, in order to study the interaction of sound waves with these structures.

The interaction of sound waves with domain boundaries creates a loss of energy in the sound field by means of two mechanisms: i) the temperature variations are reduced to practically zero at the boundary due to the much larger thermal conductivity of solids, thus stealing heat from the wave, and ii) the fluid is not allowed to slide on the boundary surface (non-slip condition), creating a sharp gradient of particle velocity where viscous losses are relevant [5]. These effects give rise to acoustic losses in thin layers of fluid, the thermal and viscous boundary layers respectively. The thicknesses of the thermal and viscous boundary layers are similar and decrease with the frequency ($2{,}5 \cdot 10^{-3}/\sqrt{f}$ m and $2{,}1 \cdot 10^{-3}/\sqrt{f}$ m in air, respectively). Since those thicknesses are much smaller than the internal dimensions of an acoustic metamaterial, acoustic losses are often neglected.

Recent work, however, has shown that acoustic losses can totally suppress the extraordinary behaviour of double-negative acoustic metamaterials [6, 7]. In this paper, we revisit the results in [7], this time with an improved Boundary Element Method (BEM) implementation with losses. We also present a model of a simpler single-negative acoustic metamaterial, initially introduced in [8].

Section 2 describes the parameters of the numerical simulation for both setups. Section 3 presents results corresponding to the single-negative metamaterial, and section 4 is dedicated to the double-negative setup. Finally, conclusions are summarized in section 5.

2. Description of the numerical models

2.1 The simulated setups

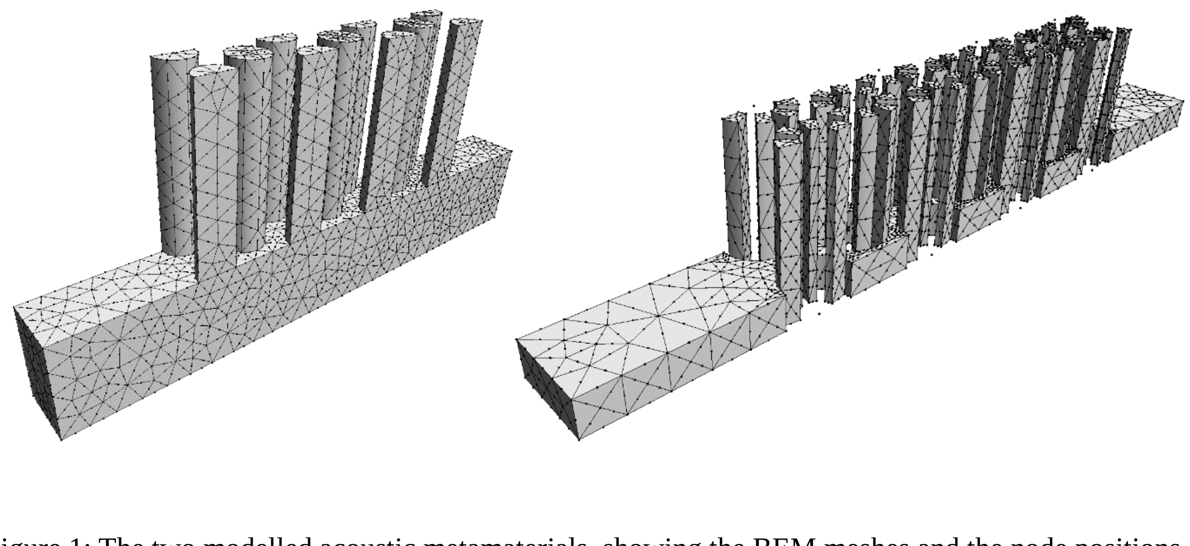

Figure 1: The two modelled acoustic metamaterials, showing the BEM meshes and the node positions. Left, single-negative, and right, double-negative [6, 8].

The two structures analysed here are shown in Fig. 1. The boundary meshes are represented, where the calculation domains are interior. The dimensions of the single-negative metamaterial (left) are as in [8], and the dimensions of the double-negative metamaterial (right) are those of sample A in [6]. To give an overall idea of the sizes, the lattice constant (distance between units) is 30 mm for the single-negative sample and 21 mm for the double-negative sample. They are meant to be inserted in an infinite waveguide, which is assumed to carry an incident plane wave. This condition is simulated in the finite setups shown in Fig. 1 by placing a moving piston on the emitting end and defining boundary impedance of ρc at both emitting and receiving ends. The piston velocity is set to *1/ρc* so that the corresponding undisturbed plane wave has an amplitude of unity.

The absorptance, reflectance and transmittance are calculated from the acoustic pressures (P_1, P_2, P_3) at three different positions along the length of the waveguide (x_1, x_2, x_3), two at the emitting end, before the sample, and one at the receiving end, after the sample. In the single-negative sample, the positions are respectively -15,07 cm, -13,9 cm and 15,07 cm, measured from the centre, while in the double-negative sample the positions are -10,55 cm, -10,20 cm and 10,55 cm, with the same reference.

$$\boldsymbol{r(\omega)} = \frac{P_2 e^{-ik_0 x_1} - P_1 e^{-ik_0 x_2}}{P_1 e^{ik_0 x_2} - P_2 e^{ik_0 x_2}} \tag{1}$$

$$\boldsymbol{t(\omega)} = \frac{P_3}{P_2} \frac{e^{-ik_0 x_2} - r(\omega) e^{ik_0 x_2}}{e^{-ik_0 x_3}} e^{-ik_0 L_{eff}} \tag{2}$$

Eqs. (1,2) are the expressions of the reflection and transmission factors, from which it is possible to calculate the reflectance $R(\omega) = |r(\omega)|^2$ and transmittance $T(\omega) = |t(\omega)|^2$. Since energy should be balanced, the absorptance is $A(\omega) = 1 - T(\omega) - R(\omega)$. Additionally, k_0 is the wavenumber in air and L_{eff} is the effective length of the metamaterial slab.

2.2 Numerical model

A BEM implementation with viscous and thermal losses is employed for the simulations [9, 10]. This implementation is based on the research BEM code OpenBEM [11]. In reference [7], this BEM with losses, albeit with a simpler mesh, is compared with the commercial finite element package Comsol, which also includes losses, on the same double-negative sample. Both methods gave very similar transmittance results and matching with measurements.

The calculation is computationally very demanding, both in terms of memory and processing time. In [7], it was observed that the memory demands were higher for the FEM model, while the BEM, which is implemented in Matlab language, took longer to calculate but with affordable memory usage.

In the new calculations, the BEM has been run in a cluster computer system. The BEM is well suited for parallelization: it is possible to obtain all frequency dependent coefficient matrices, the main bulk of the calculation, in parallel.

Table 1: Parameters of the BEM meshes

Sample	**Nr. of nodes**	**Nr. of elements**
Single-negative	4706	2352
Double-negative	9616	4810

The BEM boundary meshes in this calculation are constructed with quadratic 6-node elements, as shown in Fig. 1. The Gmsh meshing software is employed for this purpose [12]. The numbers of elements and nodes are given in Table 1. The quadratic elements adapt much better than the linear elements employed in [7] to the curved features of the setup.

3. Results: single-negative metamaterial

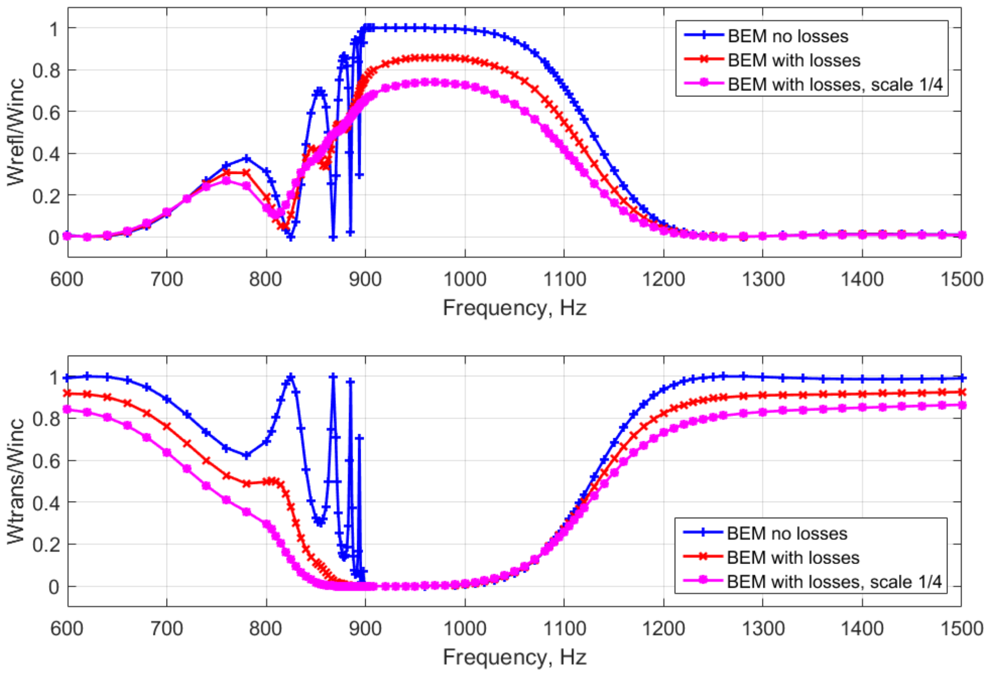

Figure 2: Reflectance (upper graph) and transmittance (lower graph) of the single-negative metamaterial shown in Fig. 1. The curves, as indicated, correspond to calculations with no visco-thermal losses, and with losses at two different scales. The curves for the scaled-down setup correspond to frequencies four times higher.

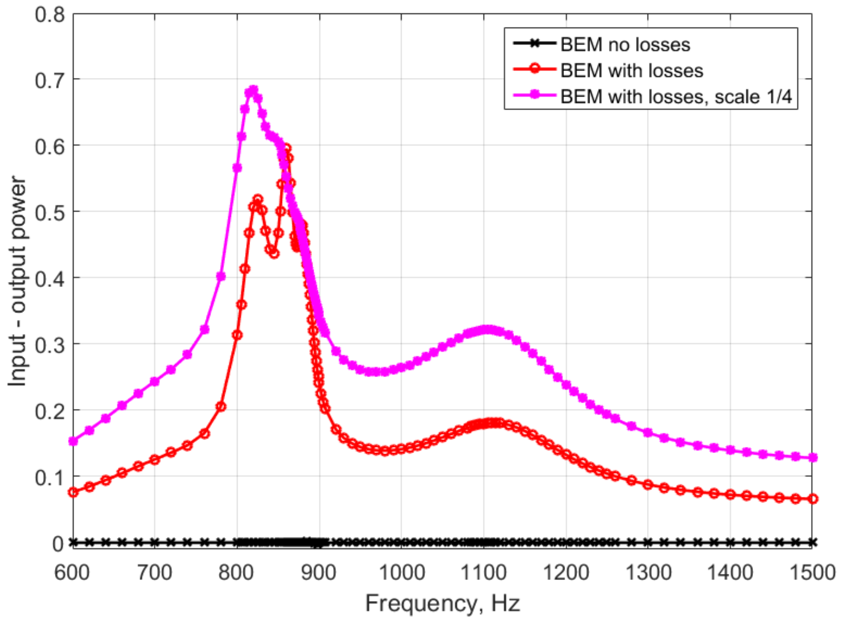

Figure 3: Absorptance of the single-negative metamaterial shown in Fig. 1. The curves, as indicated, correspond to calculations with no visco-thermal losses, and with losses at two different scales. The curve for the scaled-down setup corresponds to frequencies four times higher.

Figs. 2 and 3 show the reflectance, transmittance and absorptance results for the single-negative metamaterial setup. The calculations have been performed with no losses and with visco-thermal losses, the latter at two different scales: full and reduced to one fourth. The reasoning behind the scaling is that viscous and thermal losses do not scale in the same way as the lossless magnitudes.

The behavior of the lossless setup does not change with the scale; the results are just shifted in frequency. Viscous and thermal boundary layers, as mentioned in section 1, have thicknesses that vary as $f^{-1/2}$, so that the scaling can greatly modify the influence of losses.

It can be seen that the lossless metamaterial behaves as predicted in [8]. It shows Fabry-Perot (FP) peaks and a stop band where the effective bulk modulus becomes negative. When losses are considered, the FP peaks are reduced, almost disappearing for the reduced scale. The reflectance in the single-negative band becomes smaller than one when losses are present, indicating that part of the energy is dissipated in the slab, rather than being reflected back. The absorptance, accordingly, grows with decreasing scale. This indicates that upscaling the metamaterial can significantly reduce the effects of visco-thermal losses. Other results not shown here support this conclusion.

Note that the absorptance curve for no losses is close to zero at all frequencies. This proves that the balance of energy is correct in the calculation; an additional test for the numerical implementation.

4. Results: double-negative metamaterial

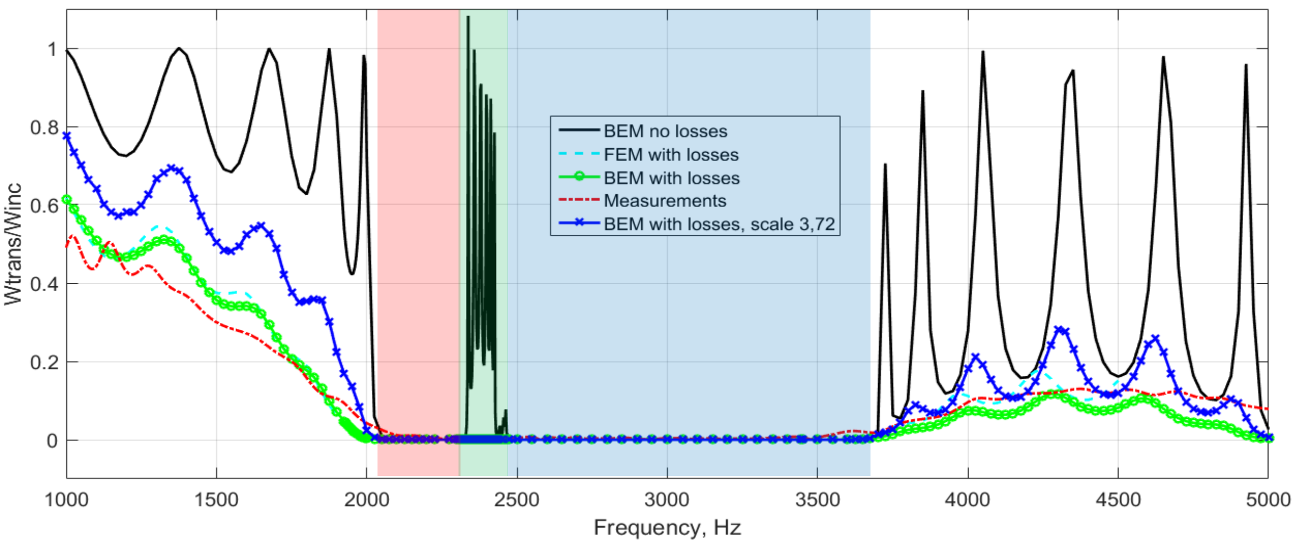

Figure 3: Transmittance of the double-negative metamaterial shown in Fig. 1. The curves, as indicated, correspond to calculations with no visco-thermal losses, with losses at two different scales and measurements. A FEM result is also included. The curve for the scaled-up setup corresponds to frequencies 3,72 times lower. The colored areas indicate single- (red/blue) and double-negative (green) frequency regions.

The results in Figs. 3 and 4 are presented for the double-negative metamaterial in a similar way as the single-negative metamaterial results in the previous section. This metamaterial behaves as single-negative with negative effective bulk modulus or density and, in a narrow frequency region from 2,33 to 2,44 kHz, as double-negative. Single-negative regions exhibit a similar behavior as the one described in the previous section: FP resonances that are attenuated more as the scale decreases. The double-negative region, however, has a different behavior. The absorptance varies very little with the scale, and it is very large. Besides, the tightly packed FP peaks in this region completely disappear, together with the extraordinary properties seen in the lossless case, for both scales. This indicates that scaling up the setup will hardly help in restoring the double-negative performance.

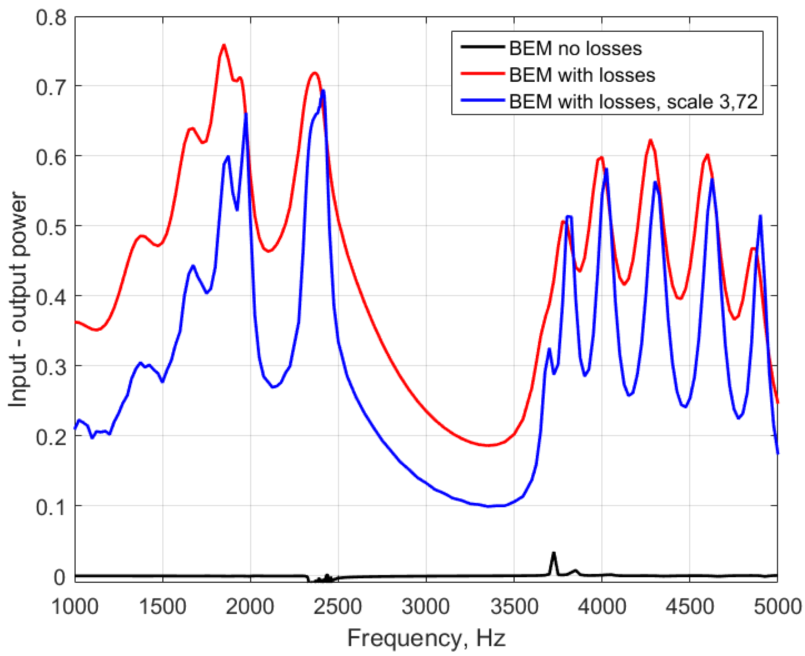

Figure 4: Absorptance of the double-negative metamaterial shown in Fig. 1. The curves, as indicated, correspond to calculations with no visco-thermal losses, and with losses at two different scales. The curve for the scaled-up setup corresponds to frequencies 3,72 times lower.

5. Conclusions

Two numerical models of a single-negative and a double-negative metamaterial are presented. The numerical tool used, BEM with losses, performs well and gives meaningful results. These metamaterials, especially the double-negative, reveal themselves as rich in features. The numerical simulation is able to show that the effect of viscous and thermal losses is very detrimental to the double-negative behavior, where the metamaterial slab tends to absorb almost all energy independently of the scale.

Acknowledgements: J. S.-D. acknowledges the support by the Spanish MINECO, and the European Union FEDER through project TEC2014-53088-C3-1-R.

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