NON-LINEAR ACOUSTICS AS A LABORATORY TOOL

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ABSTRACT

The use of non-linear propagation to generate additional frequencies of sound for use in the acoustics laboratory is considered. The advantageous characteristics of the fields generated in this way are described, as are the precautions to be taken when applying the fields to make measurements. The use of parametric arrays is illustrated with a number of examples including measurements of material transmission properties, scattering from cylinders and studies of edge diffraction. The possible use of harmonics generated by non-linear propagation is also discussed.

1. INTRODUCTION

The basic non-linearity of the differential equations describing acoustic-wave propagation in fluids has been known for a long time (see, for example, the review by Beyer [1]). However, much of the recent research on non-linear acoustics has concentrated on practical systems in which non-linear effects are significant. Some of this work has been stimulated by situations where non-linear effects occur (as a consequence of the high amplitudes used) but were not originally required or perhaps even expected! The emphasis is usually on understanding how non-linear effects might affect or degrade the system performance.

There are also, however, systems in which the non-linear effects may be used to advantage. Here the emphasis is on enhancing or using the non-linear behaviour to obtain an improved system performance. In some cases the non-linear effects can be exploited to obtain information or make measurements that would otherwise be difficult.

This division into unintentional and intentional effects is not always clear. The considerable interest in the non-linear propagation of lithotripter fields is an example. The non-linear distortion of the waveforms may just be an inconvenience that complicates calibration measurements. Alternatively the non-linear distortion and/or cavitation may be essential to the stone breaking efficiency of the system. If this is the case then a full understanding of the non-linear propagation may enable the stone breaking efficiency to be enhanced.

There are a number of phenomena associated with non-linear propagation [1] that can be exploited in practice. These include the following:

(1) The generation of a lower frequency wave by the interaction of two high frequency waves. This is exploited in parametric arrays.
(2) The generation of harmonics of a wave as it propagates leading eventually to the formation of shock fronts.
(3) Radiation pressure.
(4) The net flow, or streaming, of the fluid in the acoustic field.

(5) The generation, oscillation and potentially violent collapse of vapour bubbles within the fluid, known as cavitation.

All the above effects have potential applications in a wide range of fields. For example cavitation is widely exploited in ultrasonic cleaning baths and may possibly be the principal factor leading to the destruction of kidney stones in extracorporeal shock-wave lithotripsy [2]. In addition to these "direct" phenomena the generation of harmonics and resulting increased attenuation may lead to further advantages. These include increased heating effects, of potential use in medical applications, and enhanced streaming [3].

This paper concentrates on the use of non-linear propagation to generate additional frequencies of sound with convenient properties (by processes (1) and (2)) and their subsequent use as a tool in the acoustic laboratory. The numerous oceanographic and medical applications of these types of source will not be covered. The aim of the paper is to illustrate the wide range of applications as well as the advantages and disadvantages of these non-linear sources of sound.

2. PARAMETRIC ARRAYS

2.1 Review

A parametric array uses the non-linear propagation of primary wavefields to generate additional lower frequency (secondary) components that are then used to make measurements. The principle of a parametric array was first proposed by Westervelt in 1960 [4] and has since been used in a range of sonar applications. There are two characteristics of this type of source that make it ideal for use as a tool in the acoustics laboratory.

Firstly, it can produce beams with a narrow cross-sectional area in the nearfield region. This greatly reduces the problems associated with multiple reflections from tank walls or the water surface in small tank facilities. In addition the "region under test" can be confined, enabling measurements to be made on relatively small samples, areas or volumes without significant edge effects.

Secondly, the parametric array may be used to produce short pulses with a very wide frequency content. This greatly facilitates testing as a function of frequency by Fourier analysis and is a considerable advantage in many acoustic studies where it enables different contributions to the resultant wavefield to be isolated, identified and analysed.

So far parametric arrays have found applications in laboratory studies of a wide range of phenomena [5]. These include studies of transmission and reflection properties of isotropic and non-isotropic materials in the form of panels [6,7]; studies of scattering from rough surfaces [8]; studies of scattering from objects of different geometry [9,10]; transmission through bubble clouds [11]; scattering from fish [12]; studies of diffraction by acoustic barriers and baffles [13] and investigations of the efficiency of acoustic wedges. They have also been used as sources in model measurements of ocean propagation and as a general source for hydrophone inter-comparisons. Further areas that are being considered for investigation include transmission properties of complex structures, modelling of acoustic scattering from sediment particles and multiple scattering effects.

2.2 Experimental Systems
A range of experimental systems have been used for this type of laboratory measurement but a typical configuration is shown in Figure 1.

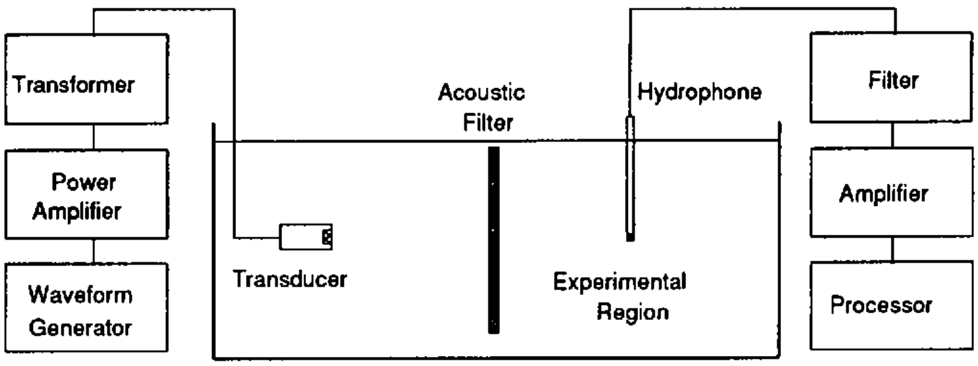

Figure 1. Typical experimental arrangement.

The parametric array is generated in the water tank by the non-linear interaction of high amplitude primary waves transmitted by a conventional transducer. The resulting secondary waves appear to come from "pseudo sources" distributed throughout the interaction region of these primaries. The wideband nature of the source depends on the fact that these sources can be altered by changing the primary frequencies transmitted within the bandwidth of the transducer. The transducer must, therefore, be reasonably efficient in order to generate adequate primary levels but must also have sufficient bandwidth. Typically the transducer is operated in the range 500kHz-2MHz, has a Q factor of about 6 and efficiency of more than 40%.

Although it is possible to drive the transmitting transducer with two frequencies to generate a single secondary frequency (useful in low signal to noise applications) most measurements are performed by transmitting a short pulse of carrier frequency (Figure 2(b)), often with a raised cosine bell envelope (Figure 2(a)). In this case the low-frequency secondary waveform generated on axis (Figure 2(c)) can be shown to be proportional to the second derivative, with respect to time, of the square of the transmitted pulse envelope [14]. Hence for a raised cosine bell envelope based on a 20kHz sine wave the resulting secondary signal has a -6dB bandwidth extending from 15kHz to 45kHz (Figure 2(d)). In addition the generated waveform shape and spectrum can easily be adjusted by altering the length and shape of the envelope function. Thus by using a raised cosine bell envelope based on a 40kHz sine wave the usable frequency range can be extended to cover 25kHz to 95kHz. A considerable range of suitable pulse envelopes may be obtained using a triggerable function generator with adjustable start phase, offset and symmetry controls. The required modulated carrier pulses are then obtained by amplitude modulating a continuous wave carrier signal with the required envelope. A peak output electrical power of 200W is typically adequate.

The resulting wavefields are detected with a small wideband hydrophone, low pass filtered and processed appropriately. The normal technique is to digitize the signals, average and transfer to a PC for subsequent analysis. Fourier analysis using FFT routines is usually used to obtain results as a function of frequency.

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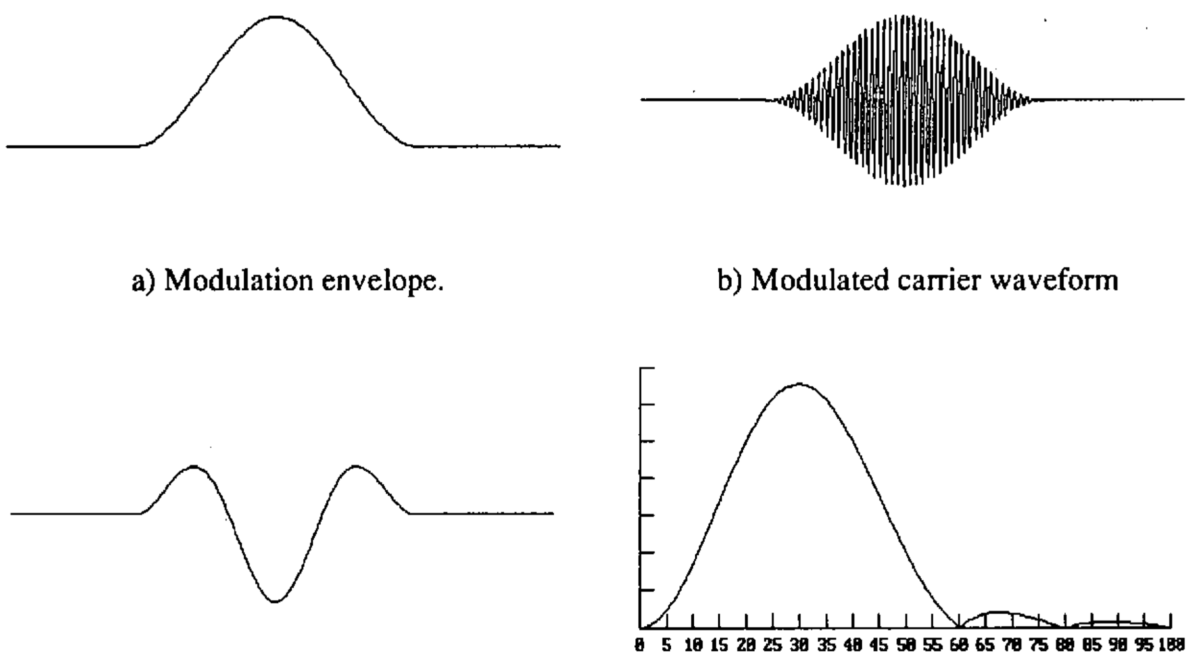

a) Modulation envelope.

b) Modulated carrier waveform

c) Secondary frequency waveform.

d) Spectrum of secondary frequency wave.

Figure 2. Signals involved in the generation of the secondary frequency wave (theoretical).

One of the principal difficulties of using a source based on a non-linear effect is that care must be taken so that other non-linear effects do not complicate or invalidate the measurement process. Hence it is important to ensure the following:

(a) That other non-linear effects, such as hydrophone non-linearity [15,16] or non-linearity of the receiving electronics are not important. The latter can be ensured by the use of a passive low-pass filter.

(b) That the required measurement can be isolated from non-linear propagation effects. So, for example, in scattering measurements it is important to ensure that non-linear generation after the scatterer is not significant [8].

These difficulties can be safely avoided in parametric array measurements by truncating the interaction region with an acoustic low pass filter, that is a material which transmits secondary waves but attenuates the primary waves. This produces a secondary source free region beyond the filter in which measurements may be easily performed without any of the above complications.

2.3 Field Characteristics

One of the main advantages of the parametric array is its characteristically narrow beam. The truncation of the array alters the characteristics of the beam, with beam broadening occurring beyond the acoustic filter. This is illustrated in Figure 3 which shows beam profiles for 50kHz produced by an array truncated at 0.52m. Measurements immediately behind the filter show the characteristic parametric array beam shape, while those measurements 0.32m beyond the filter show significant spreading with evidence of sidelobes starting to appear.

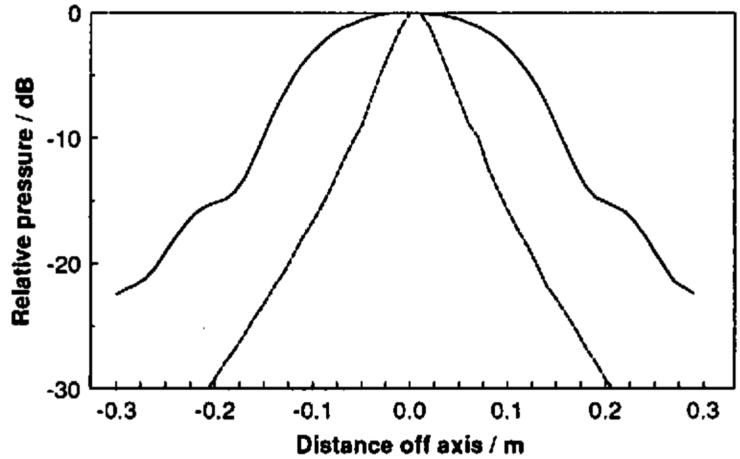

Figure 3. Beam cross-sections at 50kHz for a parametric array truncated at 0.52m; 0.025m (- - -) and 0.32m (—) beyond acoustic filter.

3. PARAMETRIC ARRAY APPLICATIONS

In this section a number of specific applications and examples of the type of results obtainable are presented.

3.1 Measurement of Transmission and Reflection Coefficients

A range of acoustic materials are required for sonar systems to act as windows, baffles and absorbers. It is important to be able to predict and measure experimentally the transmission and reflection properties of such materials over a wide range of frequencies and angles of incidence. Freefield measurements of such materials using a conventional source and receiver may be complicated by diffraction effects associated with the edge of the panel and may also require a number of sources to cover an extended frequency range. A parametric array is clearly suited to this application as the narrow beam cross-section can be used to minimise diffraction effects. In addition the wideband nature of the source enables measurements to be made over a range of frequencies [6,7].

Experimental measurements of transmission loss are made by recording the reference signal at the hydrophone $\psi(t)$ and then inserting the test panel between the acoustic filter and receiving hydrophone at an angle θ. The new transmitted waveform $\psi_t(t,\theta)$ is recorded and the respective spectra $\Psi(\omega)$ and $\Psi_t(\omega,\theta)$ calculated using a FFT. The transmission coefficient T can then be calculated using

$$T(\omega,\theta)=\frac{\Psi_t(\omega,\theta)}{\Psi(\omega)}$$

and the transmission or insertion loss TL using $TL = 10\,\mathrm{Log}(T(\omega,\theta))$. With the test panel rotated under computer control it is possible to perform a complete angular run in less than an hour obtaining detailed information over a wide range of frequencies and angles of incidence.

This technique has been used to make measurements of both reflection and transmission on a wide range of materials for the frequency range 10-250kHz [6,7]. The technique is not confined to isotropic or homogeneous materials. Figures 4 and 5 show some experimental results for a glass reinforced plastic (GRP) panel constructed with the fibres running mainly in one direction (uniaxial) and approximately 0.45x0.4m in size. For such a material the transmission depends not only on the angle of incidence from the normal (θ) but also on the angle between the plane of incidence and the fibre direction (ϕ). The results show the range and resolution obtainable. Figure 4 shows the transmission loss at normal incidence as a function of frequency while Figure 5 shows the results for 100kHz versus angle of incidence for $\phi = 45°$.

Measurements for acoustically thin isotropic panels show excellent agreement with plane wave predictions [6]. However, measurements on acoustically thicker panels show significant differences from plane wave theory [7] which can be attributed to making measurements with a source and receiver at close range in the test facility. These differences can be accurately predicted by representing the incident field in terms of its plane wave spectrum and considering the effect of the panel on each component of the angular spectrum. The resulting field beyond the panel can then be evaluated by integrating over all of the plane wave components [7]. This problem is not specific to measurements made with a parametric array but occurs for any such freefield measurement made at close range [17]!

The agreement obtainable is shown in Figure 6 which shows the transmission loss for a 12.7mm aluminium plate at 93.6kHz as a function of angle of incidence. The effect of the planewave spectrum of the source is to smooth out some of the rapid variations and shift them in angle. The agreement with the experimental results is very good. The same techniques can be applied to reflection measurements and a wide range of more complex materials and structures. The system has also been scaled up in size to enable measurements to be made at lower frequencies.

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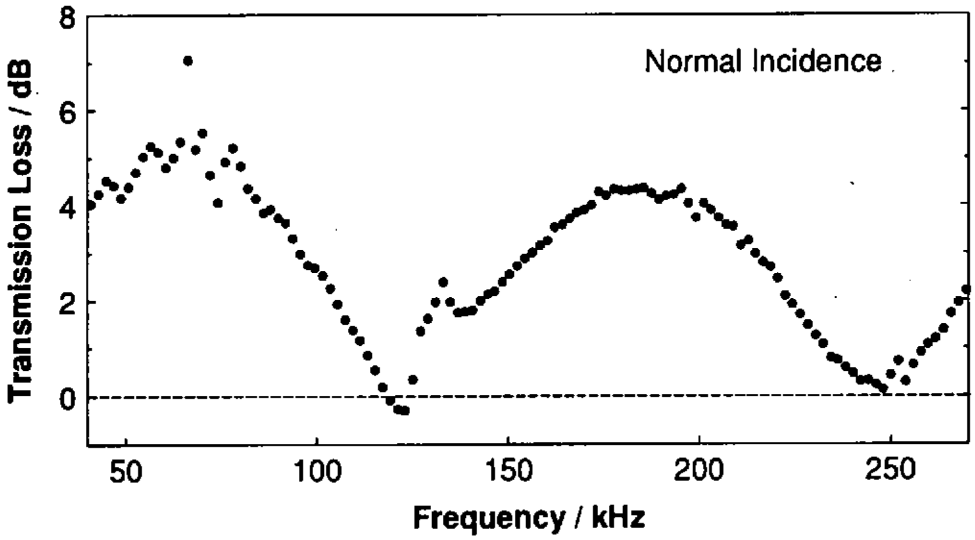

Figure 4. Transmission Loss as a function of frequency for a GRP uniaxial panel 10mm thick at normal incidence; experimental (points).

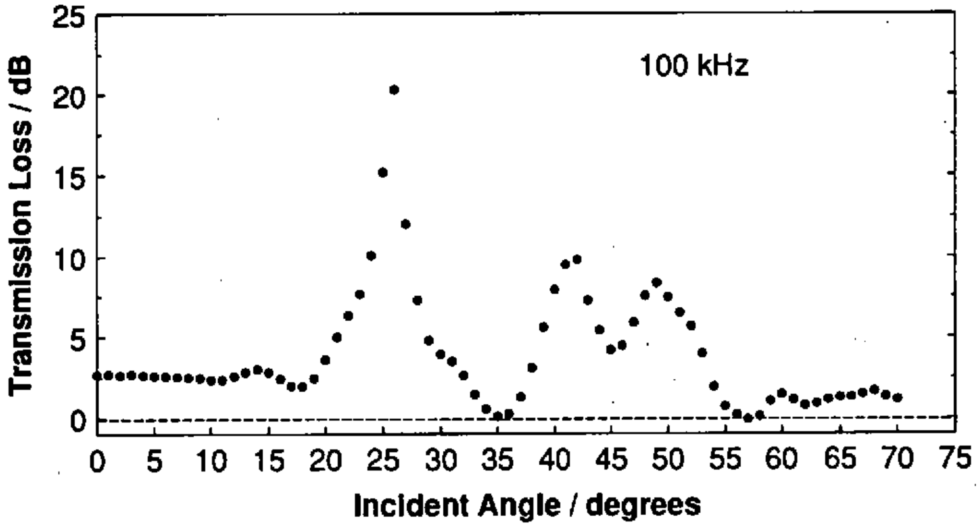

Figure 5. Transmission Loss as a function of angle of incidence for a GRP uniaxial panel 10mm thick with fibres orientated at 45° to plane of incidence; experimental (points) for 100kHz.

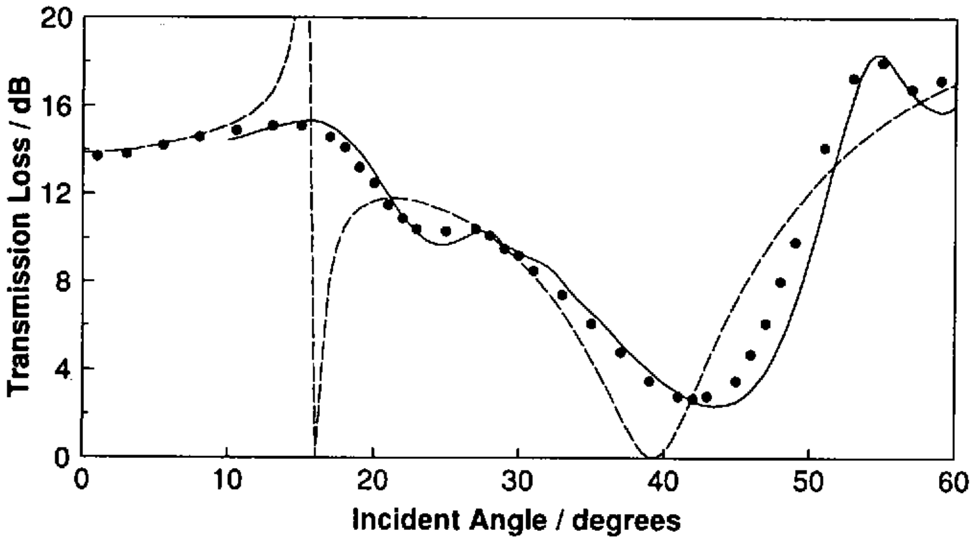

Figure 6. Transmission Loss as a function of angle of incidence for a 10.7mm thick aluminium panel at 93.6kHz; experimental (points), plane wave theory (- - -) and modified theory allowing for plane wave spectrum of source (—) [7].

3.2 Scattering Measurements

Measurements of scattering have always been of interest in underwater acoustic and other ultrasonic fields. Although theoretical predictions are available for simple regular geometries, experimental techniques are still required to investigate more complex structures, especially at frequencies where the scatterer and wavelength are of comparable size. A parametric source is ideally suited to this application as the size of the scattering objects enables some complexity to be realistically added.

The reported measurements of scattering [9,10] have utilised both the stability and narrow beamwidth of the parametric array. The stability of the source enables the low amplitude scattered signal to be isolated from any coherent background signal by subtraction of signals in the time domain. The narrow beam enables measurements to be made on finite length cylinders or cylindrical shells, for example, without end effects being significant.

As an example of the results obtainable consider a plane wave incident on an infinite cylinder of radius a at normal incidence; then the scattered field $\psi_s(\omega)$ at a range r' and observation angle ϕ is characterised by the form function $f(\omega, r', \phi)$ which is defined by

$$f = \frac{\psi_s(\omega)}{\psi(\omega)}\left(\frac{2r'}{a}\right)^{1/2} e^{-ikr'}$$

where $\psi(\omega)$ is the incident field. The form function is determined by the density and elastic properties of the cylinder and surrounding fluid and can be calculated using a normal mode solution [18].

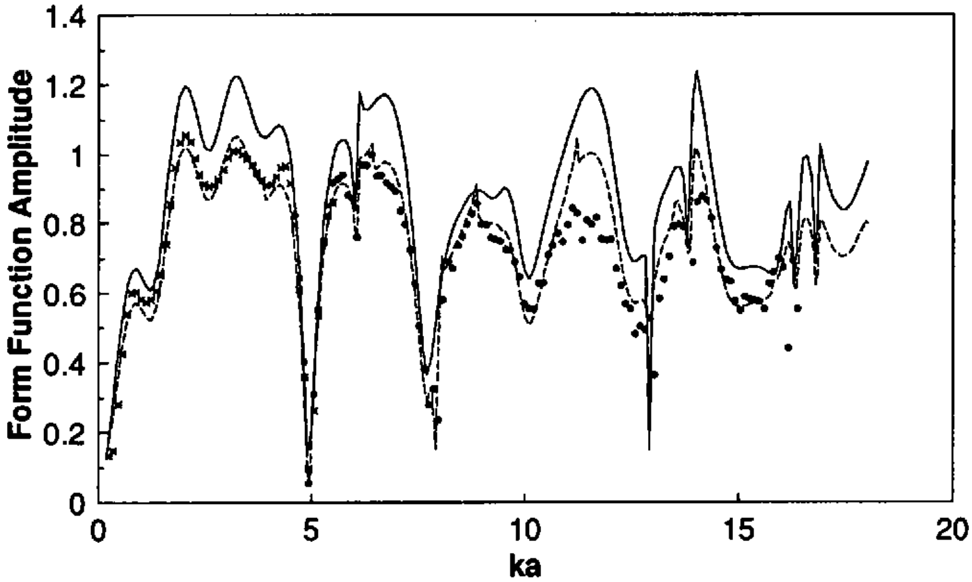

Figure 7. Form function for aluminium cylinder at normal incidence, observed at 90°; Experimental (points), plane wave theory (—) and modified theory allowing for plane wave spectrum of source (- - -) [10].

Figure 7 shows the form function measured at 90° to the incident direction for an aluminium cylinder 13mm in radius as a function of ka. The detailed broad-band nature of the results which cover the frequency range 10kHz to 270kHz should be noted. The theoretical predictions are for the single plane wave case and a modified theory which allows for the plane wave spectrum of the source [10]. The agreement with the modified theory is very good over most of the frequency range. It is important to note that the modified theory not only predicts the amplitude levels but also shows some sharp resonances seen in the experimental data but not on the plane wave result. These are attributed to helical waves excited by plane wave components incident on the cylinder at non-normal incidence. The average reduction in the amplitude of the modified form function by a factor of about 0.85 can be attributed to continued cylindrical spreading of the scattered wavefield as a result of the finite range of the source. The observed reduction agrees with that which would be expected if the effective acoustic centre of the array were half way between the transducer and acoustic filter.

3.3 Diffraction Studies

The effects of diffraction from the edges of materials are of significance in both the testing of materials and the use of baffles in sonar systems. If the baffle is a perfectly rigid or a perfectly soft half plane then the significance of diffraction can be calculated using the classic theory of acoustic diffraction obtained by Sommerfeld [19]. If the panel is elastic, and of finite thickness, no corresponding solutions exist. The need to be able to estimate edge effects for such a situation led to an experimental investigation of the phenomena [15].

In order to make this type of measurement the test panel was immersed in the tank with one edge on the axis of the truncated array. The hydrophone was arranged to move either in angle about the edge of the panel at a fixed range, or away from the edge of the panel (x) at a constant distance y from the back of the panel. The edge diffraction strength was calculated by normalising the diffracted signal by the incident signal level at the panel edge.

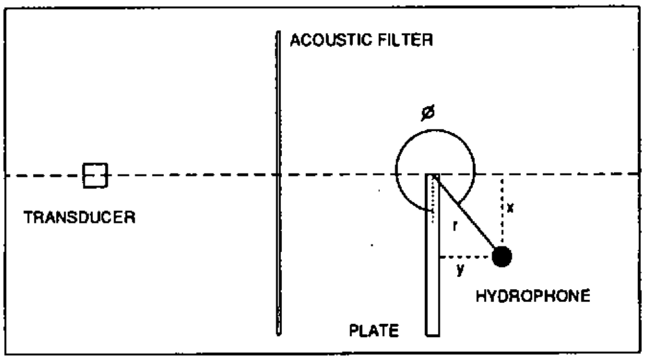

Figure 8. Experimental arrangement for making edge diffraction measurements.

Figure 9 shows the results of such an experiment on a soft half plane constructed by covering a 1.5mm aluminium plate with 4mm thick closed cell neoprene. The results obtained as a function of scattering angle (measured from the front surface) are in very good agreement with the predictions of theory except when the hydrophone approaches the back surface of the panel. This difference may be attributable to the finite (as opposed to infinitesimal) thickness of the test panel.

The complexity of the situation with real materials became apparent when the initial sample was replaced with a soft/hard panel formed by covering one side of a 10mm thick aluminium plate with closed cell neoprene. This plate was arranged with the "soft" neoprene towards the parametric array so that the transmission through the panel was minimal. This enabled the field in the shadow region to be investigated for an elastic back surface. Figure 10 shows that the situation is complex with not just the diffracted signal present. In fact the diffracted signal is preceded by two arrivals generated by plate waves travelling through the aluminium at speeds determined by the modes of the plate. These contributions are still under investigation but the key point to note from Figure 10 is that the short pulses generated by the parametric array enable the different contributions to be clearly isolated and studied.

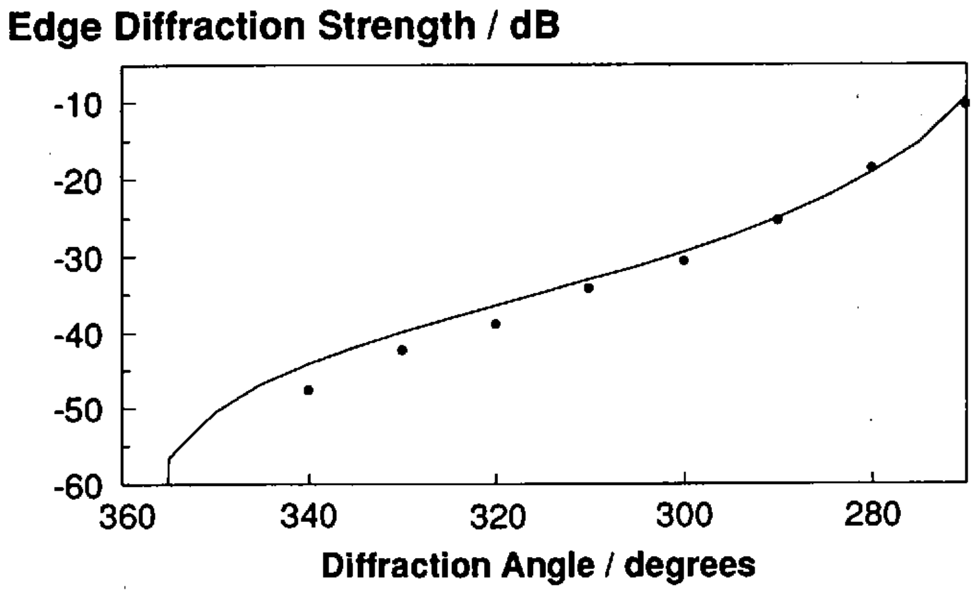

Figure 9. Edge diffraction strength for a soft/soft plate versus angle of observation measured at 0.3m from the edge at 100kHz; experimental (points) and theory (—).

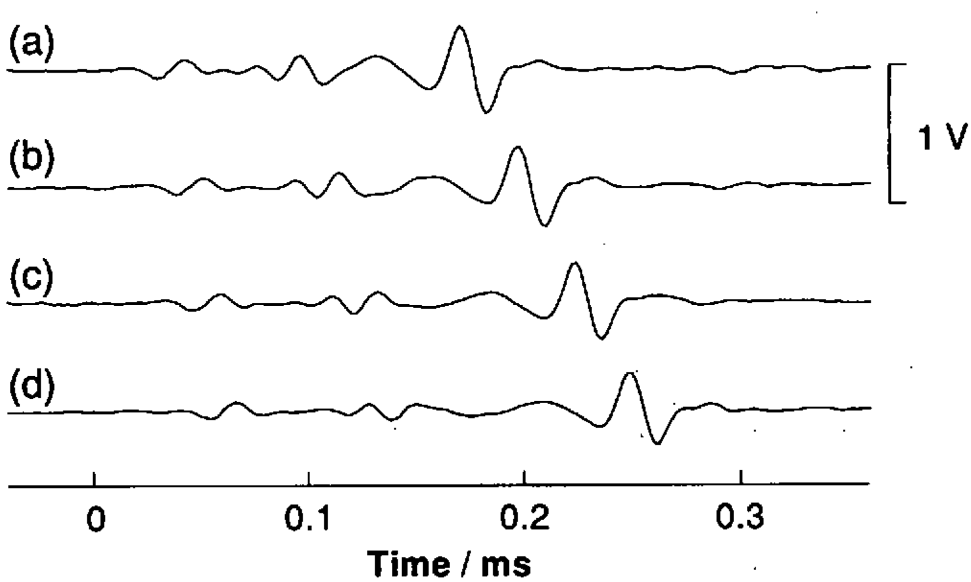

Figure 10. Observed waveforms for a neoprene/aluminium plate as a function of distance x from the plate edge (y=0.07m); (a) x=0.2m, (b) x=0.22m, (c) x=0.24 and (d) x=0.26m.

4. HARMONIC GENERATION

4.1 Field characteristics

The non-linear propagation of high frequency high amplitude waves results in the generation of significant waveform distortion and eventually in the formation of shock fronts. This distortion, which results in the generation of harmonics of the fundamental, can occur in very short distances, especially if the field is focused. Recent work in this area, stimulated by applications in medical physics, has resulted in the development of finite difference models (such as that described in [20]) capable of accounting for non-linear propagation, attenuation and diffraction in the nearfield of transducers.

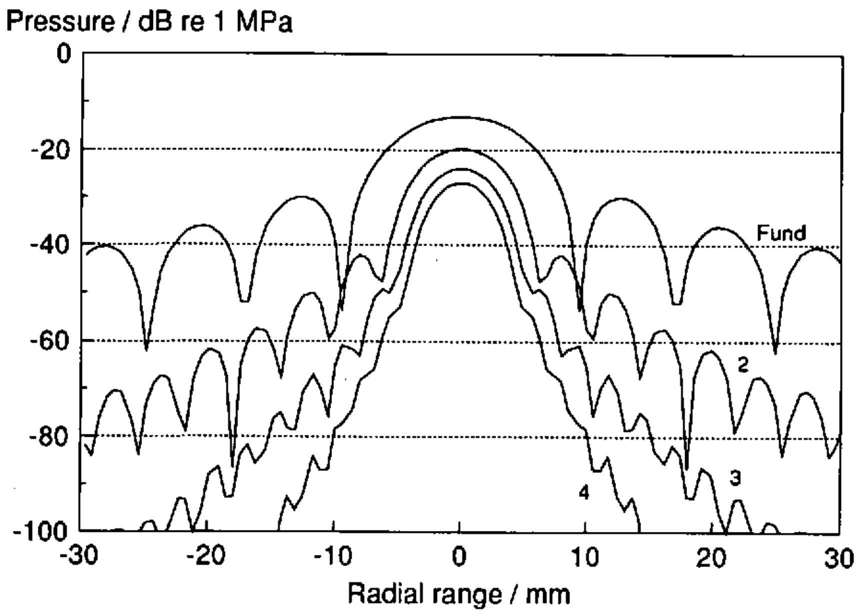

Figure 11. Theoretical beam cross-sections for the fundamental, 2nd, 3rd and 4th harmonics of a focused 2.25MHz transducer in the focal plane at a range of 440mm.

Figure 11 shows the beam cross-sections predicted by this type of model for the fundamental and first 3 harmonics in the focal plane of a 2.25Mhz transducer in water. The transducer is 19mm in radius and has a focal length of 0.44m. This figure shows the two main characteristics of the harmonic fields generated by non-linear propagation. Firstly, the harmonics have significantly narrower beamwidths with appreciably lower sidelobe levels. Secondly, a significant number of harmonics may be generated at utilisable levels since in a sawtooth waveform the harmonic level falls off as 1/n where n is the harmonic number. Thus a 2MHz transducer may be used to generate significant levels of harmonic to 50MHz and above.

Recent work has compared experimental results with these predictions and has shown them to be in good agreement [21,22] although calculations for the highest amplitude fields are still difficult and are limited by computing time.

4.2 Applications

Although the generation of harmonics has been used to measure the non-linearity parameter of materials, not a great deal of use has been made of the harmonics otherwise. One of the principal reasons for this is that it is not easy to restrict the region in which non-linear interaction occurs and so simplify the measurement of other phenomena. This contrasts with parametric array applications where an acoustic filter is very effective at restricting the non-linear interaction region.

One area in which this limitation has not been important is in the inter-comparison and calibration of hydrophones [23]. Here the generation of sawtooth waveforms rich in harmonics enables inter-comparisons to be made over a very wide frequency range using a single transmitting transducer. For example a 1MHz transducer may be used to make calibrations up to 25MHz. In addition, with suitable models of the non-linear propagation and/or a theoretical model for the hydrophone response, it is possible to obtain absolute calibrations since the harmonic generation depends on the fundamental pressure.

The characteristics of the fields generated suggest that harmonics could be employed to give improved resolution in a range of imaging applications. The low sidelobe levels indicate that this should be especially true in situations where reverberation from inhomogeneties is a limiting factor. It is interesting to note that non-linear generation appears to give significant improvements in resolving power in acoustic microscopy where the frequencies used are rather higher (2GHz) [24].

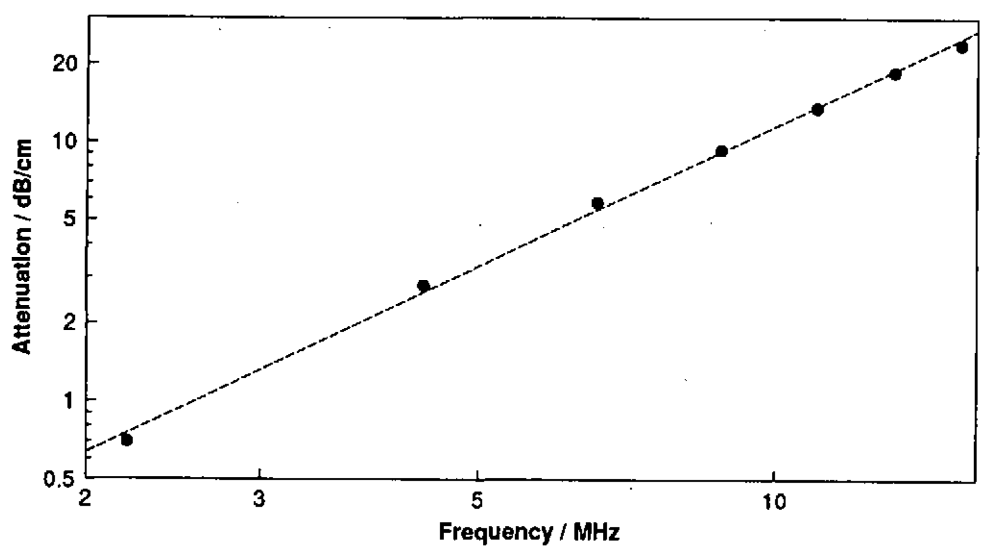

Figure 12. Attenuation of 0.2 molar 2-methyl propan-2-ol/water mixture estimated using attenuation of fundamental and non-linearly generated harmonics.

In order to assess the suitability of such fields for other applications they have been used to make some preliminary measurements of the frequency dependence of attenuation in fluids. In this case a 2.25MHz transducer was used to generate a highly distorted waveform in a water bath at some distance from the transducer. The waveforms were then observed with a membrane PVDF high frequency hydrophone and Fourier analysed. Estimates of attenuation were obtained by inserting two test cells of different thicknesses, containing the fluid under investigation, into the beam in turn. From these measurements the attenuation was calculated at the fundamental and at a number of harmonics. The results displayed in Figure 12 show that the technique can be used to obtain attenuation as a function of frequency over a significant frequency range. However interpretation can be difficult, as it is not clear how significantly non-linear propagation in the test fluid will affect the results obtained. This is an area in which comparisons with alternative techniques and with the predictions of propagation models are required.

5. CONCLUSIONS

Non-linearly generated sound displays characteristics that can be profitably exploited as a tool in the acoustics laboratory as well as other areas. Care must, however, be taken to ensure that non-linear propagation does not adversely affect the measurements obtained, this being easier to ensure for a parametric source than for the harmonics generated by finite amplitude distortion.

6. ACKNOWLEDGEMENTS

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