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CORRELATION SCANNING

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Introduction

A narrow-band acoustic wave field can be described either in terms of a directional spectrum of plane-wave components or in terms of the spatial distribution of the field over a reference plane. The function of an electronically-scanned receiver is to use an array of transducers to acquire information about the spatial field distribution over an 'aperture' plane and to convert this into information about the supposedly unknown directional spectrum of the incoming wave field. It is well-known that the aperture distribution function and the directional spectrum function can be defined in such a way that the relationship between them appears as a Fourier transformation. Because the observed part of the spatial field is limited to the physical size of the array aperture, and because the directional spectrum is limited to the range of real angles, both functions can be approximated by finite sets of discrete samples. It turns out that, for a line array for example, all the information that is practically available about the unknown directional spectrum is contained in n samples of the aperture field where n is the number of half-wavelengths in the length of the aperture. A scanning computer is then able to use the information contained in these samples to generate not more than n samples of the directional spectrum. Even if the particular scanning system is designed to produce a continuous sweep of the directional spectrum it still remains true that the fineness of detail of the reconstructed image can never exceed that implied by the available number of samples.

The samples of the aperture field are complex because they have to include both amplitude and phase information about the time-alternating field and the resulting spectral samples are also complex. But, for all practical purposes, the phase information relating to the spectrum is irrelevant; what is needed is the amplitude information only. It would obviously be a good idea therefore to seek some way of transforming the n complex samples of the aperture field into 2n real samples of the directional spectrum, at half the original spacing, thus doubling the amount of detail that can be reproduced. A hint as to how this might be achieved is provided by the Wiener-Khintchine theorem, which relates the autocorrelation of one of the functions forming a Fourier pair to the squared modulus of the other function of the pair. This suggests that what is wanted is a mthod of forming the autocorrelation of the sampled aperture function and then to apply a Fourier transformation to this in order to produce real samples of the directional power spectrum. The desire to halve the spacing of these directional samples means however that the effective aperture of the spatial autocorrelation must be twice the length of the physical array. This is not practicable. It will be shown however that a multiplicative scanning process, involving only a single multiplication followed by time-averaging, can sometimes produce results approximating to those expected from true spatial autocorrelation and can therefore lead to spectral resolutions significantly better than those obtainable with conventional additive scanning processes.

Spatial autocorrelation

The spatial correlation problem is made more complicated by the fact that the functions on both sides of the Fourier transformation are complex. This is dealt with by splitting the aperture field arbitrarily into two components which are in time-quadrature with each other and then treating each separately.

The amplitude $|P|$ and phase ϕ of a sinusoidal time-waveform can be represented by the complex number $P = |P| \exp(j\phi)$ which can be written in the alternative form $P = P_A - jP_B$. The time-waveform is then $P_A \cos \omega t + P_B \sin \omega t$.

The aperture field distribution $P(ky)$ and the directional spectrum $Q(s)$ form a Fourier transform pair

$$P_A\,(ky) \longleftrightarrow Q_A(s)$$
$$P_B\,(ky) \longleftrightarrow Q_B(s)$$

Now apply the Wiener-Khintchine theorem to each relationship separately :

$$[\text{autocorrelation of } P_A(ky)] \longleftrightarrow |Q_A(s)|^2$$
$$[\text{autocorrelation of } P_B(ky)] \longleftrightarrow |Q_B(s)|^2$$

Therefore $$[\text{sum of both autocorrelations}] \longleftrightarrow |Q_A(s)|^2 + |Q_B(s)|^2$$

Since $Q_A(s)$ and $Q_B(s)$ represent two fields which are in time-quadrature then the sum of their squared moduli must equal the squared modulus of their sum; i.e. $$|Q_A(s)|^2 + |Q_B(s)|^2 = |Q(s)|^2$$

and therefore

$$[\text{sum of autocorrelations of } P_A(ky) \text{ and } P_B(ky)] \longleftrightarrow |Q(s)|^2$$

So the required process must carry out quadrature sampling of the aperture field, form the autocorrelation of each set of samples, perform a Fourier transformation on each separately and finally add the results together.

In any practical system the whole of the function $P(ky)$ cannot be reached; all that is available is a set of samples of $P(ky)$ taken within the limits $y = \pm\frac{1}{2}\ell$ of the aperture. This means that the samples are not really of $P(ky)$ itself but of the product $P(ky)\,T(ky)$ where $T(ky)$ is equal to unity for values of ky lying within the aperture but equal to zero for all $|ky| > \frac{1}{2}k\ell$.

So the Fourier transform of interest is actually : $P(ky)T(ky) \longleftrightarrow Q(s) * U(s)$

where $T(ky) \longleftrightarrow U(s)$

The asterisk denotes convolution.

The reconstructed spectrum cannot ever be $Q(s)$ but is an approximation to the convolution of $Q(s)$ with the aperture sampling function $T(ky)$. Another way of saying exactly the same thing is that $Q(s)$ has to be <u>scanned</u> by the beam pattern $U(s)$ appropriate to that particular aperture. For an aperture length ℓ and for uniform weighting of the samples.

$$U(s) = (\sin x)/x \quad \text{where} \quad x = \tfrac{1}{2}k\ell s\,.$$

To simplify the discussion it has been assumed that the aperture has only one dimension, so that the sampling device is just a uniformly-spaced line array of transducers. It will be understood that, once the principles have been established, any conclusions reached can be extended to cases where the aperture is two-dimensional and also where the weighting of the samples is not necessarily uniform.

The autocorrelation process described above can only be applied to the limited number of aperture samples that are available; i.e. to the product P(ky)T(ky); and this leads to the power spectrum $|Q(s) * U(s)|^2$.

But this is exactly the same result as could have been obtained much more simply by carrying out an 'additive' scanning process, followed by square-law detection. At first sight it might appear that nothing has been gained and in fact this is true if it is not possible to place any restrictions on the 'unknown' directional spectrum of the waves approaching the receiver.

<u>Stationary directional spectrum</u>

Suppose however that the field is such that the mean amplitude of each sample of the directional spectrum remains constant, or at least changes only very slowly, while the phases of the samples fluctuate randomly and independently of each other. This is the kind of thing that happens in radio astronomy, for example, and conditions can be imagined under which it might also be expected to apply to some extent in underwater sonar applications. The intention now is to see whether, under such conditions, correlation techniques might possibly lead to effective apertures greater than that of the actual array and, in particular, might approach the ideal case of the doubled dimensions required to provide the 2n samples referred to in the Introduction.

<u>Time-averaging and spatial-averaging</u>

Consider the product of the outputs of two array elements, at positions $y = y_1$ and $y = y_2$, respectively, assuming for the moment that the directional spectrum of the field consists only of two components $Q_1(s)$ and $Q_2(s)$ in directions s_1 and s_2. $Q = |Q| \exp(j\phi)$.

The time-waveforms at elements 1 and 2 are, respectively :

$$|Q_1|\cos(\omega t + \phi_1 - ky_1 s_1) + |Q_2|\cos(\omega t + \phi_2 - ky_1 s_2)$$

$$|Q_1|\cos(\omega t + \phi_1 - ky_2 s_1) + |Q_2|\cos(\omega t + \phi_2 - ky_2 s_2)$$

Each of these has to be represented by two quadrature samples, one giving the coefficient of $\cos\omega t$ and the other that of $\sin\omega t$. For example, the cosine samples are :

$$|Q_1|\cos(\phi_1 - ky_1 s_1) + |Q_2|\cos(\phi_2 - ky_1 s_2)$$

and

$$|Q_1|\cos(\phi_1 - ky_2 s_1) + |Q_2|\cos(\phi_2 - ky_2 s_2)$$

Multiplication and time-averaging over one period of t (i.e. lowpass filtering to remove the alternating carrier), gives the following four terms :

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$$\tfrac{1}{2}[|Q_1|^2 \cos ks_1(y_2 - y_1)]$$
$$+\tfrac{1}{2}[|Q_2|^2 \cos ks_2(y_2 - y_1)]$$
$$+\tfrac{1}{2}[|Q_1||Q_2|\cos\{(\phi_2 - \phi_1) + ks_1\ (y_2 - y_1) + ky_1(s_1 - s_2)\}]$$
$$+\tfrac{1}{2}[|Q_1||Q_2|\cos\{(\phi_2 - \phi_1) - ks_1\ (y_2 - y_1) + ky_2(s_1 - s_2)\}] \qquad (1)$$

The following conclusions can be drawn from this :

1. the useful information is contained in the first two terms only, while the remaining terms represent 'noise',
2. the useful information depends only on the spacing $(y_2 - y_1)$ of the elements and not on their absolute position,
3. the unwanted terms are functions of the absolute positions of the elements and also of the phase difference $(\phi_2 - \phi_1)$ between the two spectral components.

A true spatial autocorrelation process requires the summation of products of pairs of aperture samples, at fixed spacings, taken in all possible positions. When this is done the unwanted components will have various phase angles and will tend to average out. So, although all the available information about the spectrum is contained in the outputs from a single pair of elements at each spacing, the process has to be repeated for all other positions in order to remove the cross-product terms.

But in the special case of a stationary-amplitude spectrum of the kind described above the phase difference between pairs of samples varies randomly with time, so the unwanted 'noise' can now be removed simply by time-averaging the products of single pairs of elements.

Although this statement is based on the assumption that the spectrum contains only two components at $s = s_1$ and $s = s_2$, it is evident that the same argument can also be used for larger numbers of spectral components.

Another useful fact is that, since the phase angle of each spectral component of this particular kind of field is assumed to vary with time, the mean-square values of the two quadrature components $Q_A(s)$ and $Q_B(s)$ will be the same

$$|\overline{Q_A(s)}|^2 = |\overline{Q_B(s)}|^2 = \tfrac{1}{2}|\overline{Q\ (s)}|^2$$

So, in this case, there is no need for quadrature sampling; the multiplication processes can be carried out directly on the signal time-waveforms.

<u>Time-averaged-product (TAP) array</u>

To simplify the explanation that follows, consider the particular example of a 6-element line array with the method of processing shown schematically in fig.1.

From eqn.(1) and the discussion that follows it each spectral component such as Q(s) will produce an output proportional to

$$|\overline{Q}|^2\,[1 + 2\cos\,2x + 2\,\cos\,4x + 2\,\cos\,6x + 2\,\cos\,8x + 2\,\cos\,10x]$$

where $x = \tfrac{1}{2}ks\ d$ d is spacing of adjacent elements.

or $\quad |Q(s)|^2 U_1(s)$

where $U_1(s)$ denotes the expression in the square brackets.

Evidently $U_1(s)$ represents the directional response of the system.

If electronic scanning is performed, by introducing an appropriate set of time-varying phase-shifts at the point indicated by the dashed lines in fig.1, the effect will be to produce the convolution of $|\overline{Q(s)}|^2$ with $U_1(s)$. So the result is $|Q(s)|^2 * U_1(s)$ instead of the expression $|Q(s) * U(s)|^2$ which would have resulted from a simple additive scanning process, followed by square-law detection.

The important point is that whereas $U(s)$ is derived from the length of the physical 6-element array, $U_1(s)$ is equivalent to the directional pattern of an 11-element additive array at the same spacing. So the scanned TAP system has effectively produced $2n-1$ real samples of the directional spectrum instead of n complex ones.

Multiplicative array

This has been adopted as a convenient name for the scheme shown in fig.2. The array is split into two parts, the signal waveforms are directly added together within the groups of elements and the two total time-waveforms are then multiplied together and time-averaged. The output for a single spectral component $Q(s)$ is proportional to

$$|\overline{Q}|^2 \left(\frac{\sin 3x}{3\sin x}\right)^2 \cos 6x$$

which can also be written as

$$|\overline{Q}|^2 \times \frac{1}{9}(\cos 2x + 2\cos 4x + 3\cos 6x + 2\cos 8x + \cos 10x)$$

This is again equivalent to the directional response of an 11-element additive array but with the following numerical weighting factors applied to the outputs of the respective elements :

1, 2, 3, 2, 1, 0, 1, 2, 3, 2, 1

Electronic scanning can be performed by introducing phase-shifts at the point indicated by the dashed line in fig.2. The result is to produce the convolution $|\overline{Q(s)}|^2 * U_2(s)$ where, apart from a numerical factor, $U(s)$ is the expression in the brackets.

Discussion

The directional patterns for $U_1(s)$ and $U_2(s)$ are plotted in figs.1(a) and 2(a). The non-uniform weighting in the latter case is responsible for the relatively large side-lobes adjacent to the main lobe. It is of interest to see whether this could be corrected to some extent by deliberately weighting the outputs from the array elements before signal-processing is carried out. A general answer can be obtained by considering a hypothetical case of multiplicative processing applied to a uniform, continuous aperture of length ℓ, split into equal parts. Defining the variable in the spectrum domain as

$x_1 = \frac{k\ell s}{4}$ the directional function appears as $\left(\frac{\sin x_1}{x_1}\right)^2 \cos 2x_1$

Treating this expression as if it were the response of an additive system, the aperture weighting of the equivalent array must be represented by its Fourier transform. The transform of $\left(\frac{\sin x_1}{x_1}\right)^2$ is a triangular pulse while that of $\cos 2x_1$ is a pair of impulse functions. So the transform of the product is the convolution, consisting of a pair of triangular pulses.

It will be noted that the 11-element discrete equivalent array already obtained for the case of the 6-element physical array is consistent with this result; it corresponds to a sampled version of the two triangular pulses.

So the aim of any compensating weighting should be to replace the two triangular distributions by ones that are more nearly uniform; i.e. by rectangular pulses. But to do this it would be necessary to give the product of the responses of the two portions of the aperture a $(\sin x/x)$ form, thus demanding that the response of each part alone should be proportional to $(\sin x/x)^{\frac{1}{2}}$. This is not practicable but the analysis does indicate that the thing to do is to depart from the idea of splitting the aperture into equal parts; in fact if we go to the opposite extreme and reduce one part to a single small element, the directional function for the other part will tend to $(\sin 2x_1/2x_1)$, giving an approximate directional function for the multiplicative array of

$$\left(\frac{\sin 2x_1}{2x_1}\right)\cos 2x_1 = \left(\frac{\sin 4x_1}{4x_1}\right)$$

This is the response of an equivalent 'additive' aperture of length 2ℓ; i.e. twice the length of the physical aperture.

Fig 3 shows this scheme applied to the example of a 6-element array. Its directional function is

$$\left(\frac{\sin (2n-1)x_1}{(2n-1)\sin x_1}\right)\cos 6x = \left(\frac{\sin 5x}{5\sin x}\right)\cos 6x$$

$$= \frac{1}{5}(\cos 2x + \cos 4x + \cos 6x + \cos 8x + \cos 10x)$$

corresponding to the response of an 11-element additive array, with the centre element missing, (see fig.3(a)).

This is practically the same as the result for the TAP system of fig.1 and it becomes clear that both 'TAP' and 'multiplicative' systems are really variations on the basic theme of what may be termed correlation scanning. The choice of equal array sections has the advantage that it maximizes the intrinsic directivity of each section, thus minimizing the effect of ambient noise in the medium.

Stationary spectrum with time-varying amplitude

It has been shown that correlation scanning can produce useful results but only if the field has a directional spectrum represented by sources, in fixed directions, with randomly-varying phases. So far it has been assumed that their amplitudes remain constant. Take the idea a step further and assume that the amplitudes of the sources can also fluctuate with time, in other words imagine that the field is due to a set of discrete sources which 'scintillate' (to borrow a term from optics), while maintaining their directional positions.

Much will now depend on the way in which the scanning and time-averaging processes are carried out. If, for example, the field represents echoes in an active sonar system the signals will be range-gated so that the spectrum for any given range annulus only exists for short intervals, widely separated in time.

This points to the fact that, to obtain the full potential advantage of a correlation scanning system, integration has to take place over a large number of successive pings. Experience with a particular 32-element multiplicative scanning sonar has been based on using the persistence of the display screen to provide a measure of ping-to-ping integration. Operators in the field have consistently judged the results to be 'better' when switched to multiplicative processing rather than the alternative additive processing which is also provided. The accepted explanation seems to be connected with amplitude scintillation, and the probability that different spectral components will predominate at different times within the same range annulus, rather than with true correlation based on the averaging-out of cross-product terms as a result of phase fluctuations.

Conclusions

It is suggested that study should now be directed to the problem of storage and integration of the output of correlation scanning systems over a larger number of successive pings, using modern circuit technology rather than relying on the persistence of a display. This should enable a closer approach to be made, for certain kinds of fields, to the ideal of an effective doubling of the array aperture. It seems likely that the large adjacent side-lobes in its directional pattern will set a limit to the performance of the symmetrical multiplicative system. If this is so it means that the trend should be away from equal array sections, accepting that the choice ultimately lies between signal-to-noise ratio and directional resolution.

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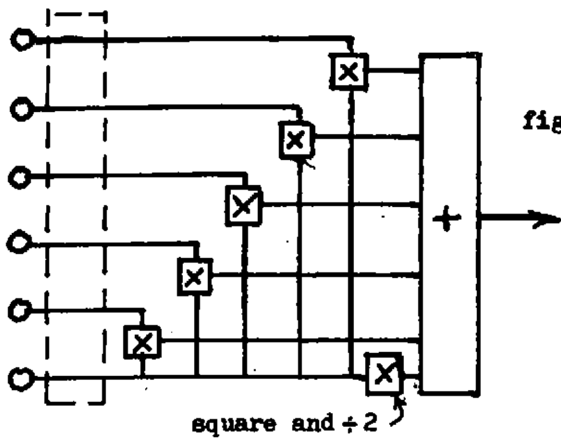

fig. 1

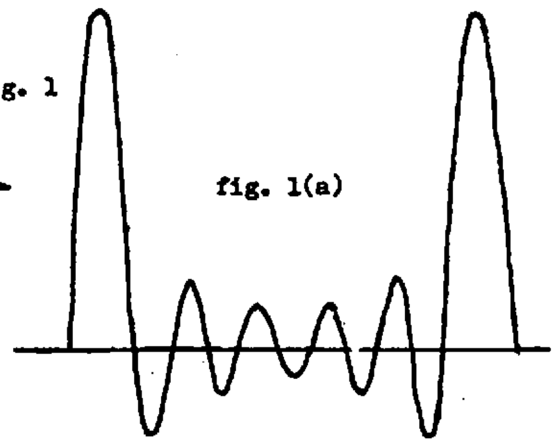

fig. 1(a)

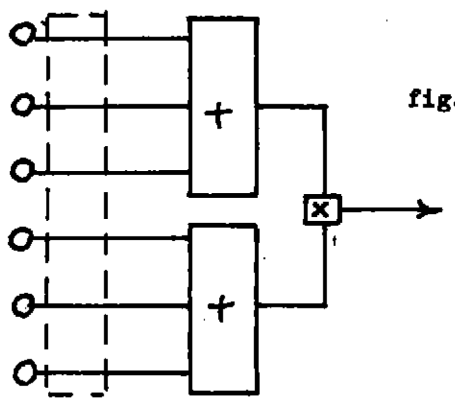

fig. 2

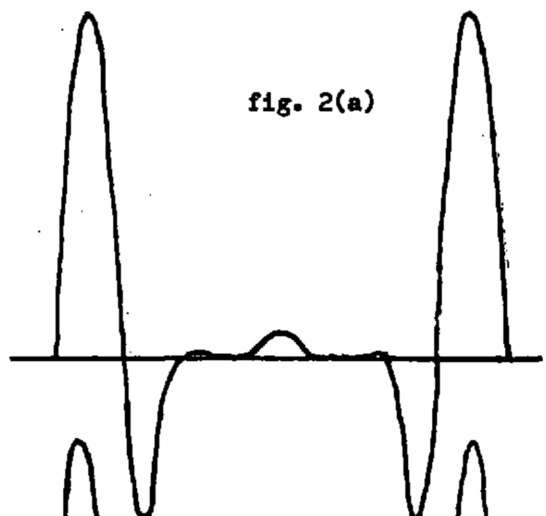

fig. 2(a)

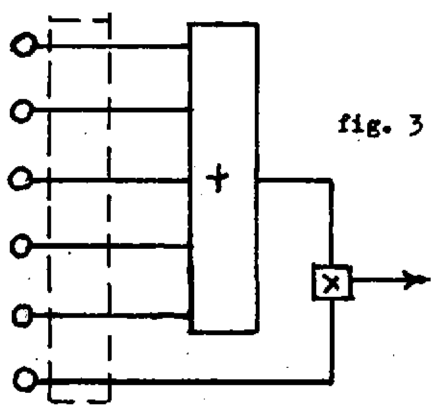

fig. 3

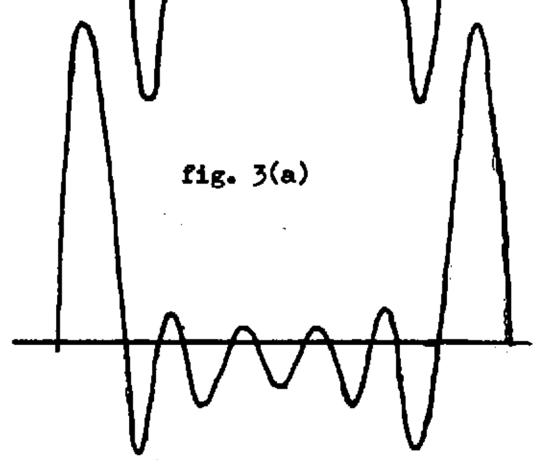

fig. 3(a)