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HARMONIC GENERATION DUE TO NON-LINEAR PROPAGATION IN A FOCUSED ULTRASONIC FIELD.

V.F. Humphrey, M. Burgess and N. Sampson

School of Physics, University of Bath, Claverton Down, Bath, BA2 7AY, United Kingdom.

ABSTRACT

A series of experimental observations of the propagation of ultrasound in a focused wavefield are presented. The wavefield studied was formed by focusing 2.08 MHz ultrasound from a planar transducer with a Perspex (Polymethylmethacrylate) lens to give a linear focusing gain of 12.5. The build up and decay of the second harmonic on the acoustic axis was observed in detail and the relative phase of the second harmonic measured. These results are compared with theoretical calculations based on the theory of Lucas and Muir (1983) [1]. The overall results are in very good agreement with the theoretical predictions in the focal region although departures are observed outside this region. These are tentatively ascribed to the influence of higher harmonics not taken into account by the theory and the non-ideal nature of the fundamental field. A linear relationship between phase and harmonic order is also noted for the harmonics in the focal region.

INTRODUCTION

The non-linear propagation of sound is a well investigated phenomenon in underwater acoustics [2]. The non-linear distortion of acoustic waves has also been shown to be important at the frequencies and intensities used in medical applications of ultrasound [3], with experimental observations of the distorted waveforms produced by medical transducers being made in a water bath [4,5] and as a result of propagation through tissue [6]. The interest in non-linear distortion in medical systems centres on three aspects: firstly, the possible interactions of finite amplitude waves with tissue; secondly, the influence of non-linear propagation on system performance; and thirdly, the increased difficulty of calibrating medical transducers in a water bath.

Recent interest in non-linear propagation in the nearfield of transducers has resulted in a number of different approaches to the problem of combining non-linear propagation and diffraction. In this paper experimental results for a focused field, comparable with those used in medical ultrasonics, are compared with the theory of Lucas and Muir [1] for the finite amplitude field of a focused transducer. Although this theory only applies to the generation of the second harmonic at moderate levels of non-linearity it is considered to be of significance for two reasons. First, the generation of higher harmonics is controlled by the level of the second harmonic and, therefore, an understanding of what happens to the second harmonic is critical to an understanding of the development of the finite amplitude waveform. Second, the theory is equivalent to that of Ingenito and Williams [7] which has been of some significance in the understanding of non-linear propagation in the field of a plane transducer and has been a useful reference point for subsequent developments.

THEORY

Consider the fundamental field of a spherically concave focusing source with radius a and radius of curvature D driven uniformly at a frequency ω. The source is taken to be large compared with the wavelength so that $ka \gg 1$, where k is the angular wavenumber, and such that $a/D < 0.3$. For this case Lucas and Muir [8] found that the axial pressure variation $p_1(z)$ is given by

$$p_1(z) = \rho_o c u_o \frac{D}{(D-z)} \left\{1 - \exp\left[iR_o\left(\frac{1}{z} - \frac{1}{D}\right)\right]\right\} \exp i(kz-\omega t) \tag{1}$$

where z is the axial range measured from the source, R_o is the Rayleigh distance $(ka^2/2)$, ρ_o is the ambient density, c is the sound velocity and u_o is the amplitude of the source velocity. The pressure amplitude at the geometric focus $(z = D)$ is given by

$$|p_1(D)| = \rho_o c u_o \frac{R_o}{D} = p_o \frac{R_o}{D} = p_o G \tag{2}$$

where p_o is the average pressure amplitude at the source and $G(= R_o/D)$ is the linear focusing gain. The maximum pressure occurs just before this geometric focus.

The behaviour of the axial pressure is illustrated in Figure 1 where the normalised amplitude of p_1 is plotted as a function of z for $R_o = 1.5$ m and $D = 0.12$ m $(G = 12.5)$. Figure 1 shows the rapid rise in the primary pressure in the region before the focus and a number of zeroes of the primary pressure that occur on axis between the transducer and focus. These are similar to the axial zeroes that occur in the field of a plane transducer. These zeroes and the associated changes in amplitude and phase of the fundamental in the near-field before the focus are very important as they prevent non-linear effects building up consistently. Thus for both plane and focused transducers the main build up of non-linear distortion does not occur until after the last axial minimum.

Figure 1 also illustrates a significant difference between the fundamental field of a focused transducer and that of a plane transducer, i.e., the possibility of axial zeroes after the focus. Equation (1) shows that the first of these occurs at a range $z = R_o/(G-2\pi)$ only if $G > 2\pi$.

The above results apply to the fundamental field in the small signal (infinitesimal) limit and represent a first order solution of the wave equation. For higher primary pressure levels it becomes important to consider the non-linear nature of the wave equation and the resulting non-linear propagation. One approach to this type of problem is to introduce a second order approximation that allows for the generation of a second harmonic wave. By writing the overall wavefield as a sum of primary and second harmonic waves and substituting into the (non-linear) wave equation it is possible to obtain an equation for the second harmonic wavefield in terms of the fundamental wavefield. The solutions obtained with this perturbation approach are only valid for moderate non-linear effects as they do not allow for loss of energy from the fundamental as a result of the non-linear process or for the generation of higher harmonics due to the interaction of the fundamental and second harmonic.

Lucas and Muir [1] applied this approach to the case of a focusing source by using a description of the primary wavefield based on a parabolic approximation of the Helmholtz wave equation. They showed that the second harmonic axial pressure variation can be written as

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$$p_2(z) = 2i\rho_o \omega q_2(z) e^{i2(kz-\omega t)} \tag{3}$$

where q_2 can be written as

$$q_2(z) = \beta \frac{U_o^2 R_o^2}{cz} \int_{p=o}^{2} p\,dp\, e^{iR_o Qp} \int_{q=o}^{1-p/2} e^{iR_o q^2 (Q-1/2z)}\, dq \times \left\{ \frac{i\pi}{4} e^{-\frac{ik_o p}{2z}} H_o^{(1)}\left(\frac{R_o p}{2z}\right) - \int_{x=o}^{q} \frac{dx}{\sqrt{x^2+2p}} e^{\frac{ik_o x}{2z}} \right\} \tag{4}$$

where $Q = \frac{1}{z} - \frac{1}{D}$, $H_o^{(1)}$ is the Hankel function of the first kind and β is the parameter of non-linearity of the medium. This triple integral was evaluated numerically for the present geometry with the integral over x being performed with an optimised routine and those over p and q by a simple routine with the integrand being evaluated in complex form. The resulting normalised pressure amplitude calculated from equation (4) is shown in Figure 2. It can be seen that the second harmonic pressure does not start to increase significantly until about z = 0.09, about 10 mm beyond the last axial minimum of the fundamental before the focus. The second harmonic peak is narrower than that of the fundamental with the peak pressure occurring near to the geometrical focus.

For a plane wave non-linear propagation eventually results in a triangular shocked wave which may be written in terms of its Fourier Series as

$$p = \sum_{n=1}^{\infty} B_n \sin(n[kz-\omega t]-\phi_n) \tag{5}$$

where $B_n = 1/n$ and $\phi_n = 0$, i.e., all of the harmonics are "in phase" if the wave is expressed as a sine series. When the wave is no longer planar it is still useful to retain the same series representation of the distorted waveform. As it is not possible to measure the phase of the fundamental absolutely the fundamental must be used as a reference by choosing the time origin to make $\phi_1 = 0$. The remaining phases ϕ_n (n = 2,3...) then represent the relative phases of the harmonics (measured in terms of the period of harmonic concerned) with respect to the fundamental.

EXPERIMENTAL

The focused wavefield investigated was produced by a Panametrics V395 plane transducer with a plano-concave lens attached to its face with silicon grease. The perspex lens had a radius of curvature of 0.054 m and a geometric focus at a range of 0.12 m in water. The transducer was driven with a tone burst centred on a frequency of 2.08 MHz. The acoustic field was measured with a 1 mm diameter bilaminar PVDF membrane hydrophone [9] that had been calibrated at the National Physical Laboratory. This hydrophone had a relatively flat response over the frequency range 2-10 MHz, and had an end of cable sensitivity of 0.14(±15%) x 10^{-6}V Pa^{-1} at 2.25 MHz and 0.131(±20%) x 10^{-6}V Pa^{-1} at 5.0 MHz. The transducer and Hydrophone were both suspended from an optical bench using mounts which provided translational and angular adjustment. These enabled the transducer and hydrophone to be accurately aligned so that the hydrophone moved along the acoustic axis.

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The fundamental field was initially investigated for a low drive level to see how accurately it followed the theoretical model. Transverse measurements near to the transducer showed that the field was reasonably uniform with no indication of significant shading effects due to the lens. The measured axial variation is shown in Figure 1 and is compared with the theoretical prediction of equation (1). The theory was fitted to the experimental data by observing the positions of the minima of the field before and after the focus. The agreement shown was obtained with a value of R_o of 1.5 m as opposed to a value of 1.59 m based on the lens diameter of 38 mm. This corresponds to an effective diameter for the source of 36.9 mm or 3% less than its physical size. The resulting agreement between experiment and theory is very good especially in the focal region, although some deviation is apparent near to the axial zero after the focus. The acoustic field for higher drive levels was recorded at a number of ranges by photographing the hydrophone output displayed on an oscilloscope screen. The photographs were digitized by hand and the resulting data analysed to produce a Fourier Series for each waveform. A number of checks were made to ensure that no distortion was introduced during the photographic and digitization process.

RESULTS

The normalised waveforms observed at six different ranges on axis are shown in Figure 3, with two whole cycles of the fundamental included. At $z = 0.046$ m the waveform is almost undistorted with a second harmonic level 30 dB down on the fundamental. Between $z = 0.096$ m and $z = 0.113$ m the distortion increases rapidly with the second harmonic pressure rising from 17 dB to only 8.9 dB down on the fundamental. The waveform at $z = 0.113$ m (where the fundamental is a maximum) is noticeably asymmetric with a narrower and higher peak than trough. This asymmetry is due to the harmonics not being in phase with the fundamental as a consequence of the changing phase of the fundamental in the nearfield. These phase changes result in the wavefield at $z = 0.156$ m becoming more like a cycloid although the relative harmonic levels are similar to these at the focus. At $z = 0.231$ m, near the post focus minimum, the fundamental and second harmonic have comparable levels and the waveform displays narrow peaks as a result of high harmonic content. At longer ranges the waveform returns to a quasi-sawtooth shape with the same positive-negative asymmetry as before. The waveforms are shown as recorded by the hydrophone and are subject to the frequency response of the hydrophone and the finite size of its active element. The hydrophone was about 2.4 dB more sensitive at the fifth harmonic than the fundamental, although this was counteracted to some extent by the finite size of the hydrophone and the smaller spot size of the higher harmonics.

The axial behaviour of the second harmonic pressure amplitude, normalised to its maximum value, is compared with the theoretical predictions of equation (4) in Figure 2. The agreement is excellent in the focal region and generally good elsewhere. The results clearly show how the second harmonic remains at low level until just beyond the last axial zero of the fundamental and its subsequent rapid rise towards the focus. The experimental data initially follow the fall off beyond the focus accurately, but deviate significantly for $z > 0.16$ m. A similar, but more pronounced deviation, was observed by Lucas and Muir [1] for their system with a gain of 40. It is possible to attribute this deviation, as they did, to the non-ideal nature of the source, although the fundamental amplitude does not appear to deviate from the ideal until significantly further out. An alternative explanation is that the theoretical predictions, although adequate to describe the focal region are not adequate to describe the post focal region due the interactions of higher order harmonics

The measured fundamental pressure at the geometric focus was 0.94 ± 0.19 MPa (rms), for which the predicted second harmonic pressure was 0.38 ± 0.15 MPa compared with a measured value of 0.38 ± 0.10 MPa. Clearly the absolute agreement was very good with the main sources of potential error being attributable to the hydrophone calibration. The above results do not include any allowance for the effect of the hydrophone size. Numerical calculations indicate that the primary pressure measurement was approximately 3% low because of this factor with the second harmonic also influenced to a slightly greater extent.

Figure 4 shows the relative phase ϕ_2 of the second harmonic as a function of range compared with theoretical predictions obtained using equations (1) and (4). The phase has been plotted over the interval -90° to 270° for convenience. Clearly the agreement is very good over the range $0.10\ m < z < 0.17\ m$ about the focus, but significant deviations occur away from this region. In the focal region the lag of the second harmonic increases from 48° at $z = 0.10$ to 90° at about $z = 0.16$. The systematic deviation in phase beyond the focal region is almost certainly related to the deviations observed in the second harmonic amplitude. Note, however, that for $z > 0.25$ m the experimental results again approach the theoretical predictions.

Figure 5(a) shows the relative phase ϕ_n of the first few harmonics plotted against the order n for two ranges in the focal region. The phases have been unwrapped and clearly show a nearly linear relationship between ϕ_n and n in this region with the best straight lines crossing the zero phase axis near to n = 1. Thus the relative phase shifts are approximately proportional to (n-1) rather than n. This indicates that the phase shifts are not simply due to a shift of the fundamental phase relative to all of the harmonics; the situation is dynamic with the harmonics attempting to keep in phase with the fundamental as its phase varies along the axis. This is emphasised by Figure 5(b) which shows ϕ_n/n plotted against n; ϕ_n/n is a measure of the delay of each harmonic in terms of the fundamental period. This shows that the lower harmonics have a smaller time delay since they respond first to changes in the phase of the fundamental. The higher harmonics respond slower and, therefore, have a greater, almost constant, time delay relative to the fundamental.

CONCLUSIONS

The experimental results presented show how the non-linear distortion of a focused wavefield builds up rapidly in the short distance between the last axial zero of the fundamental and the geometric focus. The amplitude and phase measurements of the second harmonic on the acoustic axis in the focal region are in very good agreement with the theory of Lucas and Muir [1]. The asymmetric distortion of the waveform is observed to result from the relative phase of the harmonics, with an almost linear relationship between relative phase and harmonic order existing in the focal region.

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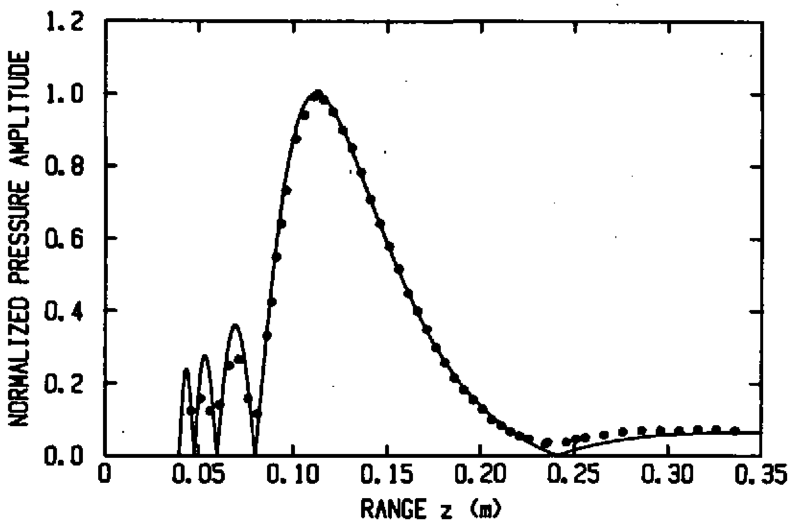

Figure 1. Normalized fundamental pressure amplitude along the acoustic axis for G = 12.5 and D = 0.12. ——, theory; ●, experimental measurements.

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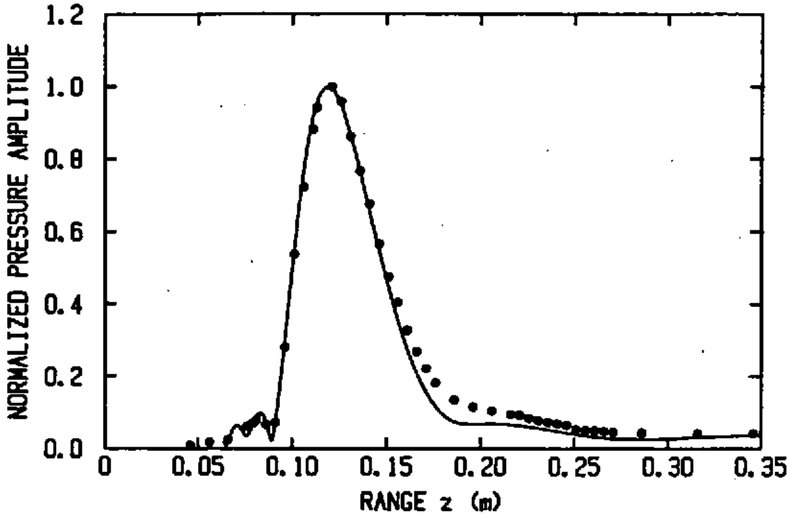

Figure 2. Normalized second harmonic pressure amplitude along the acoustic axis for G = 12.5 and D = 0.12. ——, theory; •, experimental measurements.

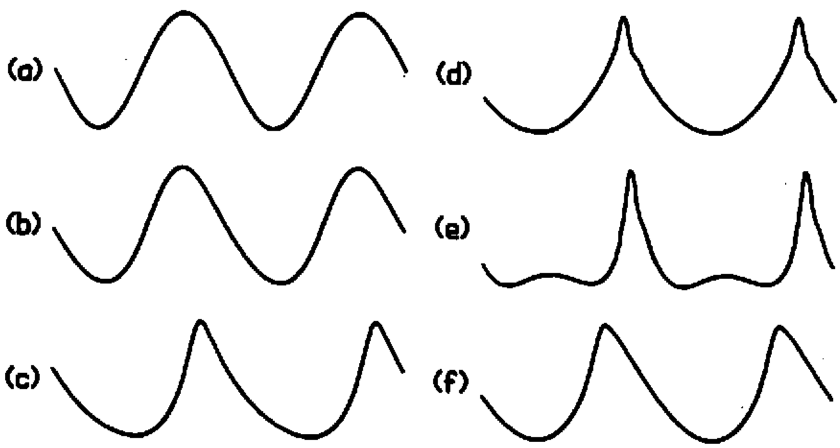

Figure 3. Normalized experimental waveforms observed at different ranges along the acoustic axis. (a) z = 0.046 m (37 mV); (b) z = 0.096 m (0.25 v); (c) z = 0.113 m (0.41 v); (d) z = 0.156 m (0.21 v); (e) z = 0.231 m (22 mV); (f) z = 0.396 m (21 mV). Figures in brackets are peak to peak output voltage of hydrophone. Two whole cycles of the fundamental frequency are shown.

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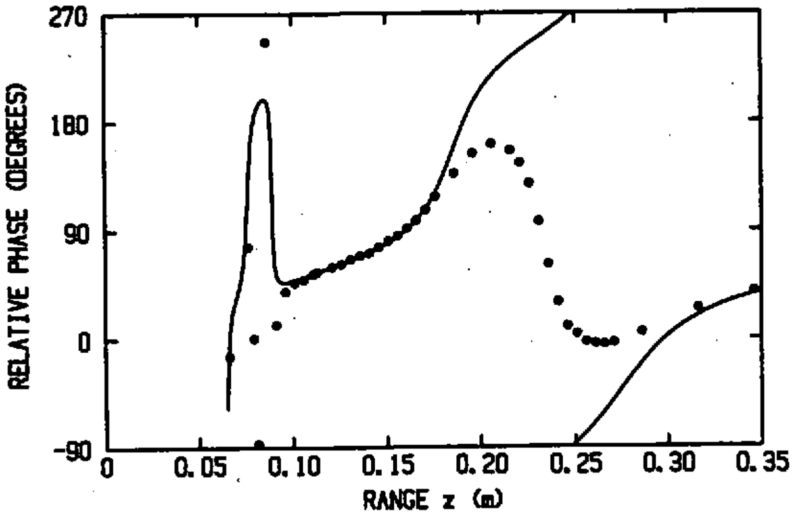

Figure 4. Relative phase ϕ_2 of the second harmonic along the acoustic axis. ——, theory; ●, experimental measurements.

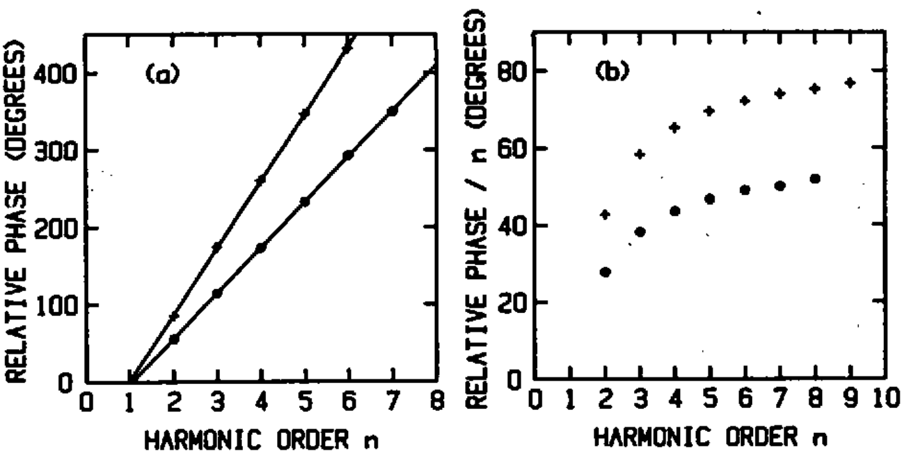

Figure 5. Relative phase of harmonic vs harmonic order n for waveforms at z = 0.113 m (●) and z = 0.156 m (+). (a) Relative phase ϕ_n vs n; (b) ϕ_n/n vs n.