THE EFFICIENCY OF ANTENNA IN WAVEGUIDE IN THE CONDITION OF NOISE WITH UNKNOWN CHARACTERISTICS

V M Baronkin, J.V.Tjutekin

Institute of Acoustics, Academy of sciences of USSR

1. INTRODUCTION

An underwater acoustic detection devices frequently works in conditions when the statistical characteristics of noise field are unknown. For example it may be unknown level of noise on array, unknown level of noise on separate phone in array, unknown parameters of an angular spectrum of noise field etc. In this case the problem of construction and analysis of the detection algorithm, which keeps invariable the level of false alarm probability (PFA), when the statistical characteristics of noise are changed, is fundamental one. In this paper the problem is solved by the asymptotically similar test($T\Delta f$ is large, where T-time of observation, Δf-bandwidth of frequency analysis). The another basic assumption in the paper is condition of weak signal. We note that these assumptions are realized in many practical situations of underwater detection. The theory of similar tests(as it is known to authors) is not used in the literature on underwater signal processing. Because size of the paper is limited we reduce part of the paper, concerning the application of similar test and we restrict ourselves by the consideration of a few simple examples.

2. METHODS OF ADAPTIVE DETECTION

Traditional Neyman-Pearson's criterion of detector performance requires stabilization of PFA when the noise characteristics are changed. Detection probability(PD), of course, depends on these characteristics. There are some books and many papers on adaptive signal processing. These works are using SNR criterion only. However, without condition of constancy of PFA under unknown noise parameters application SNR as a criterion of detector performance is not convenient. It is easy to give simple examples when SNR is large but the problem of detection under unknown noise parameters has no solution (one such example will be given in sec.4).

Let random field is observed in discrete points (for simplicity) of space- time (or space-frequency)domain. Then the field observed may be described by a collection of vectors

, say p_n, where n denotes sample size in the time region(usually $n=T\Delta f$) . Let p_n has probability density $f(p_n/\lambda,\theta)$,where λ is determined by intensity of signal and $\theta\in\Theta \subseteq R^m$ is a vector of unknown noise parameters.

The problem of detection can be formulated as the problem of testing composite hypotheses : $H_0:\lambda=0,\theta\in\Theta$ (signal is absent) and $H_1:\lambda>0,\theta\in\Theta$ (signal is present).The class of algorithms of testing hypotheses(detection),preserving invariable PDF under any $\theta\in\Theta$ is called similar algorithms.Regular methods of nontrivial similar algorithms construction are founded on application of the Neyman's structure [1] . Unfortunately , class of distributions for which we can find the structure is very narrow from practical point of view.

Detection of weak signals requires large sample size n. Then application of asymptotically similar (AS) algorithms,when the condition of preservation of PDF is implemented for large n is expedient .It allows essentially to expand class of distribution admitting construction of similar detection algorithms

In practice usually a detection algorithms

$$\Phi_n(p_n)=\begin{cases}1, \text{ if } T_n(p_n) > K_n(\alpha) \Rightarrow H_1\\ 0, \text{if } T_n(p_n) \leq K_n(\alpha) \Rightarrow H_0\end{cases} \qquad (1)$$

are used.Here $T_n(P_n)$ is the detection statistic and $K_n(\alpha)$ is a threshold chosen from the condition $M_{0,n}\Phi_n(P_n)=\alpha$ for any $\theta\in\Theta$,where α is given level of PFA and $M_{0,n}\{.\}$ is expectation under H_0.

Neyman has suggested the method of construction of AS algorithm statistic [2]. This method is associated with adaptation principles.Below we give generalization of Neyman's result .

Further for simplicity we shall omit function arguments and indexes when there was no risk of ambiguity .

Let us consider some zero-mean "basic" statistic $\eta_n(p_n,\theta)$.The choice of statistic η is determined by considerations of technical simplicity and performance of detector based on the statistic η.The statistic $T_\eta(p_n)$ (1) for AS detector is constructed in the following way. Let

$$T_\eta(\theta)=(\eta-d'I^{-1}\varphi)/\sigma \qquad (2),$$

where

$$M_0\eta=0,\ \sigma^2=M_0\eta^2-d'I^{-1}d\ ,\ \varphi=\nabla_\theta \ln f|_{\lambda=0}\ ,$$

$$\nabla_\theta{}'=(\partial/\partial\theta_1,\dots,\partial/\partial\theta_m)\ ,d=M_0\eta\varphi\ ,\quad I=M_0\varphi\varphi'$$

(I-Fisher's information matrix for θ under H_0).

Now AS detection statistic is given by $T_\eta(\tilde\theta)$,where $\tilde\theta$ is $n^{1/2}$-consistent estimate of θ. The simple sufficient condition for $n^{1/2}$ -consistency of estimate θ is the following :it should be consistent estimate,having order of dispersion $O(1/n)$.Many practical estimates such as estimates of generalized least square method ,maximum likelihood estimates and etc. have this property.It may be showed that if some conditions (central limit theorem,law of large numbers,existence of corresponding derivatives and etc.) are implemented then $T_\eta(\tilde\theta)$ has standard normal distribution for large n under H_0.Now in (1) $K_n(\alpha)=K(\alpha)\equiv\Phi^{-1}(1-\alpha)$,where $\Phi(x)$ is standard normal distribution function.We note that if $\tilde\theta$ is maximum likelihood estimate (MLE) of θ then in (2) $\varphi(\tilde\theta)=0$ .Important property of AS detectors is a speed of convergence PFA to α,when n is increased. Theoretical analysis of this problem is very difficult and therefore Monte-Carlo method for its solution is used in practice. Detection probability of AS algorithm for weak signal $(\lambda\sim1/n^{1/2})$ under large n is

$$PD=1-\Phi(K(\alpha)-\rho_\eta(AS)), \tag{3}$$

where

$$\rho(\eta,AS)=\lambda[M_0l\eta-d'I^{-1}b]/\sigma\quad ,b=M_0l\varphi,\quad l=\frac{\partial}{\partial\lambda}\ln f\,|_{\lambda=0}$$

A structure of AS (adaptive) detector is given in fig 1.We see from (3) that performance of adaptive detector is determinated by SNR $\rho_\eta(AS)$ for special way corrected statistic (2) only.In the case when θ is known

$$PD=1-\Phi(K(\alpha)-\rho_\eta)\ ,\rho_\eta=\lambda M_0l\eta/M_0^{1/2}\eta^2 \tag{4}$$

The ratio $\rho_\eta(AS)/\rho_\eta$ determines loss of efficiency detection. It is "payment" due to unknownledge of noise parameters $\theta\in\Theta$. For asymptotically optimal similar detection algorithm $\eta=l$,where l is determined in (3), i.e. l is statistic of locally-optimum algorithm.Efficiency and loss of efficiency for this case determined by (3) (4) with obvious change.

The method developed has allowed to solve several important problems of adaptive detection for different models of waveguide , including the case. of random waveguide ,and various models of noise , including the problem rejecting local sources of noise . Here we restricted ourselves by the case of deterministic waveguide model. This corresponds linear spatial processing.

3. LINEAR SPATIAL PROCESSING

Let the field $p(t,x)$ being a sum of signal and noise field ,is observed on the time interval $[0,T]$ at phones (with coordinates $x_\mu, \mu=1,..,N$) of discrete array.Assume that the signal field is generated by the point source of zero-mean gaussian stationary noise with spectral density $g(\omega)$, $\omega\in\Omega=[\omega_1,\omega_2]$. The signal propagation in waveguide is described by vector column $H(\omega,r)$ with the elements $H(\omega,x_\mu,r)$,$\mu=1,..,N$, where $H(\omega,x,r)$ is suitable Green function, with x-receiving point and r-radiation point.Parameters of Green's function during the time of observation are assumed to be invariable. The noise field is assumed zero-mean gaussian stationary field with cross-correlation matrix $G_n(\omega,\theta)$ with elements $G_n(\omega,x_\mu,x_\nu,\theta)$, $\mu,\nu=1,...,N$, where θ is vector of unknown parameters of noise which is not changed on the interval$[0,T]$. Under these assumptions it is naturally to use spectral description of the field on array by vector column $p_T(\omega)$ with elements $p_T(\omega,x_\mu),\mu=1,..N$,where $p_T(\omega,x)$ is Fourer transformation on $[0,T]$.If $n=T\Delta f>>1$,$(\Delta f=(\omega_2-\omega_1)/2\pi)$ then $p_T(\omega_i)$, $i=1,..,n$, is a sequence of approximately independent gaussian complex zero-mean vectors with correlation matrix $G(\omega_i,\lambda,\theta)=\lambda g(\omega_i)H(\omega_i)H^+(\omega_i)+G_n(\omega_i,\theta)$, $\theta\in\Theta$,where ω_i is multiple to $2\pi/T,\omega_i\in\Omega$,$i=1,..,n$, [3]. It is easy to see, that the statistic of locally-optimal (LO) detection algorithm is

$$l(p_{T,\Omega}) = \Sigma_{\omega\in\Omega}\, g|p_T^+G_n^{-1}H|^2 \qquad (5)$$

Here and further $\Sigma_{\omega\in\Omega}$ denotes summation along all ω_i multiple to $2\pi/T,\omega_i\in\Omega$, $i=1,..n$.

Why we must stabilize PFA? To answer this question we must investigate the sensitivity of detection algorithm based on statistic (5), when "true" noise correlation matrix G_n is different from supposed one. Corresponding expressions for PD and PFA may be obtained but here are not given. However, to understand how PD and PFA are changed we consider a simple example. Let a noise matrix supposed be equal to identity matrix if $G_n=\theta I$. Then, for weak signal if gH^+H=const (for example non-dispersive free-space and g=const) we have

$$PD=1-\Phi(C_\alpha K(\alpha) - Cn^{1/2}/\theta), \qquad PFA=1-\Phi(C_\alpha K(\alpha)),$$
$$C_\alpha=(K(\alpha) + n^{1/2}(1-\theta))/\theta K(\alpha);$$

Thus, if the noise level increases ($\theta>1$) PFA and PD increases also. In this case PFA exceeds the given level α and the detection device is "overloaded" relative false alarm. In the opposite case the detection device is "underloaded" relative detection performance. The typical dependence of PFA and PD on the noise level is given in Fig. 2 and Fig.3 respectively (see the curve 1). We see that small deviation θ from 1 leads to the enormous change of PFA. It shows that stabilization of PFA is necessary.

Looking at structure of statistic of LO algorithm, we choose the basic statistic η in the following form

$$\eta(p_{T,\Omega},\theta)= \Sigma_{\omega\in\Omega}\, h[\,|p_T^+A(\theta)P|^2-P^+A(\theta)G_n(\theta)A(\theta)P] \quad (6)$$

where η, A^{-1}, and P can be given as possible approximation of g, G_n and H. The choice of such approximations are connected both with requirement of simplicity of technical realization of the adaptive detector and necessity of analysis of detector's sensitivity about the signal characteristic and the model of wave sound propagation. Under some unknown signal field parameters (source coordinate, some parameters of radiated noise and Green's function) detection problem can be solved by construction of the multichannel detector where the channels correspond to different "look directions" in space of g, H or their approximations h, P.

The statistic of AS detector for linear spatial processing is determinated (2) with η given by (6). Moreover in (2) and (3) we have

$$\varphi_k(p_{T,\Omega},\theta)= \Sigma_{\omega\in\Omega}\, [p_T^+ G_n^{-1}G_{n,k}G_n^{-1}p_T-\mathrm{tr}G_n^{-1}G_{n,k}], \quad G_{n,k}=\frac{\partial}{\partial\theta_k}G_n,$$

$$d_k=\int_\Omega hP^+AG_{n,k}AP\,d\omega;\quad I_{k,l}=\int_\Omega trG_n^{-1}G_{n,k}G_n^{-1}G_{n,l}\,d\omega,\ k,l=1,..,m;$$

$$b_k=\int_\Omega g\ H^+G_n^{-1}G_{n,k}G_n^{-1}H\,d\omega;\quad M_0\eta l=\int_\Omega gh|H^+AP|^2d\omega; \tag{7}$$

$$M_0\eta^2=\int_\Omega h^2\ |P^+AG_nAP|^2d\omega;\quad M_0l^2=\int_\Omega g^2(H^+G_n^{-1}H)^2d\omega;$$

with accuracy to factor $T/2\pi$.

Here we have two basic problems: the accurate parametric description of correlation noise matrix and the effective (from the point of view of computer) method construction of $n^{1/2}$- consistent estimates of unknown parameters of the noise matrix .These problems are being solved for each the practical situation individually. As an example we consider linear dependence G from unknown parameters , i.e. $G_n(\omega,\theta)=\sum_{k=1}^m\theta_kB_k(\omega)$, where θ_k are unknown parameters and B_k are known matrices, $k=\overline{1,m}$ .In this case MLE has no explicit solution and it is necessary to use appropriate iterative procedure .In other hand, the estimate θ of generalized least square method , i.e

$$\tilde\theta=\arg\min_\theta \sum_{\omega\in\Omega} tr(p_Tp_T^+ - \sum_{k=1}^m\theta_kB_k)^2 \tag{8}$$

being $n^{1/2}$-consistent estimate, has simple form,namely,

$$\theta=V^{-1}u\ ,\quad V=(\sum_{\omega\in\Omega}trB_kB_l)_{k,l=1}^m;\quad u=(\sum_{\omega\in\Omega}p_T^+B_kp_T)_{k=1}^m$$

Many practical situations when G_n is unknown can be effectively reduced to this model of G_n .

4.EXAMPLES

As it is mentioned above,the methods offered can be used in wide class of weak signal adaptive (with control PFA) detection problems in underwater waveguide.The description of results of such investigation essentially deviates from framework of this, paper.Because of this we consider three simple examples, important from practical point of view.

4.1.Unknown noise level on the array.

Let $G_n(\omega)=\theta g_n(\omega)G(\omega)$,where θ is unknown noise level,g_n and G are known .For simplicity we assume,without loss of generality, that $g_n\equiv1$ and $G\equiv I$. To construct AS detector it is necessary to choose the basic statistic and $n^{1/2}$-consistent estimate of θ .Consider (7) with $G_n=\theta I$ and

$A=\theta^{-1}I$ as the basic statistic and MLE of θ as $n^{1/2}$-consistent estimate , which here has explicit form

$\hat{\theta}=(nN)^{-1}\sum_{\omega\in\Omega} p_T^+ p_T$ .Then statistic of the AS algorithm for weak signal is (without the terms which are independent from observation)

$$T_{\eta,AS}=(\sum_{\omega\in\Omega} h|p_T^+ P|^2) / \sum_{\omega\in\Omega} p_T^+ p_T \qquad (9)$$

It is known structure of adaptive detectorbased on normalized statistic . We note that (9) coincide with statistic of LO invariant to scale transformation algorithm which can be constructed for our problem using general approach for calculation of likelihood ratio of the maximal invariant statistic [1].

Performance of AS detector is determinated by (3) and (7) with $A=\theta^{-1}I$ and $G_n=\theta I$ .If H corresponds to plane wave with vector of arrival direction n_s ($H(\omega,x,r_s)=c^{1/2}\exp(-i\frac{\omega}{c}x'n_s)$ and P - plane wave with direction n (n-"look direction") then in (7) (with coefficient $T/(2\pi\theta^2)$)

$$d= cN \int h\, d\omega,\ I=\Delta\omega N\ ,b=cN \int g\, d\omega\ ,M_0 l=c^2N^2 \int g^2 d\omega\ ,$$

$$M_0\eta^2=c^2N^2 \int h^2 d\omega\ ,\ M_0 l\eta = c^2\int gh|D_N(n,n_s)|^2 d\omega \quad ,$$

where $D_N(n,n_s)=\sum_{k=1}^{N}\exp\{-i\frac{\omega}{c}x_k'(n-n_s)\}$is usually directivity pattern of array for far-field approximation.

Analysis of expression obtained for the performance of AS algorithm even in this case gives interesting results.Again for simplicity we put $h=g$ and $n=n_s$,i.e. consider the processing matched with the characteristic of signal.Then

$$\rho_{1,AS}=\frac{cN}{\theta}\left[T\Delta f\left(\frac{1}{\Delta\omega}\int_\Omega g^2 d\omega - \frac{1}{N}\left(\frac{1}{\Delta\omega}\int_\Omega g\, d\omega\right)^2\right)\right]^{1/2} \quad (10)$$

As can be seen from (10) the performance of AS detector is determinated by both the shape of spectral density g and the magnitude N.Let N=1, then $\rho_{1,AS}$ is determined by second central moment of g on Ω .If g=const then $\rho_{1,AS}=0$ and solution of the problem of detection under unknown noise level is impossible .It is natural with the point of view of an engineer.However , the situation changes if $N>1$ because even if g=const the magnitude of SNR is

$$\rho_{1,AS}=\theta^{-1}(T\Delta f)^{1/2}N\left(1-\frac{1}{N}\right)^{1/2}$$

and thus $\rho_{1,AS}>0$. This result is clear if we take into account that in our assumption signal field is coherent on array but noise field is incoherent.

To illustrate workability of the AS detector and for comparison AS detector and LO detector oriented on $\theta=1$ under the changed noise level we did the statistical simulation .The conditions of simulation are following :the number of phones in line array N=10 , H corresponds to plane wave, g=const, $T\Delta f=100$ and the given level of PFA equals to $\alpha=10^{-2}$ .The number of testing experiments equals to $1.5*10^4$ .The results of the simulation are given in Fig.2 and Fig.3,where PFA and PD dependence on "true" value noise level are showed.

4.2 Unknown level in separate phone in the array.

Assume that $G_n(\theta)=\mathrm{diag}(\theta_1 g_1,\dots,\theta_N g_N)$. For simplicity, consider the statistic of LO algorithm as the basic statistic and MLE of θ as $n^{1/2}$- consistent estimate , which in this case allows explicit form ($n=T\Delta f$)

$$\hat{\theta}_k=n^{-1}\Sigma_{\omega\in\Omega}\frac{|p_T(x_k)|^2}{g_k}, \qquad \overline{k=1,N}.$$

The statistic of AS detector is $T(p_{T,\Omega};\hat{\theta})$,where

$$T_{1,AS}(\theta)=\sigma^{-1}\left[\Sigma_{\omega\in\Omega}\, g\left[\left|\Sigma_{k=1}^{N}\frac{p_T^*(x_k)H(x_k)}{g_k\,\theta_k}\right|^2-\Sigma_{k=1}^{N}\gamma_k\right]\right]$$

$$\sigma^2=T\Delta f\left[\frac{1}{\Delta\omega}\int_\Omega\left(\Sigma_{k=1}^{N}g\gamma_k\right)^2 d\omega-\Sigma_{k=1}^{N}\left(\frac{1}{\Delta\omega}\int g\gamma_k d\omega\right)^2\right];\ \gamma_k=\frac{|H(x_k)|^2}{g_k\theta_k}$$

Now structure of AS algorithm is not so obvious as in the case above . Its performance is determined as $\rho_{1,AS}=\sigma$. To understand the dependency of $\rho_{1,AS}$ upon the shape of noise level on array we again made simplicity assumption,namely, we assumed $\frac{g}{g_k}|H_k|^2\equiv\mathrm{const}$ (for example,when $g\equiv g_k,\overline{k=1,N}$,and H corresponds to plane wave). Then

$$\rho_{1,AS}\sim\left\{T\Delta f\left[(\Sigma_{k=1}^{N}1/\theta_k)^2-\Sigma_{k=1}^{N}1/\theta_k^2\right]\right\}^{1/2}$$

The statistical simulation of the AS detector for different shape of noise level on line array has given results analogous as above.

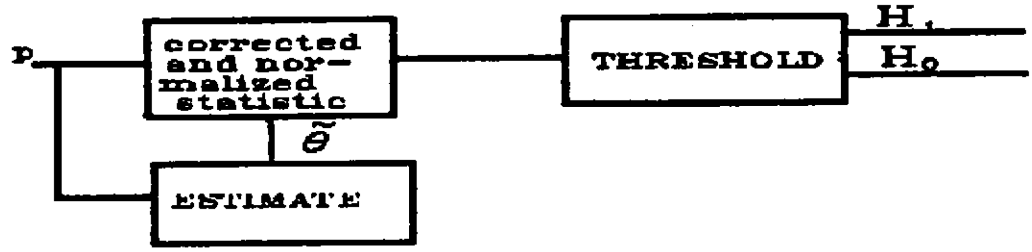

Fig.1 Simplified block diagram

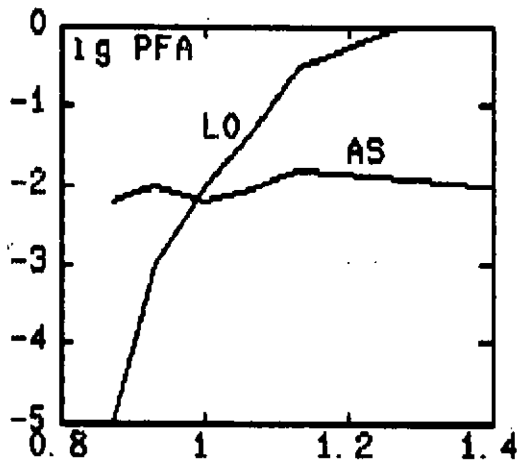

Fig. 2 PFA sensitivity

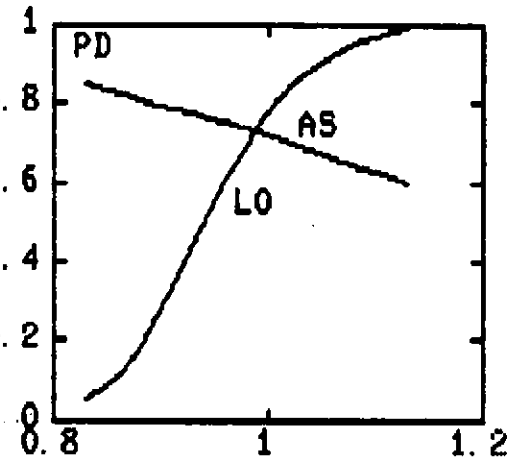

Fig. 3 PD sensitivity

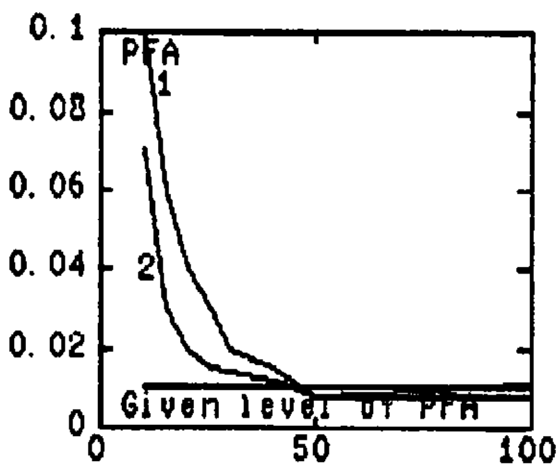

Fig. 4 PFA convergence speed

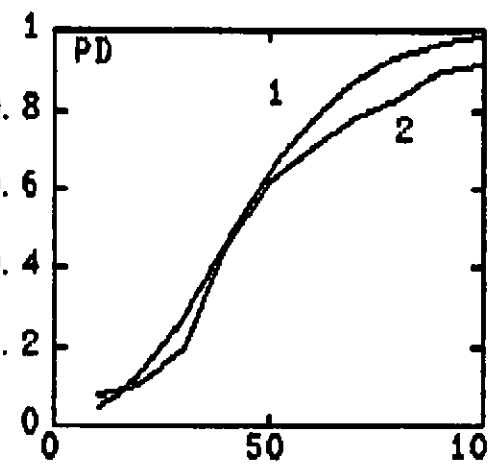

Fig. 5 PD comparision

4.3.Unknown angular spectrum of noise.

Consider the case of vertical line array in waveguide , assuming that noise correlation matrix G_n is determined by angular spectrum i.e. G_n is Toeplitz matrix.This approximation is appropriate for relative short antenna when the WKB method can be used [4]. In opposite case it is necessary to work with G_n directly.

Analysis of theoretical and experimental works [5,6,7,8] on structure of noise angular spectrum has led to the thought that the most useful approximation (from practical point of view) is the approximation by a step function. Thus assuming that $G_n(\psi)$ has weak dependency on frequency on Ω we have

$$G_n(\psi)=\Sigma_{k=1}^{m} \theta_k \chi_{E_k}(\psi) \quad , \psi\in\Psi\subseteq [-\pi/2,\pi/2] \qquad (11) ,$$

where θ_k are unknown parameters. $\chi_A(\psi)=1$ if $\psi\in A$ and $\chi_A(\psi)=0$ if $\psi\notin A$; $U_{k=1}^{m} E_k=\Psi$ and $E_k\cap E_l=\emptyset$ $k\neq l$.

Using (11) we easily can determine matrix V and vector d in (8) by both directivity pattern of array and Fourer spatial transformation of observation vector $p_T(\omega)$ for construction of $n^{1/2}$-consistent estimate θ . This expression we omit. The structure and efficiency of the AS detector are obtained from (2), (3) and (8) by substitution of this corresponding $n^{1/2}$-consistent estimate .

As it mentioned in sec 2 , important characteristic of AS is speed of convergence of PFA to given value α , when $T\Delta f$ is increases. We has investigated this characteristic for given AS detector by the statistical simulation. The result's of the simulation for the line array (N=30) and m=3 (the number of testing experiments equals to 1.5 10^4)are given in Fig.4 and Fig.5 ,where dependency of PFA and Pd versus to $T\Delta f$ is represented . In Fig.4 the curve 1 corresponds to PFA for $\alpha=10^{-2}$and the curve 2 - 10 PFA for $\alpha=10^{-3}$.In Fig.5 the curves 1 and 2 correspond theoretical and experimental PD . As it can be seen from the Fig.4 the AS detector has good speed of convergency PFA to α ,calculated by the asymptotical approximation.

5.CONCLUSION

The paper concerns only a part of general problem of the creation of signal processing algorithms under unknown signal and noise characteristics .It is impossible to enumerate all directions of theoretical and experimental investigations which are necessary for solving the problem .

However , we can pick out two of these ,which is most important from our point of view in underwater acoustics detection problems . The first one is construction and analysis of detection algorithm when statistical structure of noise is similar to structure of signal .For example , signal and noise have modal structure but they distinguish by degree of smoothness of their modal spectra . The second problem connects with "automatic" choice of parametric description of statistical characteristics of noise (for example, choice of size of parametric space) by appropriate procedure of estimation of noise model complexity for example by Akaike's criterion .The investigation in these direction are carrying out .

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