24th INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
23–27 July 2017, London

USE OF STOCHASTICALLY ROUGH SURFACES FOR CONTROL OF TRAFFIC-INDUCED GROUND VIBRATIONS

Victor Krylov

Department of Aeronautical and Automotive Engineering, Loughborough University, Loughborough, UK
email: v.v.krylov@lboro.ac.uk

In the present work, a potential use of stochastically rough surfaces for damping ground vibrations from rail and road traffic is considered. Two types of surface roughness are explored in this work: two-dimensional roughness formed by parallel grooves of randomly varying width and depth, and three-dimensional roughness formed by combinations of randomly positioned and randomly sized pits and hills. Recommendations are made on the values of the parameters of rough surfaces to be used in practice. Calculations of Rayleigh seismic wave attenuation are carried out for recommended parameters and typical sizes of rough surface areas at the frequency range of 0-100 Hz typical for traffic-induced ground vibrations. The results are compared with the available data for open trenches used as seismic barriers. Comparisons are also made with the reduced-scale model experiments carried out for rough surfaces at ultrasonic frequencies. Although statistically uneven surfaces for typical values of the parameters are less efficient as seismic barriers than trenches, the advantage of their use is that they can be easily made and maintained, in comparison with trenches, and they can be used for planting trees and other vegetation, while remaining accessible for people and animals.

Keywords: ground vibrations, vibration control, rough surfaces

1. Introduction

Ground vibrations generated by railway and road traffic are predominantly Rayleigh surface waves propagating away from railways and roads. Therefore, it is natural to consider possible effects of ground surface topography on traffic-induced Rayleigh waves, in particular on possible reduction in the levels of associated ground vibrations at the points of observation. In the present work, a potential use of stochastically shaped ground surfaces for damping Rayleigh waves from rail and road traffic is considered.

It should be noted that the adverse effects of rough surfaces on propagation of Rayleigh waves have been widely investigated at ultrasonic frequencies with regard to their applications in solid state physics, non-destructive testing of materials and signal processing (see the monograph [1] for a historical review and general discussion). However, there was little attention paid to the use of rough surfaces for damping traffic-induced ground vibrations, where the effects of roughness play a positive role (see, for example, the paper [2], where, among other things, the initial reduced-scale experimental investigations of using rough surfaces as seismic barriers have been carried out).

In the present work, a more detailed analysis of using stochastically rough surfaces for control of traffic-induced ground vibrations will be carried out. Two types of surface roughness are considered: two-dimensional roughness formed by parallel grooves of randomly varying width and depth, and three-dimensional roughness formed by combinations of randomly positioned and randomly sized pits and hills. Recommendations are made on the values of the parameters of rough surfaces to be used in practice. Calculations of Rayleigh wave attenuation are carried out for recommended parameters and typical sizes of rough surface areas at the frequency range of 0-100 Hz typical for traffic-induced ground vibrations. The results are compared with the available data for open trenches used as seismic barriers. Comparisons are also made with the reduced-scale model experiments carried out for rough surfaces at ultrasonic frequencies.

2. Theoretical background

In what follows we will consider two distinctive cases of stochastically rough surfaces adjacent to a railway or a road: the case of two-dimensional roughness described by the random function $f(x)$, and the case of three-dimensional roughness described by the random function $f(x,y)$ (Fig. 1). Both these functions describe the height (or depth) of the rough surface in respect of the ideally flat surface, and it is assumed that statistically averaged values of these functions are zero.

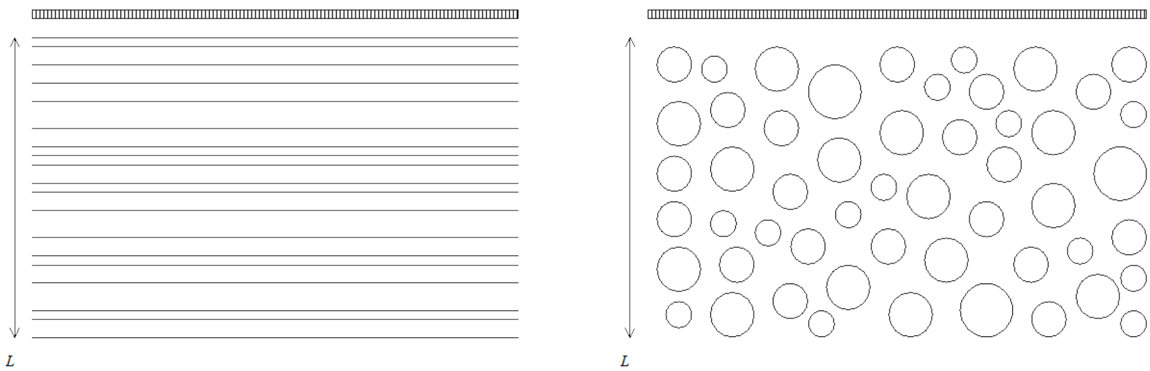

Figure 1: Two-dimensional (left) and three-dimensional (right) stochastically rough surfaces adjacent to a railway or a road shown on the top.

During propagation of Rayleigh waves over rough surfaces, the processes of multiple scattering of Rayleigh waves into bulk longitudinal and shear waves, as well as into other Rayleigh waves, occur. As a result of the above-mentioned scattering processes, the propagating Rayleigh waves are attenuated with distance due to the energy losses via scattering. In addition to attenuation, the propagating Rayleigh waves also experience dispersion (due to multiple scattering), i.e. their velocities become frequency-dependent.

Being interested in roughness-induced attenuation of Rayleigh waves, we will consider propagation of plane Rayleigh waves over the rough area in perpendicular direction x in respect of a railway track or road. For such plane waves, the distance-dependent attenuation can be described by the exponential functions, for two- and three-dimensional rough surfaces respectively:

$$A^{(2)}(x) = exp(-\alpha^{(2)}x), \tag{1}$$

$$A^{(3)}(x) = exp(-\alpha^{(3)}x). \tag{2}$$

Here $\alpha^{(2)}$ and $\alpha^{(3)}$ are the corresponding attenuation coefficients. For completeness, we will include into these coefficients also the effect of material losses in the ground described by the ground loss factor γ via the exponential function $exp(-\gamma k_R x)$. Here $k_R = \omega/c_R$ is the wavenumber of Rayleigh waves in an ideally flat elastic half space without material losses, c_R is their velocity, and $\omega = 2\pi f$ is the circular frequency. It should be mentioned that in the majority of practical situations of generation of ground vibrations by rail and road traffic the generated Rayleigh waves are not plane, but cylindrically divergent, so that non-exponential attenuation due to geometrical spreading should be taken into account as well. However, since taking geometrical spreading into account is very easy to do, we will deal here with plane Rayleigh waves only.

Following the results of the most comprehensive theoretical investigations of attenuation and dispersion of Rayleigh waves propagating on two- and three-dimensional stochastically rough surfaces [3-6] (see also the monograph [1]), one can write down the expressions for the attenuation coefficients $\alpha^{(2)}$ and $\alpha^{(3)}$ in the form:

$$\alpha^{(2)} = k_R\gamma + k_R B\left(\frac{\sigma}{a}\right)^2 (k_R a)^3, \tag{3}$$

$$\alpha^{(3)} = k_R\gamma + k_R G\left(\frac{\sigma}{a}\right)^2 (k_R a)^4. \tag{4}$$

Here σ is the root mean square deviation of the stochastically rough surfaces from the flat surface describing the average height/depth of irregularities, and a is the correlation length describing the average horizontal size of surface irregularities. Non-dimensional coefficients B and G depend on the Poisson's ratio ν of the elastic medium. They are constants for $k_R a \leq 2\pi$, i.e. at relatively low frequencies ω corresponding to the values of a being equal or smaller than the Rayleigh wave lengths $\lambda_R = 2\pi/k_R$. At higher frequencies, the values of B and G are reduced with frequency, limiting the power-law growth following from Eqs. (3) and (4). Note that the roughness-induced attenuation in Eqs. (3) and (4) is proportional to the fourth power of frequency for two-dimensional roughness (Eq. (3)) and to the fifths power of frequency for three-dimensional roughness (Eq. (4)), whereas the material-induced attenuation in the same equations is proportional to the first power of frequency.

It should be mentioned that the case of two-dimensional roughness is topographically anisotropic, and for oblique incidence of Rayleigh waves on the elements of roughness there are some specific features in the behaviour of Rayleigh wave scattering, depending on the angle of incidence [7, 8]. In this paper, we limit our consideration by considering only the case of Rayleigh wave propagation in the perpendicular direction to stochastically positioned parallel grooves and ridges, which is the most interesting for practical applications.

3. Numerical results and discussion

As follows from Eqs. (3) and (4), the effect of natural roughness of the ground surface, without landscaping, on Rayleigh wave attenuation is negligibly small for typical frequencies of traffic-induced ground vibrations, which is the sequence of very small values of $k_R a$ in this case. Therefore, stochastically rough surfaces specifically designed for ground vibration control should be created artificially, using appropriate excavating machines. In this way, both two-dimensional and three-dimensional rough surfaces can be created. The convenient choice of the correlation length a and of the mean square deviation σ could be, for example, 2 m and 0.3 m respectively. Such parameters of random surface would allow people to walk freely across the landscaped area, and it would be possible to plant threes or shrubs over there. In what follows we present some calculations for the above-mentioned roughness parameters, for illustration purposes.

The Rayleigh wave amplitude attenuation coefficient $\alpha^{(2)}$ for a two-dimensional rough surface was calculated according to Eq. (3) in the range of frequencies f = 0-100 Hz. Calculations have been carried out for the value of the ground Poisson's ratio ν = 0.25 and for the value of Rayleigh wave velocity c_R = 200 m/s. The value of the coefficient B has been determined using the results of the paper [5] obtained for ν = 0.25 - by adding up the partial coefficients accounting for the contributions of scattering into bulk elastic waves and into other Rayleigh waves, and by dividing the sum by two. This gave the value of $B = 0.138$. It should be noted that the contribution of the scattering into bulk elastic waves is predominant in the frequency range considered.

Calculations of the Rayleigh wave amplitude attenuation coefficient $\alpha^{(3)}$ for a three-dimensional rough surface have been carried out using Eq. (4). For calculations, the same values of roughness parameters were considered: $a = 2$ m, $\sigma = 0.3$ m, and the same values of the elastic parameters: $c_R = 200$ m/s and $\nu = 0.25$. The value of the coefficient G has been determined using the results of the paper [6] for $\nu = 0.25$. This gave the value of $G = 0.068$.

The calculated attenuation coefficients for two- and three dimensional roughness as functions of frequency are shown in Fig. 2. For convenience, the results are presented in Fig. 2 simultaneously for the combined attenuation coefficients (caused by the effects of roughness and material losses) for two- and three-dimensional roughness - by solid and dashed curves respectively, and for the attenuation coefficient caused by material losses alone - by a dotted curve.

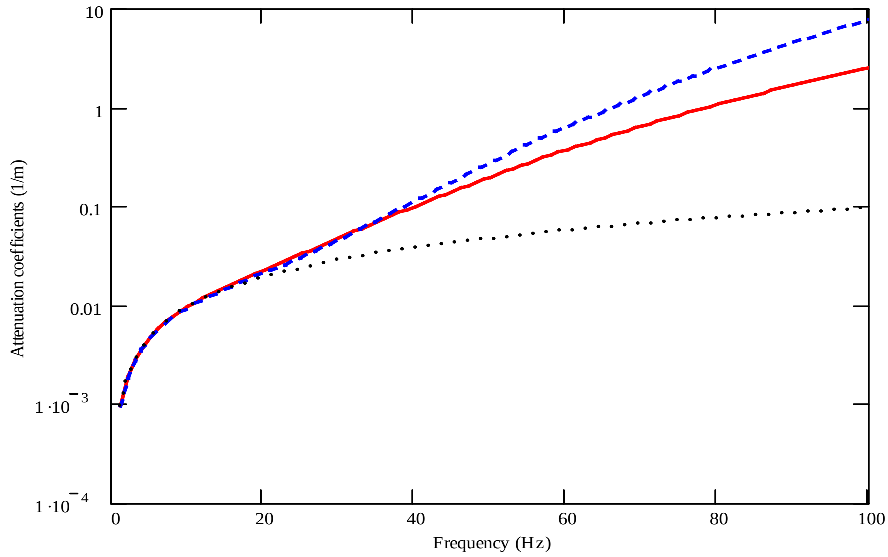

Figure 2: Combined attenuation coefficients for two-dimensional (solid curve) and three-dimensional (dashed curve) rough surfaces; a dotted curve shows the ground material attenuation alone.

It can be seen from Fig. 2 that at frequencies below 20 Hz the effect of roughness is negligible, and the combined attenuation coefficients are determined largely by the ground material attenuation. Above 20 Hz, the contributions of roughness to Rayleigh wave attenuation become dominant. In the case of three-dimensional rough surfaces, the roughness-induced attenuation grows faster than in the case of two-dimensional roughness, as expected.

The total amplitude attenuation (reduction) of Rayleigh waves passing through a strip of two- and three-dimensional rough surfaces of length L (see Fig. 1) is shown in Fig. 3. Calculations have been carried out using Eqs. (3), (1) and (4), (2) respectively. The parameters of rough surface used in the calculations were the same as in Fig. 2. The value of length L has been chosen to be 20 m. As in Fig. 2, the combined effects of roughness and material losses are shown by a solid and a dashed curves for two- and three-dimensional cases respectively, whereas the contribution of the material losses alone is shown by a dotted curve. It can be seen that, in agreement with the results for attenuation coefficients shown in Fig. 2, the roughness-induced suppression of Rayleigh waves becomes substantial at frequencies exceeding about 20 Hz, and the almost total suppression takes place at frequencies above 55 Hz - in the case of two-dimensional roughness, and above 50 Hz - in the case of three-dimensional rough surfaces.

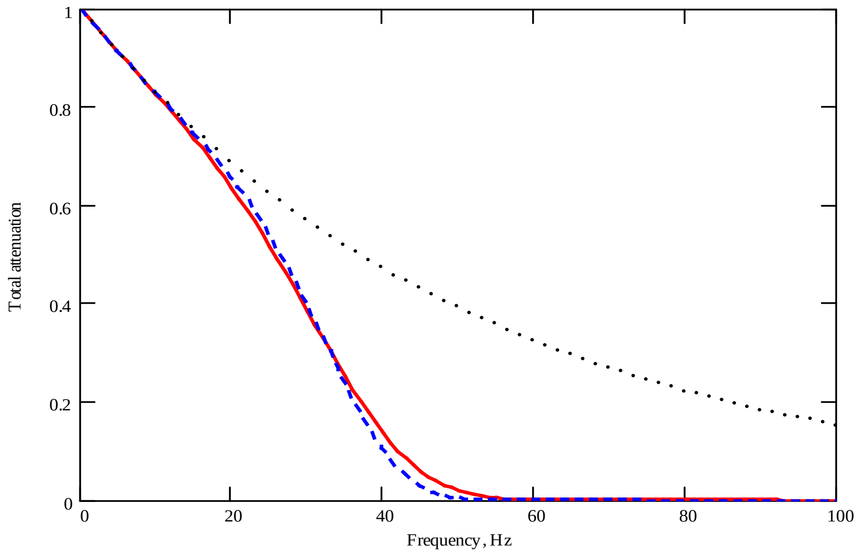

Figure 3: Total amplitude attenuation of Rayleigh waves passing through two-dimensional (solid curve) and three-dimensional (dashed curve) rough surface areas of length L= 20 m; a dotted curve shows the effect of the ground material attenuation alone.

4. Reduced-scale ultrasonic measurements

It is useful to compare the expressions for attenuation coefficients of Rayleigh waves propagating along two- and three-dimensional stochastically rough surfaces that have been used for calculations in the previous section, Eqs. (3) and (4), with the experiments.

In particular, in the paper [9] investigations of Rayleigh wave propagation over two-and three-dimensional rough surfaces have been carried out at ultrasonic frequencies 0.6 - 1.5 MHz, which corresponded to the values of $k_R a$ in the range 0.08 - 0.20. The two-dimensional rough area having the length L = 12 cm was prepared on the surface of a sample made of Aluminium by scratching the surface with a sand paper. The three-dimensional rough area of the same length was prepared by impacting the same sand paper into the surface. The roughness parameters were evaluated using a microscope. The estimated values of σ were 20 µm and 30 µm for two- and three-dimensional rough areas respectively. The value of a was 60 µm in both cases.

Measurements have been carried out at 12 different frequencies using 12 pairs of piezoelectric wedge transducers. Attenuation coefficients α_a were calculated in dB/cm using the formula $\alpha_a = (20/L)lg(U_0/U)$, where U and U_0 are the amplitudes of the Rayleigh wave signals passing over rough and smooth (reference) surfaces respectively. The obtained values of the attenuation coefficients in two- and three-dimensional cases, $\alpha_a^{(2)}$ and $\alpha_a^{(3)}$ respectively, are shown in Fig. 4 using a logarithmic scale. It can be seen from Fig. 4 that the measured values of $\alpha_a^{(2)}$ are grouped along the straight line with the tangent of an angle m = 4, whereas the values of $\alpha_a^{(3)}$ are gathered along the straight line with the tangent of an angle m = 5. Such a behaviour illustrates proportionality of the attenuation coefficients to the fourth and fifths powers of frequency for two- and three-dimensional rough surfaces respectively, in agreement with the theory (see Eqs. (3) and (4)). However, the absolute values of the measured attenuation coefficients in Fig. 4 are substantially larger than those predicted by Eqs. (3) and (4). One of the possible reasons for such a discrepancy, as it was mentioned in the paper [10], could be the effect of additional scattering of Rayleigh waves on subsurface inhomogeneities formed as a result of mechanical damage during the preparation of rough surfaces used in the experiments.

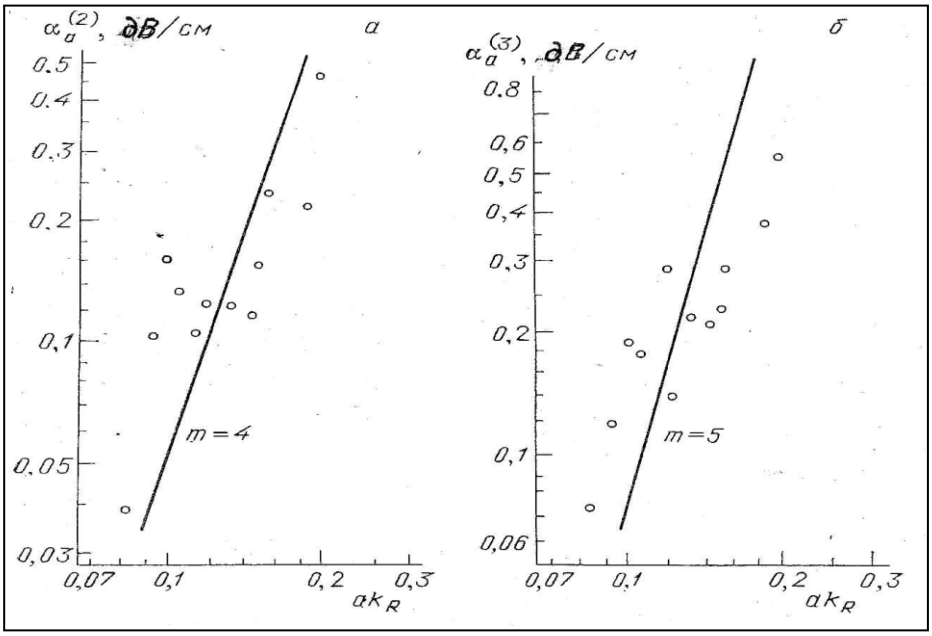

Figure 4: Attenuation coefficients of Rayleigh waves propagating over two-dimensional (left) and three-dimensional (right) rough surfaces of an Aluminium sample [9].

In the paper [11], measurements of Rayleigh wave reflection from the straight line boundary between smooth and rough areas of surfaces have been carried out, also at ultrasonic frequencies. One of the aims of that work was to check experimentally if the well-known simple scalar model of acoustic reflection from the boundary between two liquids (Fresnel's formula) can be applied to describe Rayleigh wave reflection in the case under consideration. Another aim was to evaluate the combined contribution of the roughness-induced attenuation and dispersion [1, 3, 4, 8, 10, 12] to the reflection coefficient, in agreement with the above-mentioned simple scalar model.

According to the scalar model of sound reflection and transmission, the reflection coefficient of acoustic waves incident at angle θ on the boundary between two liquids that differ from each other only by the values of sound velocity has the form

$$R(\theta) = \frac{\cos\theta - (n^2 - \sin^2\theta)^{1/2}}{\cos\theta + (n^2 - \sin^2\theta)^{1/2}}. \tag{5}$$

Here $n = c_1/c_2$ is the refractive index, where c_1 and c_2 are sound velocities in the first and second media respectively. Applying this model to Rayleigh waves propagating through the boundary between smooth and rough surfaces, one can express the associated refractive index as $n(\sigma,a,k_R) = 1 + \Delta n$, where Δn is the non-dimensional complex function describing the relative change in the complex Rayleigh wave velocity on a rough surface due to the effects of both attenuation and dispersion caused by roughness. The expressions for Δn are not given here for shortness. For the roughness parameters used $|\Delta n| << 1$. Therefore, Eq. (5) can be simplified as

$$R(\theta) = -\frac{\Delta n}{2\cos^2\theta}. \tag{6}$$

Measurements have been carried out on an Aluminium sample. The dimensions of the area with a three-dimensional roughness were 19 x 3.5 cm. The parameters of roughness were: σ between 20 and 50 μm, and a - between 100 and 140 μm. Generated Rayleigh wave pulses had the value of central frequency of 2.1 MHz. The results of the measurements are shown (by circles) in Fig. 5, along with the theoretical dependence $|R(0)|/cos^2\theta$ (solid curve) following from Eq. (6). The value of $|R(0)|$ for plotting the above curve was taken from the experimental values of $|R(\theta)|$ extrapolated to the area of $\theta \approx 0$.

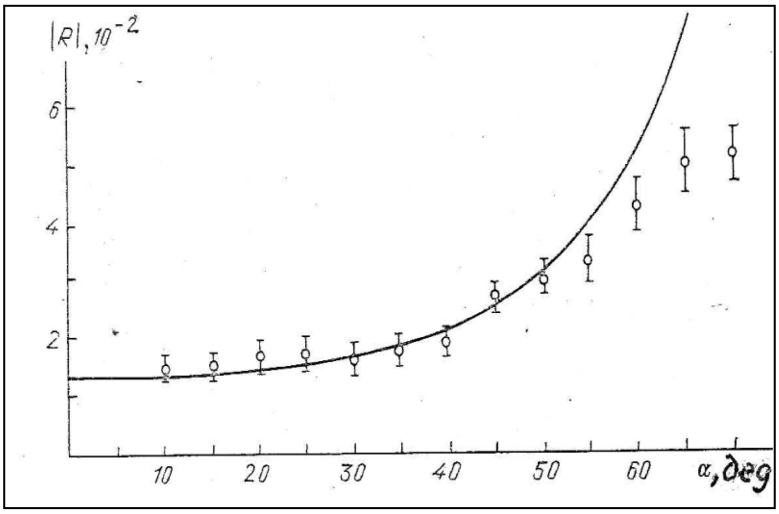

Figure 5: Angular dependence of the reflection coefficient of Rayleigh waves from the boundary between smooth and rough areas on the surface of an Aluminium sample [11].

It can be seen from Fig. 5 that the observed behaviour of $|R(\theta)|$ agrees well with the theoretical curve following from the simple scalar model. The values of $|R(\theta)|$ in Fig. 5 increase from about $1.5 \cdot 10^{-2}$ at small values of θ to about $5 \cdot 10^{-2}$ at angles about 70-80 deg. Such values of the reflection coefficient, although small, contribute to the overall amplitude reduction of transmitted Rayleigh waves.

5. Comparison with trenches

It is important to compare the efficiency of stochastically rough surfaces for reduction of traffic-induced ground vibrations with that of typical trenches used for the same purpose. Reduced-scale ultrasonic measurements of Rayleigh wave transmission through open trenches have been carried out in the recent paper [2] at a fixed frequency of 1 MHz. The depth h of the model trench made on the surface of an Aluminium sample was 0.7 mm, which was about 0.24 of the Rayleigh wavelength at this frequency (λ_R = 2.9 mm), and its width w was 1.5 mm. The measured value of the amplitude reduction factor (the transmission coefficient) for such a trench was 0.3. Assuming that the typical depth of a real trench in the ground is 2 m, this level of reduction would have been achieved for λ_R about 8 m, which for the ground with c_R = 200 m/s corresponds to the frequency of about 25 Hz.

The above mentioned value of reduction factor 0.3 indicates that at this frequency a typical single trench is more efficient than the rather wide strips of rough surfaces under consideration (with L = 20 m), where at the same frequency the reduction is only about 0.5 for both two- and three-dimensional rough areas (see Fig. 3). However, the efficiency of roughness rapidly increases for higher frequencies, whereas the amplitude reduction factor for a single trench does not decrease below about 0.1, as it follows e.g. from the paper [13] describing experimental measurements conducted at ultrasonic frequencies corresponding to the values of h/λ_R between 0.8 and 3.0.

6. Conclusions

It has been demonstrated in this paper that, for typical values of the parameters, stochastically rough surfaces as seismic barriers are less efficient than trenches at lower frequencies, but they are more efficient than trenches at higher frequencies.

Although stochastically rough surfaces are less efficient at lower frequencies, the advantage of their use is that they can be easily made and maintained, in comparison with trenches, and they can be used for planting trees and other vegetation, whilst remaining accessible for people and animals.

7. Acknowledgements

Part of the research reported here has been supported by EPSRC grant EP/K038214/1.

REFERENCES

1 Biryukov, S. V., Gulyaev, Y. V., Krylov, V. V. and Plessky, V. P., *Surface Acoustic Waves in Inhomogeneous Media*, Springer, Berlin (1995).

2 Azbaid El Ouahabi, A. and Krylov, V. V. Reduced-scale ultrasonic modelling of Rayleigh wave transmission over seismic barriers, *Proceedings of the International Conference on Noise and Vibration Engineering (ISMA 2016)*, Leuven, Belgium, 19-21 September, (2016).

3 Eguiluz, A. G. and Maradudin, A. A., Frequency shift and attenuation length of a Rayleigh wave due to surface roughness, *Physical Review B*, **28** (2), 728–747, (1983).

4 Huang, X. and Maradudin, A. A., Propagation of surface acoustic waves across random gratings, *Physical Review B*, **36** (15), 7827–7839, (1987).

5 Kosachev, V. V., Lokhov, Y. N. and Chukov, V. N., Theory of attenuation of Rayleigh surface acoustic waves on a free randomly rough surface of a solid, *Soviet Physics - JETP*, **67** (9), 1825–1830, (1988).

6 Kosachev, V. V., Lokhov, Y. N. and Chukov, V. N., On the theory of scattering the Rayleigh surface acoustic waves by a two-dimensional statistical roughness of a free solid surface, *Solid State Communications*, **73** (8), 535–539, (1990).

7 Mayer, A. P. and Lehner, M., Effect of random surface and interface roughness on the propagation of surface acoustic waves, *Waves in Random and Complex Media*, **4** (3), 321–335, (1994).

8 Kosachev, V. V. and Shchegrov, A. V., Dispersion and attenuation of surface acoustic waves of various polarizations on a stress-free randomly rough surface of solid, *Annals of Physics*, **240**, 225–265, (1995).

9 Krylov, V. V. and Popel, I. V., New experimental data on Rayleigh wave attenuation by rough surfaces, *Proceedings of the XIV All-Union Conference on Acoustoelectronics and Physical Acoustics of Solids*, Part 2 (in Russian), Kishinev, Moldova, (1989).

10 Krylov, V. V. and Smirnova, Z. A., Experimental study of the dispersion of a Rayleigh wave on a rough surface, *Soviet Physics - Acoustics*, **36** (6), 583–585, (1990).

11 Krylov, V. V., Experimental investigation of surface acoustic wave reflection from the rib of an obtuse-angled wedge and from the boundary of a rough surface area, *Proceedings of the XIII All-Union Conference on Acoustoelectronics and Quantum Acoustics*, Part 1 (in Russian), Chernovtsy, Ukraine, (1986).

12 Krylov, V. V. and Lyamov, V. E., Dispersion and damping of a Rayleigh wave propagating along a rough surface, *Soviet Physics - Technical Physics*, **49** (11), 1424–1425, (1979).

13 Zharylkapov, S.K. and Krylov, V.V., Scattering of Rayleigh waves by a groove of arbitrary depth, *Soviet Physics - Acoustics*, **33** (5), 509-511, (1987).