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A STUDY ON THE RELATION BETWEEN THE ACTUATOR PLACEMENT AND THE CHARACTERISTICS OF THE SOUND RADIATED BY A PLATE STRUCTURE

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One of the parameters influencing the performance of active control systems for the reduction of vibration or noise is the placement of the active components of the system (actuators) on the host structure. In active vibration control systems the actuators are normally placed following a criterion related to the kinetic energy on the structure, whereas in active noise reduction systems they are placed taking into account a sound pressure related metric. This numerical study investigates the placement of point force actuators on a generic plate structure within an active structural acoustic control system. Two setups are tested: a plate radiating in free field, and a plate radiating into a rectangular cavity. In both setups the plate is excited in its first natural frequencies by an incident diffuse sound field. The optimal placements of the actuators on the plate determined for a minimal sound power radiated in free field or a minimal acoustic potential energy radiated into the cavity are compared to the optimal actuator placements determined for a minimal mean loudness over the area of interest. The effect of the different actuator placements on the attenuation of the dominant radiation modes is analysed. The possible benefits of using the psychoacoustic loudness as an actuator placement criterion are discussed.

Keywords: ASAC, actuator placement, loudness

1. Introduction

The placement, along with the type, number and size of actuators is a significant factor in the design of active control systems. Different criteria can be defined to optimise the locations of the actuators in an active control system depending on the type of control and goal of the system (e.g. [1]). In active structural acoustic control (ASAC) a commonly used criterion for the placement of the control actuators on the structure to be controlled is the radiated sound power [2, 3].

The perception of sound and acoustic comfort play an increasingly important role .n numerous applications. In order to reliably describe the perceived characteristics of sound, psychoacoustic measures have been developed and are being progressively applied not only in sound design, but also in the field of active control of noise (e.g. [4]).

This numerical study investigates how the placement of actuators in an ASAC system based on a loudness criterion affects the sound attenuation and compares the results to more "conventional" placement criteria such as sound power or acoustic potential energy.

Section 2 briefly presents the theory used in this study to describe the sound radiation from vibroacoustically excited structures, as well as the formulation of an error criterion for an active control system. The models used to conduct the numerical study are described in Section 3, while the results for the two setups examined are presented and discussed in Section 4. Finally, some concluding remarks are given in Section 5.

2. Theoretical background

The goal of active control of vibroacoustically excited structures is to globally attenuate the system response with respect to a suitable measure. In the case of ASAC the global error criterion is a measure of the radiated sound power (or energy) expressed as a function of structural velocities. Thus the global error criterion J can be formulated as

$$J = \mathbf{v}^H \mathbf{A} \mathbf{v}, \tag{1}$$

where $\mathbf{v}$ is the vector of complex velocity amplitudes on the structure, H is the conjugate transpose operator, and $\mathbf{A}$ is a real, symmetric, positive-definite error weighting matrix [5].

For a structure radiating sound in free field the error criterion to be minimised is normally the radiated sound power L_w, which can be described as a function of the structural velocities and the radiation resistance matrix $\mathbf{R}$ [6]:

$$L_w = \mathbf{v}^H \mathbf{R} \mathbf{v}. \tag{2}$$

$\mathbf{v}$ is the vector of complex velocities of each elemental radiator that the structure is divided into.

For a structure radiating sound into a weakly coupled, fluid-filled contiguous cavity the error criterion for active control is the acoustic potential energy inside the cavity. The acoustic potential energy can be calculated using the modal interaction approach, according to which the response of the system can be modelled in terms of the in vacuo structural mode shapes of the vibrating structure, the acoustic mode shapes of the rigid-walled cavity, and the Kirchhoff-Helmholtz integral that links the sound pressure inside the cavity to the structural velocities on the vibrating structure. Using this theory the acoustic potential energy E_p inside the cavity can be expressed as

$$E_p = \mathbf{v}^H \mathbf{\Pi} \mathbf{v}, \tag{3}$$

where $\mathbf{v}$ is the vector of complex velocities of each elemental radiator that the structure is divided into and Π is the error weighting matrix for the acoustic potential energy [5, 7].

Assuming that the primary disturbance excitation of the structure is known, a feed-forward control strategy with secondary actuators on the structure can be used to minimise the formulated error criteria. Under the idealised assumptions of linearity and steady-state behaviour for the system, as well as total coherence of the signals (absence of other uncorrelated noise sources), an optimal feed-forward control can be expressed as the minimisation problem of a quadratic error criterion:

$$J_c = \mathbf{y}^H \mathbf{W}_y \mathbf{y} + \mathbf{v}_c{}^H \mathbf{W}_c \mathbf{v}_c, \tag{4}$$

where $\mathbf{y}$ is the error signal, $\mathbf{W}_y$ is the error weighting matrix, $\mathbf{v}_c$ is complex amplitude of the voltage applied to the secondary actuators, and $\mathbf{W}_c$ is the effort weighting matrix [3]. Since the radiated sound power in Eq. (2) and the acoustic potential energy in Eq. (3) already have the form of the first term in Eq. (4), they can be used as an error criterion for optimal feed-forward control.

3. Simulation models

As already mentioned, two setups are examined in this study: a thin plate excited by a diffuse sound field, radiating sound in free field, as well as the same thin plate radiating into a contiguous rectangular, air-filled cavity. The aluminium plate used for the numerical study has the dimensions L_x = 0.5 m, L_y = 0.3 m and thickness 0.005 m and is assumed to be baffled at its edges. For the second setup, the plate is bounded by a rectangular, acoustically rigid-walled cavity with the dimensions L_x, L_y and L_z = 0.23 m, which is filled with air (ρ_{air} = 1.204 kg/m^3, c_{air} = 343 m/s). The plate is modelled in ANSYS using four-noded shell elements (*shell181*) with an edge length of 0.01 m, while the air inside the cavity is modelled using eight-noded acoustic fluid elements suitable

Table 1: Structural modes of the plate (left) and acoustic modes of the cavity (right). The modal indices (a, b) correspond to the plate dimensions (L_x, L_y) and the modal indices (l, m, n) to the cavity dimensions (L_x, L_y, L_z) respectively.

Mode shape (a, b)	Eigenfrequency $\omega_{a,b}/(2\pi)[Hz]$	Mode shape (l, m, n)	Eigenfrequency $\omega_{l,m,n}/(2\pi)[Hz]$
(1,1)	352.77	(0,0,0)	0
(2,1)	507.31	(1,0,0)	343.06
(3,1)	775.47	(0,1,0)	571.93
(1,2)	890.70	(1,1,0)	666.93

for fluid/structure interaction problems (*fluid30*) with the same edge length. The first four (in vacuo) structural modes of the plate, as well as the first four acoustic modes of the cavity are listed in Table 1.

In order to ensure a good observability of the system for the active structural acoustic control the velocities of the plate are (virtually) sensed at the nodes of every finite element that the plate model is divided into. Those are also the possible positions for the placement of secondary actuators on the plate, which are examined within the optimisation problem of the actuator placement. The secondary actuators are assumed to be ideal point forces. The placement optimisation problem is solved using a genetic algorithm due to its non-convexity.

The excitation used to excite the plate in both setups is a diffuse sound field. The diffuse sound field is modelled as a superposition of a great number of acoustic monopole sources with randomly distributed phases located on a hemisphere with a sufficiently large radius spanning over the receiving side of the plate [8]. For the current study 286 monopole sources are homogeneously distributed on a virtual hemisphere with a radius of 100 m and its centre located at the centre of the plate. The resulting pressure on each element of the plate is calculated using the monopole source strength and the free space Green's function for each monopole source [6] and subsequently adding up the contribution of each monopole source to calculate the total pressure for each structural element. Knowing the pressure, and thus the force on each element, the differential equation of motion can be formulated to describe the response of the plate to the diffuse excitation.

In order to investigate the effects of the ASAC system on each mode of the plate (or the cavity) one by one, different band-limited diffuse sound field excitations with different upper cutoff frequencies are simulated. For the plate radiating in free field the cutoff frequencies chosen are 490 Hz, 630 Hz, 850 Hz, and 1000 Hz, so that only one, two, three and four structural modes are excited respectively. According to the same principle for the acoustic modes in the cavity, the cutoff frequencies for the second setup are set to 300 Hz, 430 Hz, 620 Hz, and 675 Hz.

The presented structure models and excitations are used to calculate the performance of an optimal feed-forward ASAC system in terms of the resulting radiated sound power in the case of the plate radiating in free field, and in terms of acoustic potential energy in the case of the plate radiating into the rectangular cavity. A secondary path model describing the control path dynamics for an actuator configuration is calculated. The optimal control signals for the actuators, in this idealised delayless case of a known excitation, are estimated in the frequency domain by minimising the quadratic functions in Eq. 2 or Eq. 3 respectively, where the structural velocity comprises the disturbance part as well as the secondary excitation through the actuators. Using these equations the estimated performance of the ASAC system in terms of radiation attenuation can be calculated. A genetic optimisation algorithm retrieves the actuator positions that minimise the selected error function. Even though such an optimal feed-forward control algorithm cannot be employed in realistic applications, the result provides an estimation of the maximal performance limit of the active control system.

Since this study is also concerned with the perceived characteristics of the radiated sound, the

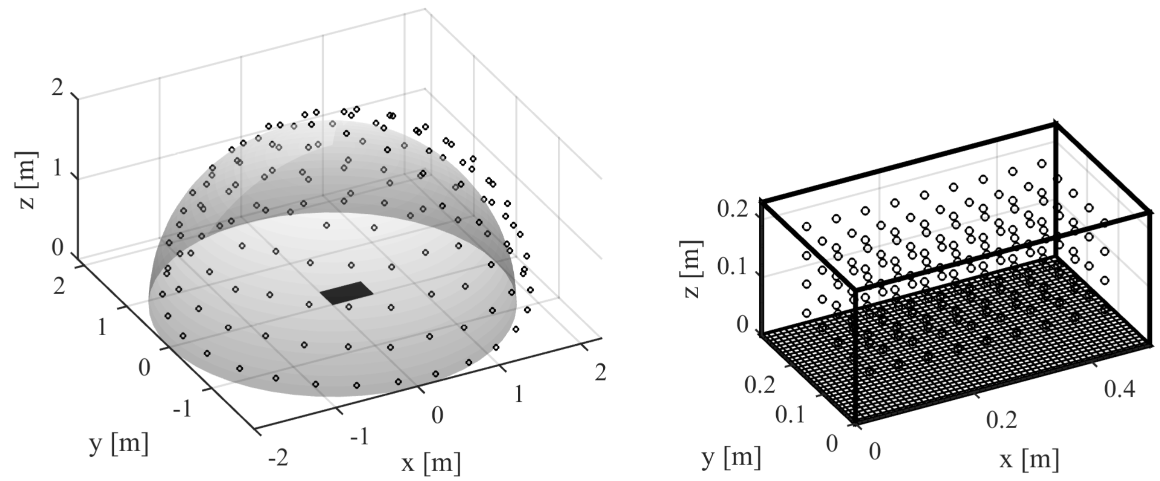

Figure 1: Loudness evaluation points for the plate radiating in free field (left) and the plate radiating into the rectangular cavity (right).

psychoacoustic loudness of the radiated sound is calculated. Loudness is a sound pressure related metric, thus an evaluation area for the sound pressure and hence the loudness has to be defined for each of the two setups. For the plate radiating in free field the sound pressure is calculated for 138 positions homogeneously distributed on a hemisphere with radius 2 m spanning over the radiating side of the plate using the Rayleigh integral formulation [6]. The selected radius is big enough to avoid being in the near field of the radiating plate and small enough to avoid measuring too small loudness values. For the coupled plate-cavity system the sound pressure is calculated for 180 positions inside the cavity using the modal interaction theory [6]. The evaluation areas are shown in Fig. 1. For the plate radiating in free field 77 points distributed on a plane twice as large as the plate parallel to the plate at a distance of 2 m from it are additionally used for the evaluation. The resulting loudness is subsequently calculated according to DIN 45631 [9]. The mean loudness over the corresponding evaluation areas is also tested as an optimisation criterion for the actuator placements. It should however be noted that the controller design for the ASAC system does not change in this optimisation case, i.e. it still minimises Eq. 2 or Eq. 3 respectively.

4. Results

Using the theory presented in Section 2 and the models from Section 3 an idealised optimal feed-forward ASAC system, where all the transfer paths and signal data are known, can be simulated. A series of numerical experiments is conducted in order to find the optimal placement for the secondary actuator(s) with respect to two different criteria. The results for the two numerical setus are presented and discussed in the following parts.

4.1 Plate radiation in free field

For the model of the aluminium plate excited by a diffuse sound field on one side and radiating in free field on its other side, the optimal feed-forward control is designed to minimise the radiated sound power. For the actuator placement the criteria used in the genetic optimisation algorithm are a minimal radiated sound power on the one hand, and a minimal mean loudness over a selected evaluation area on the other hand. Two different evaluation areas are considered for the loudness of the radiated sound: a hemisphere with radius 2 m spanning over the plate, and a parallel plane 2 m above the plate (s. Section 3). Four different diffuse sound field excitations that incrementally excite one to four structural modes of the plate are examined.

The results regarding the optimal actuator placement on the plate and the resulting loudness for a diffuse sound field excitation with an upper cutoff frequency of 850 Hz (which excites the first three structural modes) are shown in Fig. 2 for one actuator and in Fig. 3 for three actuators. At the bottom of each column the resulting sound power level (SWL), mean loudness on the hemisphere ($N_{mean,sph}$), and mean loudness on the evaluation plane ($N_{mean,pl}$) are given. The edges of the plate are outlined underneath the evaluation areas.

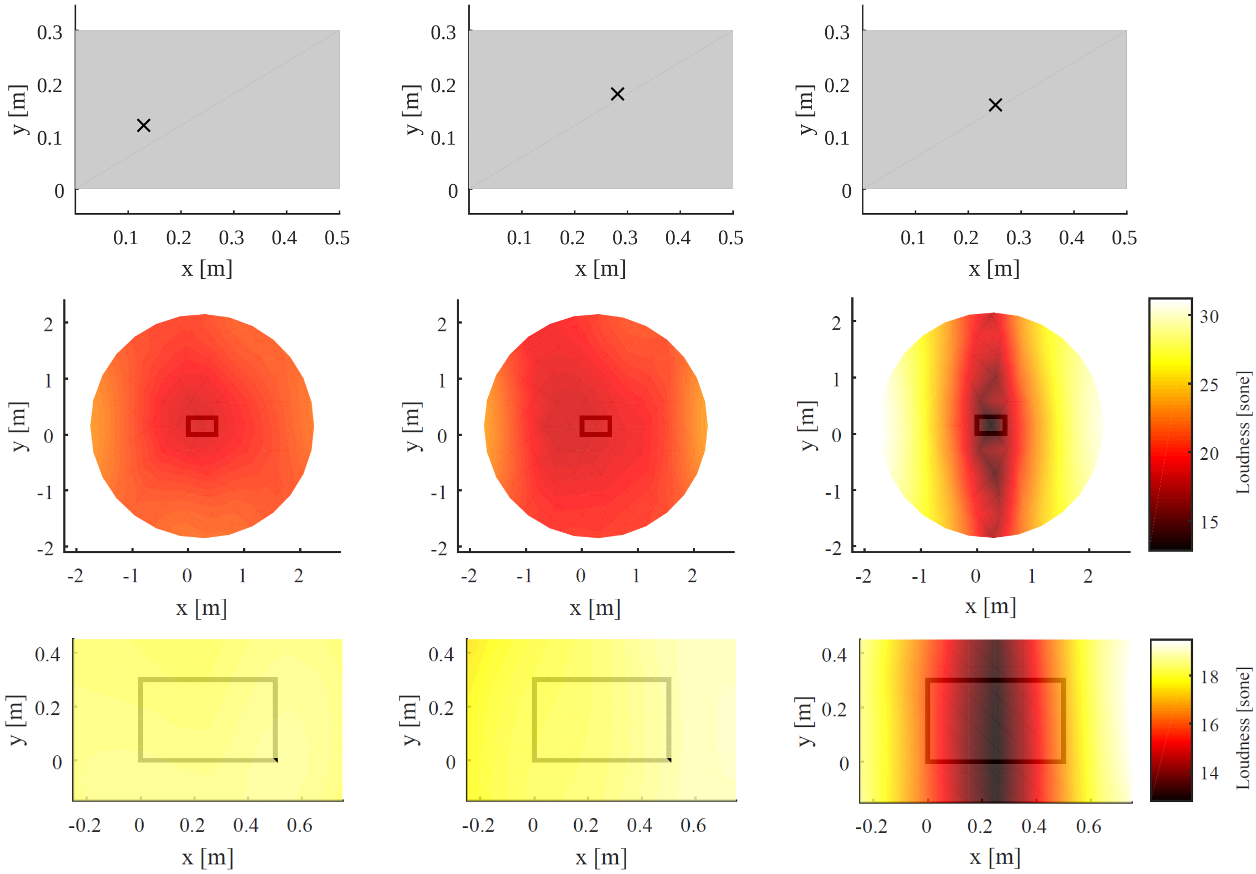

Figure 2: Optimal position for one point force actuator on the plate for a diffuse sound field excitation with an upper cutoff frequency of 850 Hz (first row) and the resulting loundess on the evaluation hemisphere (second row) and plane (third row). The columns correspond to the different placement optimisation criteria: first - minimal sound power; second - minimal mean loudness on hemisphere; third - minimal mean loudness on plane.

As can be observed the resulting actuator placements for the first two optimisation criteria (minimal sound power and minimal mean loudness on hemisphere) are very similar; thus is the resulting loudness. A similar behaviour can be observed for the other excitation ranges examined. The difference in the actuator placement (and hence the loudness) is slightly bigger when the number of actuators used is smaller than the number of modes excited by the primary disturbance (s. Fig. 2). In this case the actuator placement is chosen to suppress the radiation modes in a way that leads to a smaller loudness value. However, since the optimal control leads to a drastic attenuation of the radiated sound, the resulting differences in sound power and loudness are very small.

A different behaviour of the system can be observed for the third placement criterion, a minimal mean loudness on the evaluation plane (third column in Fig. 2 and 3). Here the resulting actuator placement and loudness distribution are different from the previous two cases. This can be explained

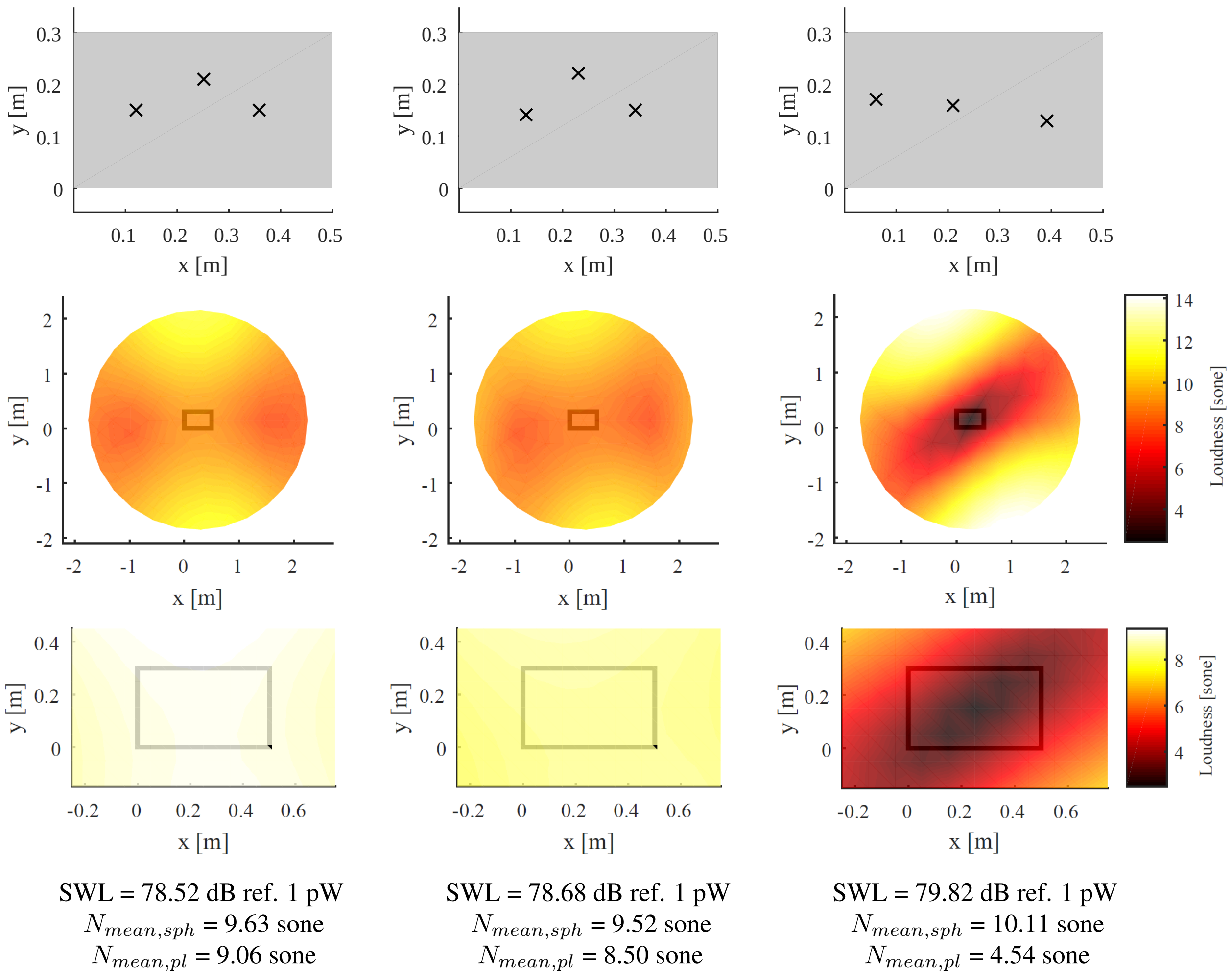

Figure 3: Optimal position for three point force actuators on the plate for a diffuse sound field excitation with an upper cutoff frequency of 850 Hz (first row) and the resulting loundess on the evaluation hemisphere (second row) and plane (third row). The columns correspond to the different placement optimisation criteria: first - minimal sound power; second - minimal mean loudness on hemisphere; third - minimal mean loudness on plane.

by the fact that the evaluation plane is more sensitive to the directivity characteristics of the radiated sound in comparison to the hemisphere, which gives a better representation of the global radiation characteristics of the structure, since it spans at a distance where the far field radiation area of the plate starts to form.

4.2 Plate radiation into contiguous cavity

For the model of the aluminium plate excited by a diffuse sound field and radiating into a contiguous air-filled rectangular cavity, the optimal feed-forward control is designed to minimise the radiated acoustic potential energy inside the cavity. For the actuator placement the criteria used in the genetic optimisation algorithm are a minimal radiated acoustic potential energy and a minimal mean loudness over the 180 evaluation points inside the cavity. Four different diffuse sound field excitations that incrementally excite one to four acoustic modes of the acoustic volume are examined.

The results for the optimal actuator placement on the plate and the resulting loudness for a diffuse sound field excitation with an upper cutoff frequency of 675 Hz (which excites the first four acoustic modes) are shown in Fig. 4 for one actuator and in Fig. 5 for four actuators. At the bottom of each column the resulting acoustic potential energy level (APE), and mean loudness inside the cavity (N_{mean}) are given.

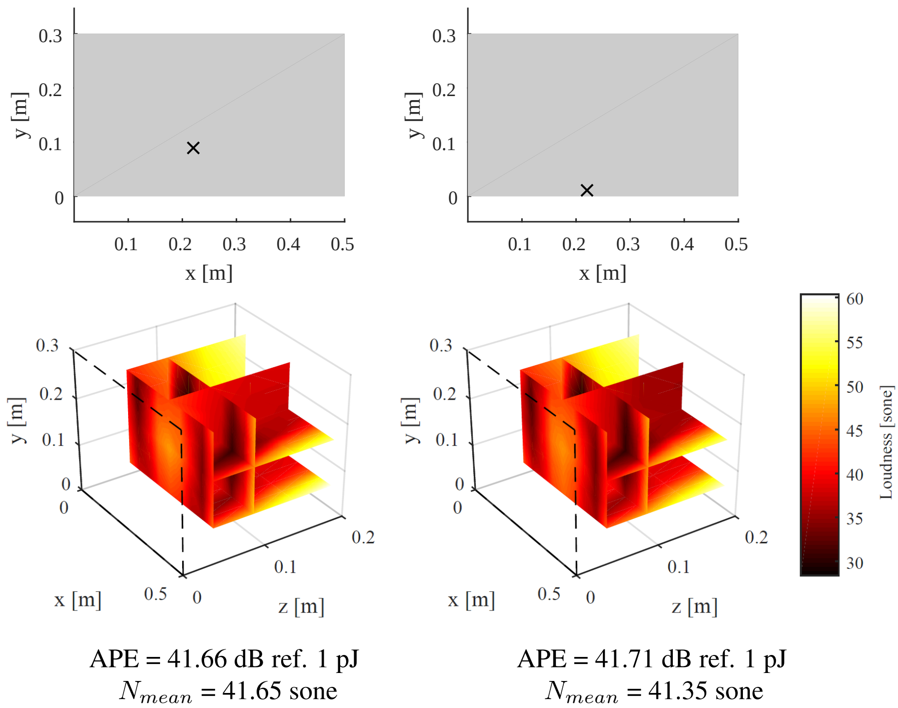

Figure 4: Optimal position for one point force actuator on the plate for a diffuse sound field excitation with an upper cutoff frequency of 675 Hz (first row) and the resulting loundess inside the cavity (second row). The columns correspond to the different placement optimisation criteria: left - minimal acoustic potential energy; right - minimal mean loudness in the cavity.

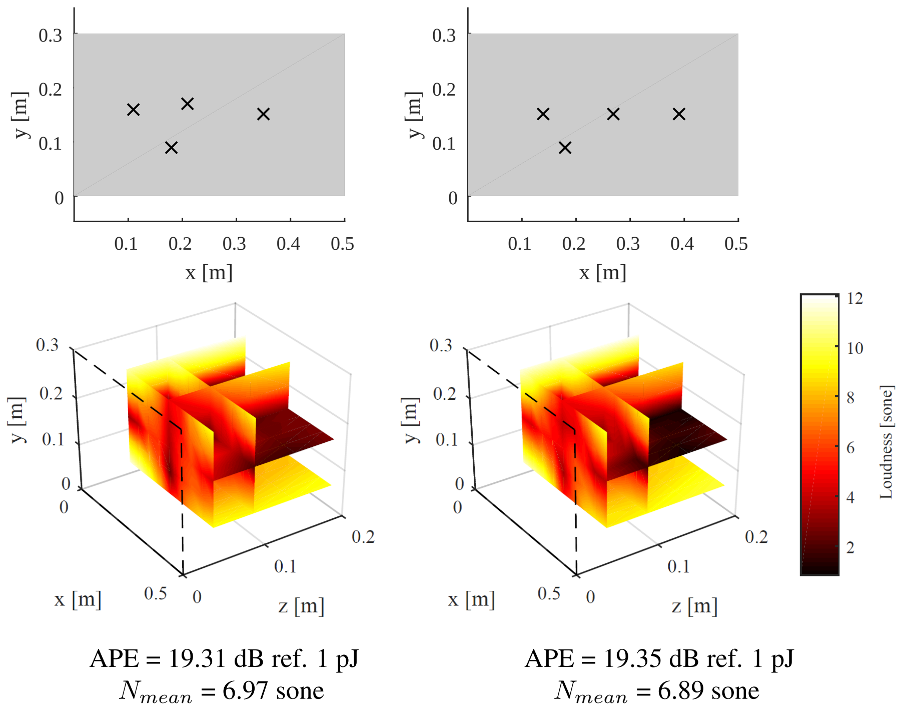

Figure 5: Optimal position for four point force actuators on the plate for a diffuse sound field excitation with an upper cutoff frequency of 675 Hz (first row) and the resulting loundess inside the cavity (second row). The columns correspond to the different placement optimisation criteria: left - minimal acoustic potential energy; right - minimal mean loudness in the cavity.

The resulting actuator placements for the two optimisation criteria, minimal acoustic potential energy and minimal mean loudness inside the cavity, are similar. This is particularly the case when four actuators are used, which means that the control system has very good authority over the four acoustic modes that are excited in this instance. When a smaller number of secondary sensors is used the resulting placements and loudness values show slightly bigger differences, which means that the actuators position is chosen to lead to a more effective suppression of the radiation modes that contribute more to the resulting loudness. This tendency can be observed for all the tested excitation/actuator configurations. As in the free field model these results and the absence of bigger differences can be explained by the extreme attenuation achieved by the optimal control.

5. Conclusions

This study investigates the influence of the criterion used for the optimisation of the actuator placement within an ASAC system for two different numerical setups: a plate radiating sound in free field, and a plate radiating into a contiguous rectangular cavity. In particular the mean loudness over a certain evaluation area is chosen as a placement criterion and the results are compared to criteria such as minimal sound power or minimal acoustic potential energy.

The different actuator placement criteria lead to similar results when the loudness evaluation areas describe the global radiation characteristics of the structure (hemisphere). Slightly bigger differences can be observed when the control authority of the ASAC system is limited due to an insufficient number of control actuators. Since the idealised optimal control used here leads to an unrealistically high attenuation of the radiated sound, these results are not adequate to make a reliable prediction of the behaviour of a realistic ASAC system. The observed phenomena should be investigated in an experimental setup as part of future works.

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