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CHAOS IN MUSICAL SOUNDS

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1. INTRODUCTION.

The main purpose of this paper is to show how popular mechanical systems such as woodwinds or bowed strings have become a fascinating tools of experiments closely related with a more fascinating topic: chaos.

The interest in non linear dynamical systems study has grown recently in electronics, mechanics, and fluid dynamics. Now chaos covers the whole science and each day one find new subjects in which we can apply the rules and the tools used in it [1].

Musical instruments are conveniently divided in two families. The first one is known as percussive instruments, in which we include piano, harpsichord and plucked instruments and self sustained instruments which contains brass, woodwinds and bowed strings. It is now well known that this second family is only a particular class of non linear dynamical systems. We can schematically describe the production of sound as follow: a continuous flow (or strength, or pressure) is fed into a mechanical system and produces a periodic motion of air which is transmitted and gives the periodic sound emitted by the instrument. Such a system is obviously non linear. one know that (at least for woodwinds) the main part of the instrument (the resonator which defines the pitch) often characterized by its input impedance is linear. The non linear part of the instrument is very well localized, and we know for a long time [2] that this strong non linearity is the exitator of the instrument. In woodwinds it is the reed or the flue edge system. In bowed strings the non linear part stays in the bow friction characteristic.

Because of this non linear characteristic these musical instruments are able to produce bifurcations. They easily give after steady continuous signals (that means no sound) periodic sounds in normal playing. In jazz and now most in contemporary music composers ask players for to play multiple sounds or multiphonics. These sounds are heard as a non harmonic chord. A chord which components do not lie on a tempered chromatic scale. In another hand, one know that transients and slightly overblown pipes produce anharmonic sounds. With in mind the fact that such systems are closely related with NLD systems it becomes obvious that such sounds should be analysed with the same tools.

There are two ways to perform such an analysis. First, one can mimick the musical instrument or better the non linear part with an electronic or numerical device. Second, one can find or produce natural situations where the signals are not strictly periodic. Following Maganza and al. [3] we have tried the first way and used some tools such as Phase Space Representations [4,5] and Poincaré Sections, then trying the second way, we have shown that the acoustical events produced by increasing the blowing pressure into a recorder are biperiodic and chaotic. Because these signals are also known as multiphonics, we have studied multiphonic sounds produced by oboe, saxophone and clarinet. These sounds were found to be chaotic or at least biperiodic. On strings we studied some "woolf-note related" sounds produced on a cello which give similar results.

2) SOME DEFINITIONS.

In order to present and define the tools we use in what follows, we need some simple examples. For more sophisticated details one can refer to [4,5]. Phase Space are well known in mechanics. The true Phase space of a mechanical system implies the knowledge of the equations of the system. In experimental situations we do not know these equations. Fortunately it has been shown that one can obtain the same topological information from derivative Phase Space obtained with a measured signal $f(t)$ and its various time derivative $\frac{df^n(t)}{dt^n}$ and with time delayed Phase Space where the coordinates are time delayed replications of the signal $f(t)$, $f(t+\tau)$, $f(t+2\tau)$.... . We know also that even when the theoretical dimension of the space is infinite that only a few number of dregrees of freedom are important. In practice only two or three dimensions are nedeed, and we represent our signals in a Time Delayed Phase Space.

$$f(t),\ f(t+\tau),\ f(t+2\tau)$$

Choosing τ is very important, if τ is two short the signal describes a straight line in the TDPS, if τ is too large the delayed values may be uncorrelated, and no information is carried by the trajectory in the Phase Space.

The Poincaré sections of these Phase spaces are defined for 3-D PS as the intersection of the trajectory with a 2-D space. Since we have studied zero mean value signal it is convenient to use the plane $f(t) = 0$. These representations have the following properties:

trajectory of a periodic signal gives:
a closed curve in PS, a point as Poincaré S.

Biperiodic signal:
a 2.D torus in PS, a closed curve as Poincaré S.

multiperiodic signal:
a n.D torus in PS, a n-1.D torus as Poincaré S.

chaotic signal:
a strange attractor with a non integer dimension d in PS,
a d-1 strange attractor as Poincaré S.

3) CHAOS AND THE MAGANZA EXPERIMENT.

Maganza and al. [3] obtained a clarinet like system by replacing the mouthpiece and the reed by a microphone followed by a non linear function an amplifier and a loudspeaker (fig.1).

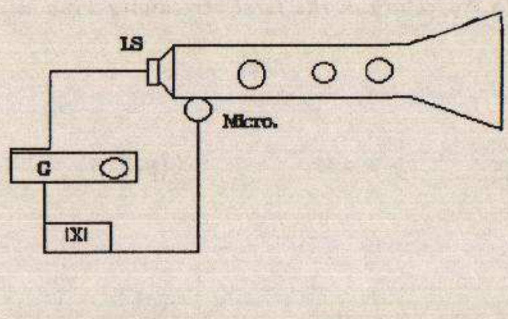

Figure 1
Set – up used to mimick a woodwind.

The non linear function has a shape which looks like those of single reed instrument. They succeeded in obtaining a Feigenbaum scenario through chaos. With a similar setup of higher acoustical power we have obtained also some period doubling, but for one fingering we have gone after chaos. It is known that a period doubling scenario exhibits some periodicity windows. We present here a period tripling which lies in the first of these windows. The attractor has obviously a non integer dimension of about 1.3. It shows a particular spliting related with the period tripling (fig. 2).

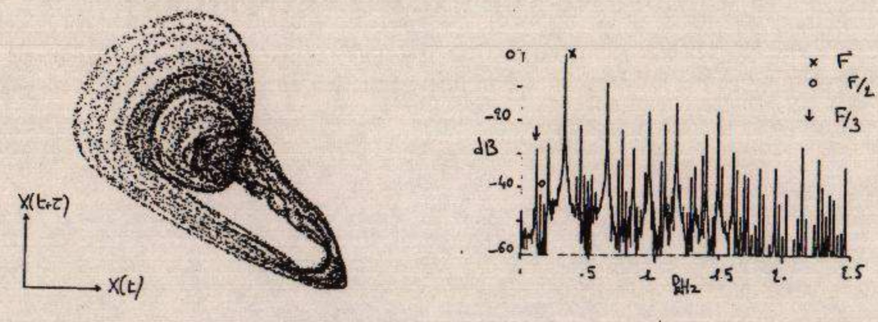

Figure 2
The PS trajectory in the first periodicity window and its FT.

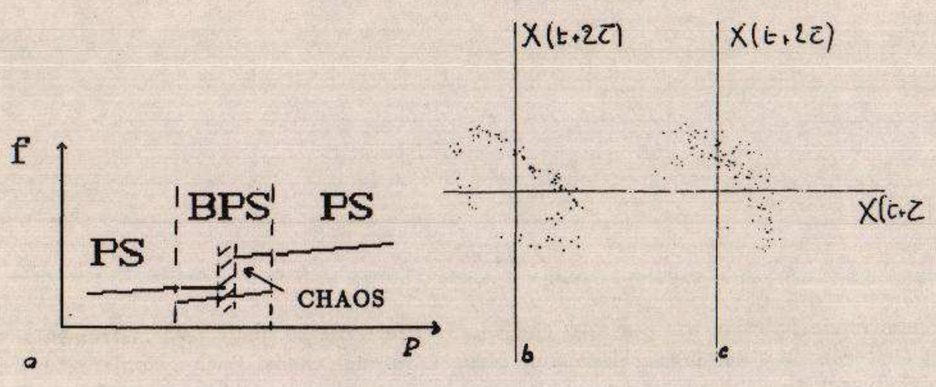

Figure 3
a) Evolution of the acoustical signals with pressure.
b) Poincaré Section of the first biperiodic signal.
c) "Chaotic" behavior of the recorder.

4) TRANSITIONS ON RECORDER.

The recorder is an interesting "subject "of experiment because it offers a convenient control parameter with the blowing pressure. By increasing slowly, step by step and without decreasing the pressure we can explore the dynamic of an eventual bifurcation. One obtain (fig. 3) for a sufficient pressure a periodic and normal signal then a more complicated signal which has a biperiodic Poincaré Section. For higher pressures one obtain an even more complicated Poincaré section and then another biperiodic state followed by the periodic second regime of the instrument. the biperiodic state is built with the non linear interaction of two frequencies. The first one is directly related with the frequency of the regime and the second with the travelling time a small perturbation needs to propagate from the flue to the edge.

Determining if the section drawn on fig 3c is or not chaotic is a problem and there is two different answers depending of the fingering. For "non harmonic" fingerings which give non harmonically related regimes, the trajectory in Phase Space is stable and the signal is chaotic. For fingerings where the two regimes are rather harmonically related one expect more a 3-D torus than a chaos. The only one fingering which is harmonic is the A fingering. We present the Poincaré Section of its transition on fig 4. Even if the signal seems to be chaotic there is a great probability for a 3-D torus. For others fingerings the "chaotic" Poincaré Section is stable. This indicate more the characteristic of a chaotic transition via a biperiodic scenario. The third frequency involved in these cases is obviously the frequency of the second regime. When decreasing the pressure from any state, one find again the same signals for lower pressures. This indicates a quite normal hysteresis phenomenon.

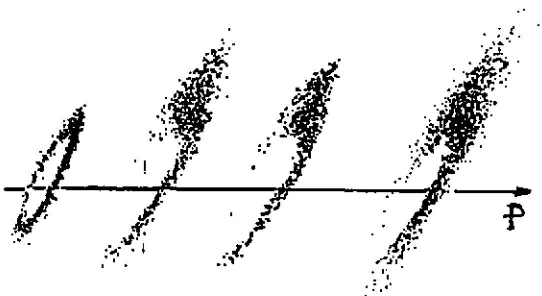

Figure 4
Poincaré sectionsof the chaotic part of an harmonic fingering.

The topological dimensions of the attractors have not been computed. The short durations recorded did not allow precise calculations.

5) MULTIPHONICS.

We have studied various multiphonics sounds emited by woodwinds. The signals produced by the recorder are also multiphonics and we have shown they could involves chaos and multiperiodic signals. These sounds are mostly produced with particular fingerings whose impedance curves are highly non harmonic, but one can also obtain them with normal fingerings. They are always produced when the reflexion function associated with the fingering has the long tail of the fig 5.

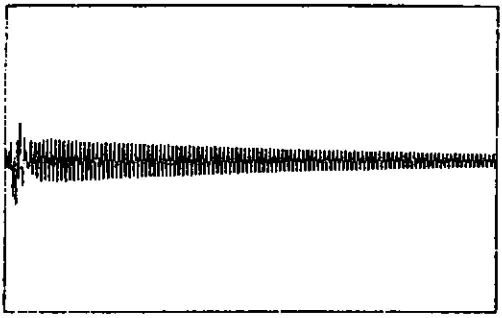

Figure 5
Reflexion function of a multiphonic fingering.

We have met some difficulties with sounds played by musicians. The musician who plays a multiphonic has no pitch to refer. He only heard a complex sound and has his personnal knowledge of this sound. He is always trying to produce a particular sound and the results is (in a physical meaning) an unsteady signal which explore the dynamic of the system without any knowledge of the control parameter. Nevertheless on oboe signals (fig. 6) we have obtained some beautiful Poincaré sections which show that the multiphonics are at least biperiodic signals and sometimes chaotic one's. Because of the lack of control parameter with professional musicians we produced ourself some multiphonics on saxophone trying to be as steady as it was possible. We present in figure 7 the Phase space representation and the Poincaré section of a signal for which J. Holzfuss has computed a dimension of 2.3. Nevertheless one must keep in mind that this kind of result only indicates a chaos, it does not prove it.

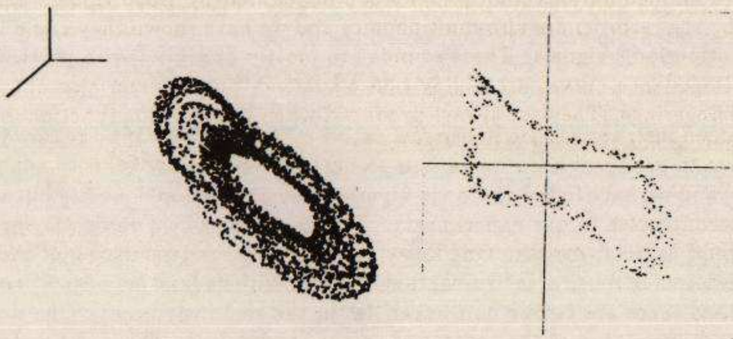

Figure 6
PSR of an oboe multiphonic and its Poincaré Section

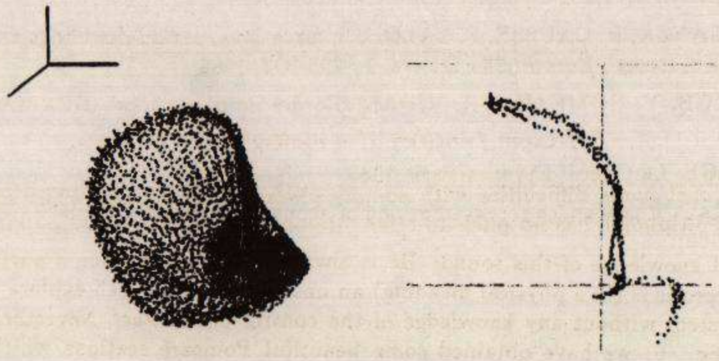

Figure 7
PSR of a saxophone multiphonic and its Poincaré Section

6) BOWED STRINGS AND TRANSIENTS.

Bowed strings have the same fundamentals properties than woodwind, the resonnances of the body are non harmonic with those of the string. Some cellists play multiphonics. We have recorded and studied some of these signals. The results are the same than for woodwinds: often biperiodicity and sometimes chaos.

7) CONCLUSIONS

Self sustained instruments are linear dissipative systems. There is no reason which could forbid them to produce chaos via various scenarios. We have used this to analyse musical sounds [6] and we have found chaos via biperiodicity and period doubling but we have not found intermitencies in our experiments. The trombone offers some playing situations which should be related with. One know know that an understanding of the particular functionnements of musical instruments such as transitions (and transients) needs to keep in mind that there are strong non linearities in the real instrument. One needs also to have a good knowledge of the tools used in the study of non linear dynamical systems, tools which are in fact nothing more than new signal analysis or representations methods.

8) REFERENCES.

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