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SOUND SCATTERING BY VISCOUS FLOW IN THE VICINITY OF MOVING BODY

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1. INTRODUCTION

The problem of sound scattering on isolated immobile body remains classical, and for a number of bodies of most simple forms, corresponding accurate analytic solutions are described in multiple monographs and manuals on acoustics [1,2]. But for moving bodies, even moving with small velocities in media, a sequence of new problems, strongly complicating procedure of accurate and approximate solution search, arise. First of all, a lot of mathematical difficulties are met due to the fact, that boundary conditions should be fulfilled on body moving surface, and second, which is most qualitatively important, in surrounding medium, which is initially at rest, corresponding flow arises. Medium particles, situated in the vicinity of body surface, are forced into regular motion, that is why sound scattering takes place now not only on the moving surface of body, but on fluid flow velocity inhomogeneities, formed in the vicinity of body, as well. In paper [3] the simplest situation of that sort was analyzed, i.e. the case, when sound scattering takes place on ideal fluid potential flow inhomogeneities, related to sphere motion with constant velocity, much less than velocity of sound. Scattering amplitude corrections for the field scattered on small inhomogeneities of corresponding flow in longwave approximation are shown to be small, proportional to Mach number, scattering itself was found to be practically isotropic in space and containing in equal degrees monopole, dipole and quadropole components. For opposite case, when sound wavelength is much smaller, than characteristic flow dimensions, scattered field acquires sharp forward directivity, and for zero angle scattering amplitude becomes relatively large. Although in the papers [4,5] it was already shown, that division of scattered field in general solution of such problems on partial components, corresponding to flow and body scattering, is just conditional and does not bear any physical sense, nevertheless, this division is sometimes very convenient mathematically to simplify solution analysis. In more complex problems solution it is important to remember, that total phase gain in the vicinity of moving body due to multiple scattering of shortwave sound on potential flow to be calculated in ray approximation, is zero theoretically, at least in the part not concerning shadow wave propagation, where it is finite. At the same time, corresponding calculations for fluid with non-zero vorticity show, that phase gain to be found in practical cases has always finite value. That is why suspicion arises, that if viscosity of fluid and related vorticity of flow would be taken into account, it could substantially influence results of scattered field calculations. It is shown below, for example of sphere moving in viscous fluid, that this suspicion has very good fundament, and in a sequence of practical situations sound scattering amplitude for viscous flow inhomogeneities increases substantially in comparence with scattering amplitude found for the case of potential flow.

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2. DESCRIPTION OF SCATTERING

2.1 Viscous flow in the vicinity of sphere description

In the beginning of the paragraph let us consider sphere motion in viscous fluid with constant velocity V, which is much less not only in comparence with sound velocity in the fluid c, but with parameter ν/a as well, where ν - kinematic viscosity of media, a - sphere radius. It is possible in this case to consider fluid as incompressible, then resulting flow will be of Stocks type, since corresponding Reinolds number will be very small. According to paper [6], velocity distribution in the fluid could be described by expression U=rotrot(gV), where unknown function g(r') could be found from equation $\mathrm{grad}\Delta^2 g=0$ solution with corresponding boundary conditions, written in the coordinate system, where sphere is at rest. For absolutely rigid sphere velocity distribution U(r') in the media could be written in moving frame of reference r'=r-Vt in following form

$$U(r') = -V + \frac{3a}{4}\frac{(Vn)n+V}{r'} - \frac{a^3}{4}\frac{3(Vn)n-V}{r'^3} \quad (1)$$

where n=r'/r' - unit vector, directed from the sphere center to the observation point r'. It is worth to notice, that in laboratory system r=r'+Vt flow velocity equals to U+V, while distribution (1) is valid up to distances of an order a/Re. For greater distances Oseen's flow regime would take place, and velocity decrease with distance would become exponential, i.e. more intensive, than proportional to 1/r' [7]. In the case, when sphere is moving with velocity changing in time, function V(t) could be expanded in Fourier integral, and for arbitrary Fourier-component V_Ω the same problem could be solved independently. Velocity distribution in viscous fluid in the vicinity of sphere oscillating with frequency Ω in that case could be presented as before by expression $U=\mathrm{rotrot}(V_\Omega g)$. But unlike to the previous case, unknown function g(r') would obey equation $\Delta^2 g+\kappa^2 g=0$, in which squared wave number κ would have purely imaginary value $\kappa^2=i\Omega/\nu$ and $\kappa=(1+i)/\delta$. Parameter $\delta=\sqrt{2\nu/\Omega}$ is of length dimensions and has physical sense of viscous boundary layer thickness. Solution of equation mentioned above with corresponding boundary conditions on sphere surface r'=a and infinity, yield for absolutely rigid sphere

$$g(r') = \frac{3a}{2\kappa^2 r}\left[e^{i\kappa(r-a)} - 1 + i\kappa a + \frac{\kappa^2 a^2}{3}\right] \quad (2)$$

One could notice, that for condition $|\kappa a|\to\infty$ boundary layer thickness is approximating to zero, and solution (2) acquires simple form $g=a^3/(2r)$ corresponding to potential flow of ideal fluid near sphere. For the limiting case of very low oscillation frequency of sphere, viscous layer thickness becomes infinitely large ($|\kappa a|<<1$), and for particle velocity expression similar to (2) describing Stocks flow regime could be obtained.

2.2 Sound governing equation

Sound propagation in the vicinity of sphere moving in viscous fluid could be

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described, as before in papers [3-5], by means of Lighthill equation. For monochromatic wave with frequency ω this equation could be written in the form

$$\Delta p + k^2 p = \frac{2i}{\omega}\frac{\partial}{\partial x_\alpha}\left(U_\beta \frac{\partial^2 p}{\partial x_\alpha \partial x_\beta}\right) \tag{3}$$

where $k=\omega/c$ - wave number, p - acoustic pressure. It is well known, that equation (3) is valid with accuracy up to terms linear in hydrodynamical Mach number $M=V/c$, but to be accurate, in its derivation viscosity was not taken into account, as well as entropy changes related to dissipative process arising due to medium heat transfer. As it is known, taking into account zero order in M dissipation processes leads to conventional attenuation of propagating waves and could be considered by wave number renorming, for in this case it acquires additional imaginary part. More general than (3) Blockhintsev-Howe equation comprises additional cross terms linear in Mach number and proportional to first degree of dissipation factors as well. When these factors together with Mach number are small, additional terms become small as well in comparence with terms already presented in equation (3) and could be safely neglected in first order approximation. More accurate conditions for equation (3), valid for sound propagation description in moving viscous medium, could be found in monographs [8,9] and are related to cases, when dissipative process could be neglected, as well as to situations where vorticity value of flow could be considered small in comparence to frequency of propagating sound ($\mathrm{rot}U \ll \omega$). Below, we shall use equation (3) for sound scattering description in viscous flow, taking into account its vorticity, while velocity distribution U(r) will be taken in the form (1) or (2).

2.3.1 Sound scattering on Stocks viscous flow inhomogeneities. Using equation (3), one could find wave field, scattered on flow (1) inhomogeneities, produced by sphere motion with constant velocity. Let us suppose, that plane monochromatic wave $p_i(r)=p_o\exp(ik_o n_o r - i\omega_o t)$ is coming to sphere from infinity, then, using Born approximation, it is possible to find solution for plane incident wave scattered on flow inhomogeneities, considering all remarks mentioned above and taking into account the sense of partially scattered field.

$$F_f = \frac{ik}{2\pi c}\int d^3r' \; e^{iqr'}\left[(n_o\nabla)(Un_o) - iknU\right] \tag{4}$$

Here n_o - unit vector, coinciding with incident wave direction, $q=k(n_o-n)$ - impulse vector transmitted by the wave to the medium, its module $q=2k\sin(\theta/2)$, where θ - scattering angle, corresponding to equality $\cos\theta=nn_o$. Integral (4) with velocity distribution U(r) in the form (1) could be calculated for arbitrary value of (ka) parameter, and its detailed calculation is fulfilled in paper [10]. It is worth to notice, that formally integral (4) may diverge due to weak decreasing of velocity in subintegral expression on infinity. But one should remember, that, as it was mentioned in paragraph 2.1, distribution (1) is valid up to distances of a/Re order, and that is why integration in (4) should be limited to these values of r'. Simple analysis of expression (4) for small values of (ka) shows, that when irrotational part of flow, described by the last term of expression (1), is close to potential flow, then corresponding component of scattering amplitude behaves in the same way as before, i.e., it is proportional to k^2a^3M, according to [3]. In the same time, order of magnitude of scattering amplitude vortex component is different, it is proportional to $a(Mn+Mn_o)$. Comparison of these expressions shows, that scattering amplitude for vortex component of flow for small (ka) increases in $(ka)^{-2}$ in relation to potential component for wide range of scattering angles, except of backward direction, where it is zero. If sound frequency is decreased, this relation magnitude, according to expression (4), is quickly increasing. Thus, one could see in particular, that taking viscosity into account is necessary not only for ka<<1, but for ka>>1 as well, for in the latter case it also leads to substantial amplification of scattering, for instance, in forward direction.

2.3.2.1 Sound scattering in the vicinity of oscillating sphere. Calculation of integral (4) with velocity distribution U(r) in the form (2) could also be done analytically for arbitrary value of (ka) parameter. Detailed procedure of corresponding integrals calculation, which in that case are all converging and have finite values, was fulfilled in paper [10]. For expressions obtained there are of very complex form, we will use below just corresponding limit expressions. For small values of (ka) scattering amplitude on viscous flow, taking place in the vicinity of oscillating sphere, has the form

$$F_f \cong k^2a^3 \left(\frac{2}{3} (Mn_o) - \frac{1}{2} (nn_o)(Mn_o + Mn) \left[1 + 3 \frac{1-i\kappa a}{q^2a^2-\kappa^2a^2}\right]\right) \qquad (5)$$

When viscous layer thickness is small compared to sphere radius, $|\kappa a|$>>1, second term of expression in the square brackets of (5) is negligible and this expression becomes close to unity. In this case a type of flow is very close to potential, and scattering amplitude (5) is proportional to k^2a^3M, as we already saw before. In opposite case, for low frequency sphere oscillations, when κa parameter is substantially smaller than unity, scattering amplitude is increasing not simply in comparence with potential flow case, but continue to increase with decreasing frequency of oscillations Ω too. Two possibilities are available here. In the first case, when $|\kappa|$>>k, wavelength λ is large compared with δ, and scattering amplitude is increased in $|\kappa a|^{-2}$ times in comparison with ideal fluid flow case and becomes equal

$$|F_f| \cong 3a \left(\frac{\delta}{4\pi\lambda} \right)^2 (n_o n)\ (Mn_o+Mn) \qquad (6)$$

In the second case, when $|\kappa|$<<k, scattering amplitude (5) for finite, not too small, scattering angles, is increased up to value which is equal to Ma in the order of magnitude.

$$F_f \cong \frac{3}{2} a \ \frac{nn_o}{(nn_o-1)} (Mn_o + Mn) \ ; \qquad (n \neq n_o) \qquad (7)$$

Expression (5) analysis shows, that when scattering angle decreases, field amplitude (7) in the limit θ 0 is finite and equals to expression (6). It is worth to notice, that scattering amplitudes (6) and (7) substantial values in forward direction, explained by rotational type of scattering flow, are angle dependent and come to zero in backward direction. That is why, for the case of backward scattering the order of magnitude of corresponding field amplitude will be k^2a^3M due to second term of expression (1) as for potential flow.

2.3.2.2 Sound scattering by large scale flow inhomogeneities. Analysis of sound scattering amplitude for the case of large scale viscous flow inhomogeneities observed in the vicinity of oscillating sphere shows, that here F_f behavior is determined by Bessel spherical functions of argument $qa=2ka\sin(\theta/2)$. At finite scattering angle values, parameter (qa) is very large together with (ka), and scattering amplitude is small enough, for scattering itself is of pronounced anysotropic character. In angle region $\theta<1/(ka)$ one could use Bessel series expansion for small argument, then scattering amplitude for θ 0 takes the form close to expression (5). But unlike to the case analyzed in previous section, dimensionless parameter (ka) here should be supposed to be large. Fast oscillations of scattering amplitude F_f could be observed, when scattering angle is increased, and for θ of the order of $1/(ka)$ its value practically comes to zero. For small compared to sphere radius boundary layer thickness flow is close to potential, and it is obvious from expression (5), that for $ka\gg1$ and $\theta\ll1/(ka)$ scattering amplitude F_f, as before, is proportional to k^2a^3M. While in opposite case, when inequality $|\kappa a|\ll1$ takes place, scattering amplitude F_f increase is possible, even for small sphere ($a\ll\lambda$), in $(\kappa a)^{-2}$ times for small angle scattering.

2.4 Multiple scattering.

Fluid viscosity influence on sound scattering for large (ka) values ($ka\gg1$) is especially obvious, when multiple scattering is taken into account. It leads, in fact, to ray refraction in the vicinity of moving body. In the frame of multiple wave scattering theory and slow perturbation method problem solution could be presented as follows

$$p = p_o\, e^{ikn_o r+iS} \qquad (8)$$

Physical sense of eikonal S here is additional phase gain, which wave acquires in the vicinity of localized flow, for local speed of sound here is equal not to c, but to $c+n_oU(r)$. In slow perturbation approximation method eikonal S could be written in form the same as expression (4). But to calculate accurately Born approximation correction for $(ka)\gg1$, it is necessary to take, as zero order solution, not incident plane wave alone, but additionally wave reflected from sphere surface as well. Taking into account scattered field directivity, it is possible to take Green function in subintegral expression (4) in approximate form $G(r)=z^{-1}\exp(ikz+ikr_\perp^2/2z)$ and thus at once perform integration over transverse coordinates $r_\perp=(n_o\times r)\times n_o$. This procedure is available due to slow change of $U(r',r)$ in comparence to fast oscillations of exponential factor $\exp[ik(r_\perp-r'_\perp)^2/2(z-z')]$. Thus velocity distribution could be safely excluded from integration with respect to $dr_\perp$, according to mean theorem. Remaining integral with respect to longitudinal coordinate $z=r_\perp r_o$ differs from expression (4) and could be presented in the form

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$$S = -k/c \int^{z} dt' \; (Un_0) \qquad (9)$$

Total phase gain for incident wave in the vicinity of moving sphere could be obtained for z value equal to infinity, and for potential flow, where $U=\nabla\Phi$, integral (9) becomes zero, at least at the part, not concerning shadow wave. But for viscous fluid, where to express flow velocity vector potential is necessary, and U=rotrot(gV), integral (9), as it follows from previous expression for total phase gain, is not zero for sure, moreover, in the moving viscous fluid rays are bent and their deflection is proportional to vorticity of flow Ω= rotU [6]. In this case integration in (9) should be fulfilled not along the straight line parallel to n_0, but over the trajectories of bent rays. In calculating scattered sound intensity it leads to substantial correction in comparence with potential flow.

3. CONCLUSION

Thus it is necessary to state, that fluid viscosity influence on moving body sound scattering in a row of practical cases could be substantial. It could be observed almost always for longwave and shortwave sound scattering. Analysis carried out above shows, that viscosity influence on field scattering could be explained in two ways. First of all, due to slower change of velocity with distance from the body, as, for instance, in Stocks flow, integration region in expression resembling expression (4) would be expanded, thus corresponding integral value, which is the measure of scattering, is increased. Second, due to non-negligible vorticity observed in fluid flow field, which, in its turn, leads to increase of scattering amplitude as well. The latter property becomes most obvious in the case of large parameter (ka) values, where eikonal S of scattered wave, with shadow wave exception, could become zero for potential flows, while for rotational flows it still remains finite.

4. REFERENCES

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