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ACQUISTIC SCATTERING BY HIGHWAY NOISE BARRIERS

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## INTRODUCTION

Barriers are commonly constructed beside highways to screen residential areas from traffic noise. These barriers are constructed from natural or man-made materials which reduce the directly transmitted sound appreciably. However, some of the acoustic energy is diffracted over the barrier or scattered by corners, so that part of the acoustic energy is deflected to nomes behind the barrier. The effectiveness of the barrier depends on its shape and height, the material with which it is constructed and its distance from the source of noise. In this paper, the T-matrix method [1,2] and the finite element approach [3] in conjunction with the method of images are employed to solve the scattering problem and the numerical results obtained are compared with scale model experimental results.

## FORMULATION

The ground and the highway barrier are modelled by rigid boundary conditions which for acoustic wave scattering translates to Neumann boundary conditions. The highway barrier is modelled as an infinitely long cylinder of arbitrary cross-section oriented parallel to the z-axis. If a harmonic line source parallel to the z-axis or a plane harmonic wave propagating in the x-y plane is incident on the barrier, both the exciting and scattered sound field are independent of the z-coordinate and the problem is truly two dimensional.

The scattering problem in the x-y plane is done by the method of images. It is well known that the Green's function in a half space with Neumann boundary conditions in the  $z_{-0}^{\infty}$ 0 plane is given by

$$g(\vec{r}, \vec{r}') = g_0 + \tilde{g}_0 \text{ in } \{\Pi_0(k|\vec{r} - \vec{r}'|) + H_0(k|\vec{r} - \vec{r}'|)\}$$
 (1)

where  $\overset{\rightarrow}{r}$  is the field point,  $\overset{\rightarrow}{r'}$  is the location of the line source in

the x-y plane and  $\vec{r}'$  is the mirror image of  $\vec{r}'$  about the n-axis which is parallel to the ground (see Fig. 1 below). In Eq. (1)  $g_0$  is the free space Green's function for the line source.

Let ; be the scalar potential in region  $\Omega$  which is the sum of the incident excitation ;  $_0$  and the scattered field  $\phi_S$ . All time factors of the form  $\exp(-i\omega t)$  where  $\omega$  is the frequency of the excitation are suppressed for notational convenience.

The Helmholtz integral representation for the problem on hand can be written as

$$\frac{1}{4\pi} \int_{SU\Gamma} \{ \varphi_{+}(\vec{x}^{+}) \hat{n}^{+} \cdot \nabla^{+} g(\vec{x}, \vec{x}^{+}) - g(\vec{x}, \vec{x}^{+}) \hat{n}^{+} \cdot \nabla^{+} \varphi_{+}(\vec{x}^{+}) \} ds(\vec{x}^{+}) 
+ \frac{1}{4\pi} \int_{SU\Gamma} \{ \varphi_{+}(\vec{x}^{+}) \hat{n}^{+} \cdot \nabla^{+} g(\vec{x}, \vec{x}^{+}) - g(\vec{x}, \vec{x}^{+}) \hat{n}^{+} \cdot \nabla^{+} \varphi_{+}(\vec{x}^{+}) \} ds(\vec{x}^{+}) 
+ \frac{1}{4\pi} \int_{SU\Gamma} \{ \varphi_{+}(\vec{x}^{+}) \hat{n}^{+} \cdot \nabla^{+} g(\vec{x}, \vec{x}^{+}) - g(\vec{x}, \vec{x}^{+}) \hat{n}^{+} \cdot \nabla^{+} \varphi_{+}(\vec{x}^{+}) \} ds(\vec{x}^{+})$$

$$= \frac{1}{4\pi} \int_{SU\Gamma} \{ \varphi_{+}(\vec{x}^{+}) \hat{n}^{+} \cdot \nabla^{+} g(\vec{x}, \vec{x}^{+}) - g(\vec{x}, \vec{x}^{+}) \hat{n}^{+} \cdot \nabla^{+} \varphi_{+}(\vec{x}^{+}) \} ds(\vec{x}^{+})$$

$$= \frac{1}{4\pi} \int_{SU\Gamma} \{ \varphi_{+}(\vec{x}^{+}) \hat{n}^{+} \cdot \nabla^{+} g(\vec{x}, \vec{x}^{+}) - g(\vec{x}, \vec{x}^{+}) \hat{n}^{+} \cdot \nabla^{+} \varphi_{+}(\vec{x}^{+}) \} ds(\vec{x}^{+})$$

$$= \frac{1}{4\pi} \int_{SU\Gamma} \{ \varphi_{+}(\vec{x}^{+}) \hat{n}^{+} \cdot \nabla^{+} g(\vec{x}, \vec{x}^{+}) - g(\vec{x}, \vec{x}^{+}) \hat{n}^{+} \cdot \nabla^{+} \varphi_{+}(\vec{x}^{+}) \} ds(\vec{x}^{+})$$

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$$= \frac{1}{4\pi} \int_{SU\Gamma} \{ \varphi_{+}(\vec{x}^{+}) \hat{n}^{+} \cdot \nabla^{+} g(\vec{x}, \vec{x}^{+}) \hat{n}^{+} \cdot \nabla^{+} \varphi_{+}(\vec{x}^{+}) \} ds(\vec{x}^{+})$$

$$= \frac{1}{4\pi} \int_{SU\Gamma} \{ \varphi_{+}(\vec{x}^{+}) \hat{n}^{+} \cdot \nabla^{+} \varphi_{+}(\vec{x}^{+}) \hat{n}^{+} \cdot \nabla^{+} \varphi_{+}(\vec{x}^{+}) \} ds(\vec{x}^{+})$$

We require that

$$\hat{\mathbf{n}} \cdot 7; (\mathbf{r}) = 0; \quad \mathbf{r} \in \mathbf{SU}$$
 (3)

and we know from Eq. (1) that

$$\hat{\mathbf{n}} \cdot \nabla \mathbf{g}(\vec{\mathbf{r}}, \vec{\mathbf{r}}^*) = 0; \quad \hat{\mathbf{r}} \in \Gamma \tag{4}$$

and hence Eq. (2) reduces to

$$\frac{1}{4\pi} \int_{SUS} \hat{\varphi}_{+}(\vec{r}') \hat{n}' \cdot 7' g_{0}(\vec{r}, \vec{r}') ds(\vec{r}')$$

$$= (\hat{\varphi}_{0}(\vec{r}) + \hat{\varphi}_{0}(\vec{r})); \quad \vec{r} \not\geq v + \tilde{v}$$

$$\hat{\varphi}_{0}(\vec{r}); \quad \vec{r} \not\geq v + \tilde{v}$$
(5)

where  $\tilde{S}$  is the reflection of the prismatic barrier about the n-axis and V+V is the region enclosed within.  $\phi_O$  and  $\tilde{\phi}_O$  are the incident excitation and its mirror image respectively.

Thus the equivalent scattering problems is as shown in Fig. 2. Now the problem can be handled by any of the standard techniques for infinitely long rigid cylinders of arbitrary cross section. We have chosen to solve the problem using the T-matrix and finite element approaches as detailed in Ref. [1-3]. In this approach both the incident and scattered fields are expanded in cylindrical wavefunctions and the T-matrix calculated using the two forms of the integral representation given in Eq. (5) relates the unknown scattered field coefficients to those of the incident field. For both plane waves as well as a line source coefficients of expansion of  $\phi_0$  in cylindrical wavefunctions are well known.

## NUMERICAL AND EXPERIMENTAL RESULTS

The diffracted field is computed for plane wave incidence with a correction to compare with experimental results for a point source. Computations were also performed for a line source excitation. The model we had assumed is a rectangular barrier. We had considered two different noise source heights, h=3.5", 19.2", to represent the noise from cars and trucks, respectively. To get an informative picture of the effectiveness of the barriers, we had performed calculations over a field of receiver points as shown in Fig. 1. The results are presented in Tables I and II in the form of 20 log pr/po for both cars and trucks, where pr is the diffracted pressure and po is the incident pressure. The agreement between our present theory and scale model experiments performed at Pennsylvania State University [4] is good and very encouraging for further study in this area assuming impedance boundary conditions. The T-matrix theory employed here is so general that a variety of barrier shapes can also be considered without much difficulty.

#### REFERENCES

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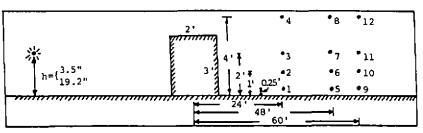


Fig. 1. Rectangular highway barrier and source and receiver positions.

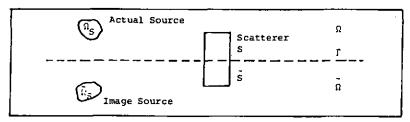


Fig. 2. Equivalent scattering problem by using method of images.

Table I. Comparison of theory and experiment for a source located at a height, h=3.5" above the ground at two different frequencies.

Frequency	Experiment(T)	Height of receiver		
500 H2	- 6.0(-5.4)	-11.0(-7.3)	-11.8(-7.25)	48"
630 H2	- 8.3(-7.92)	-15.7(-9.84)	-15.5(-11.32)	
500 Hz	- 7.5(-8.1)	-11.0(-10.1)	-8.8(-7.128)	24"
630 Hz	-10.5(-9.2)	-11.5(-10.91)	-12.0(-12.01)	
500 H2	- 4.8(-4.0)	-10.3(-9.4)	-15.0(-7.2)	12"
630 Hz	- 8.3(-7.92)	-12.8(-11.18)	-13.8(-12.18)	
500 Hz	- 5.5(-5.1)	-9.3(-8.92)	-14.5(-7.2)	3"
630 Hz	- 6.8(-8.25)	-10.0(-11.26)	-17.0(-12.23)	
	24 '	48'	60'	Hor. Loc. of Rec.

Table II. Numerical results for line source excitation located at heights h=3.5" and 19.2" above the ground at two different frequencies.

		Height of					
Frequency	h=3.5"	h=19.2"	h=3.5"	h=19.2"	h=3.5"	h=19.2"	receiver
500 Hz	-6.78	-7.81	-6.95	- 8.90	-7.58	- 9.63	48"
630 Hz	-9.04	-9.56	-7.73	-10.48	-8.19	-11.20	
500 Hz	-4.00	-5.88	-6.27	- 8.42	-7.15	- 9.32	24"
630 Hz	-4.77	-7.47	-6.69	-10.00	-7.53	-10.88	
500 Hz	-3.26	-5.40	-6.10	- 8.29	-7.06	- 9.24	12"
630 Hz	-3.68	-6.96	-6.44	- 9.84	-7.36	-10.80	
500 Hz	-3.04	-5.25	-6.04	- 8.25	-7.0	- 9.22	3"
630 Hz	-3.35	-6.80	-6.35	- 9.80	-7.3	-10.76	
	24 '		48'		60'		Loc. Rec.