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ON SOUND PROPAGATION NEAR MOVING BODY

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1. INTRODUCTION

Recent years special attention was drawn to problems concerning sound propagation in media with localized flow [1, 2]. Practical needs and constant necessity of definite experiments results explanation in the field of moving bodies sound scattering in the beginning of the 80ths led us to assumption on possible focusing action of flows produced by moving bodies and their important role in the sound scattering phenomena. Bodies movement in practical cases is accompanied by adjacent particles of media involvement, so it is not surprising, that sound scattering takes place not only on the moving surface of the body, but on fluid flow velocity inhomogeneities as well. It was shown in papers [3, 4] that for bodies of dimensions small compared to the wavelength of sound, division of total scattered field on partial components is just artificial, and for accurate taking motion into account it is necessary to solve the problem as a whole, not by parts. It was shown as well, that for small hydrodynamical Mach numbers, corrections to scattered field due to body motion are also small and linear in Mach number. Nevertheless, in paper [5] it was shown already, that scattered field in the vicinity of big moving body is reconstructed in such a way, that in definite regions of space substantial field amplification, even in the case of otherwise homogeneous media, for relatively small Mach number values, is possible. In this work we are going to analyze with necessary accuracy diffraction problem of shortwave sound scattering on body of big dimensions. For that purpose, we shall begin with a type of ethalone problem - the simplest case of sound scattering on sphere moving in ideal fluid, when surrounding fluid flow is potential.

2. SINGLE SCATTERING OF SOUND ON BIG SPHERE

2.1 Problem statement.

Let us consider that in infinite homogeneous fluid, which is at rest on infinity, absolutely rigid sphere of radius a is moving with constant velocity V, much less than sound velocity c in the medium. Supposing that surrounding fluid is ideal, while flow in the vicinity of sphere is potential, one can find velocity distribution in medium U(r,t), which will be described by well known expression [6]

$$U(r,t) = \frac{a^3}{2} \frac{3(Vn)n - V}{|r - r_o(t)|^3}, \qquad (1)$$

where r_o - sphere center position vector, $n=[r-r_o(t)]/|r-r_o|$ - outer normal to the sphere surface unit vector, directed to the observation point r. Further it would be taken into account as well, that plane monochromatic wave of the form

ON SOUND PROPAGATION NEAR MOVING BODY

$p_i(r,t)=p_o\exp(ik_o r-i\omega_o t)$ is coming to the sphere from infinity along direction n_o where wave number vector k_o is parallel to n_o while its magnitude is connected to sound frequency ω_o by conventional relationship $k_o=\omega_o/c$. For approximation linear in hydrodynamical Mach number $M=V/c$, sound propagation in the fluid with flow will be described, as in papers [3-5], by Lighthill equation

$$\frac{\partial}{\partial t}\left(\Delta p - \frac{1}{c^2}\frac{\partial^2 p}{\partial t^2}\right) = 2\frac{\partial}{\partial x_\alpha}\left(U_\beta \frac{\partial^2 p}{\partial x_\alpha \partial x_\beta}\right) \qquad (2)$$

For simplicity we shall analyze the case of absolutely rigid sphere with unique boundary conditions on its surface $|r-r_o|=a$ - absence of normal component of fluid velocity in sound wave. In this case it is not necessary to solve sphere inner problem for $|r-r_o|\le a$, while general problem solution becomes much easier.

2.2 Problem solution method.

In solution of problem formulated above it is convenient to make transition to moving frame of reference $r'=r-r_o(t)$, where sphere is at rest. After that transition apparent temporal dependence could be met only in formulation of conditions on infinity in the form of exponent $\exp(-i\omega t)$ temporal product. New frequency ω would be shifted in relation to the initial sound frequency ω_o on small value proportional to M and equals $\omega=\omega_o(1-Mn_o)$. Transforming equation (2) to moving frame of reference,it is possible to formulate in the beginning, as in papers [3,4], equation for scalar potential $\varphi(r,t)$ related to acoustic pressure by means of conventional Euler equation, written for moving medium, and then to introduce calibration potential $\psi(r')$. In accordance with [4], potential ψ is connected with scalar velocity potential $v=\nabla\varphi$ by relationship $\varphi(r,t)=\psi(r')\exp(ikr'M-i\omega t)$. After its introduction and coordinate system transformation mentioned above, equation (2) acquires the form close to Helmholtz equation form. Equation with additional conditions obtained thus, allows to find unique solution of the problem, which would be searched in the beginning by means of conventional perturbation theory, considering Mach number M to be small. For boundary conditions and governing equation analyzed are written with accuracy up to linear terms in Mach number, solution serial expansion in M would have a sense only with same accuracy, i.e. it is of use to search the solution in the form $\psi=\psi^{(0)}+\psi^{(1)}$. From the other hand, general solution of the problem could be expanded, as in paper [4], in the form of incident and scattered waves superposition

$$\psi(r') = \psi_i(r') + \psi_{sp}(r) + \psi_{sf}(r) \qquad (3)$$

Incident wave $\psi_i(r')$ is known function here, which differs from incident wave $p_i(r)$ in laboratory system by correction $\psi_i^{(1)}$ linear in M. The form of this correction is connected with the form of flow (1) and calibration potential ψ pressure p relationship, obtained from Euler equation. For zero order approximation in M,

ON SOUND PROPAGATION NEAR MOVING BODY

general problem solution obtained by means of $\psi_i^{(0)}$ describes sound scattering on the sphere at rest for the case when fluid flows nearby are absent. It is known as well, that in the latter case, besides incident field $\psi_i^{(0)}$, scattered field ψ_s should be also introduced. That is why in general expression (3) second term ψ_{sp} describing field scattered by moving sphere surface is also presented and governed by homogeneous Helmholtz equation. Substituting general solution (3) in governing equation for calibration potential, one could find, at last, equation for ψ_{sf} field, describing sound scattering on flow inhomogeneities

$$\Delta\psi_{sf} + k^2\psi_{sp} = -\frac{2ik}{c} U_\alpha \frac{\partial\psi}{\partial x_\alpha} - \frac{2ik}{c} n_{o\alpha} n_{o\beta} \frac{\partial U_\alpha}{\partial x_\beta} \psi_i^{(0)} \qquad (4)$$

here $k=\omega/c$ - renormed wave number, and primes in coordinate expression $r'=r-r_o(t)$ are temporally omitted.

2.2.1 Scattered field ψ_{sf} search procedure. For unique equation (4) solution obtaining it is necessary to formulate additional boundary condition on the surface of the sphere for ψ_{sf}, but it was shown already in paper [4], that decomposition of unique boundary condition for total field in two partial conditions for ψ_{sp} and ψ_{sf} could not be done in unambiguous way. That is why it is proposed to use for equation (4) unique solution its partial solution which could be obtained by means of free space Green function $G(R)=R^{-1}\exp(ikR)$. In Born approximation equation (4) solution could be presented in the form of two terms sum.

$$\psi_{sf} = \frac{ik}{2\pi c} \int d^3r' G(r - r')[(n_o\nabla)(\psi_i^{(0)} n_o U) + (U\nabla)\psi_{sp}^{(0)}] \qquad (5)$$

where integration is performed over the outer region of sphere r>a. It is important to notice, that solution for ψ_{sf} written above is also nonunique, for if other form of Green function would be chosen, then other solution for ψ_{sf}, differing from (5), could be obtained. But if we still choose function ψ_{sf} in the form (5) and consider it formally as new additional incident field in the problem of sound scattering on moving sphere surface r=a, then taking into account unique boundary conditions for r=a, corresponding Helmholtz equation unique solution for scattered field $\psi_{sp}^{(1)}$ could be found. If still, in solving differential equation (4) other partial solution is used, with other Green function in expression (5), then definite forms of field expressions obtained before will be changed not only for ψ_{sf} field only, but for ψ_{sp} field as well. Nevertheless together, as a sum, newly found general solution ψ would still obey initial governing equation and additional conditions on infinity and sphere surface. Due to uniqueness of problem all solution expressions (3) for total field ψ obtained by various methods should coincide. Proposed method could be considered as alternative to well known Green function, satisfying equation and boundary conditions, method, which is used frequently for partial problems solution [7].

2.2.2 Scattered field ψ_{sp} search procedure. To find ψ_{sp} field, scattered on moving sphere surface, one could use well known Green integral relationship, connecting value of arbitrary function obeying Helmholtz equation in the observation point with its value and its derivative value on the surface of sphere $r'=a$. Using this relationship for ψ_{sp}, one could obtain integral equation, in which derivative $\partial\psi_{sp}/\partial x$ value on the boundary $r'=a$ could be expressed by means of initial boundary conditions, written for total field ψ, through known values of ψ_i and ψ_{sf} fields. To find total potential ψ value on the boundary $r'=a$, classical method described in well known monograph [7] could be used. For that purpose all integration region - surface of sphere in Green relationship - should be divided by two parts: "illuminated" part, where $n_o n'<0$ and shadow part, where $n_o n'>0$. In shortwave approximation used here, each elementary portion of sphere surface dS could be approximated as plane, while it is known that, for reflection of sound from absolutely rigid plane, amplitude of reflected wave equals to the incident wave amplitude. That is why, on the "illuminated" part of sphere, equality $\psi_{sp}(r'=a)=+(\psi_i+\psi_{sf})$ would take place, while on the shadow part, where total field ψ on the sphere surface $r'=a$ equals to zero, equality $\psi_{sp}(r'=a)=-(\psi_i+\psi_{sf})$ would be valid. Substituting field and its derivative values thus obtained in Green relationship, one could write solution ψ_{sp} in general form

$$\psi_{sp} = \frac{1}{4\pi}\int[dS\nabla(G\psi_i + G\psi_{sf}) + dSM2ikG\psi_i^{(0)}] + \\ + \frac{1}{4\pi}\int dS[\ G(\nabla\psi_i + \nabla\psi_{sf}) - (\psi_i + \psi_{sf})\nabla G], \qquad (6)$$

where integration in first term is fulfilled over the part of sphere surface $ds=n'a^2d\Omega$, for $n_o n'<0$, while in second term over the shadow part of sphere for $n_o n'>0$.

2.3 Problem solution results and discussion.

Scattered field ψ_{sp} calculation in Fraungopher scatterer zone shows, that corresponding scattering amplitude could be represented, as well as expression (6), in the form of two term sum: isotropic term, connected with the wave reflected from sphere surface and described by first term of expression (6) and anisotropic term, connected with shadow wave and described by second term of (6). In zero Mach number approximation scattering amplitude for reflected wave is proportional to sphere radius a, while for shadow wave it is proportional to ka^2, and for condition $ka\gg1$ it exceeds reflected wave substantially. Shadow wave propagation takes place mainly in forward direction - in angle θ region $\theta\le1/ka$, while resulting contribution of both waves in total crossection turns out to be approximately the same and the sum equals to $2\pi a^2$. When sphere motion is considered simply in the frame of Helmholtz equation, and surrounding fluid motion is not taken into account, then reflected wave acquires correction with relative value proportional to Mach number. In this latter case practically the form of shadow scattered wave is not changed, to be accurate, it is changed just slightly due to wave number $k=k_o(1-Mn_o)$ Doppler renorming connected with sphere motion. Calculation of ψ_{sf} field, scattered on surrounding ideal fluid potential flow inhomogeneities, was already fulfilled in paper [8], where it was shown that partial scattering amplitude for incident wave in the case ka>>1 is strongly anysotropic - forward directed - as well and proportional to k^2a^3M. Thus for big sphere relative correction for total scattering amplitude, due to sphere and medium motion, is shown to be proportional to M, just like in the case of Helmholtz equation, but very large additional dimensionless factor $(ka)^2$, multiplying M in relative scattered amplitude, obtained in solution of Lighthill equation, more closely related to the nature of problem, changes comparative contributions of body and fluid drastically. It is worth to notice here, that double scattering of sound on flow inhomogeneities is taken into account, i.e. besides of sound directly scattered on moving fluid, sound reflected from sphere surface and then scattered by flow, is also accounted to calculate corresponding total scattering amplitude. That is why in far field of the scatterer, presumably in forward direction, directivity of scattered field is conserved, even if fluid is taken into account, while total scattering amplitude in the forward direction, formed presumably by shadow wave, turns out to be of the order of $ka^2[1+Ck^2a^2(Mn_o)]$, where C is numerical factor independent, in zero order approximation, of M and (ka) parameters. Total sound scattering crossection on moving sphere, just like in previous papers [3,4], turns out to be not additive in relation to partial crossections of body and fluid alone and equals

$$\sigma \cong 2\pi a^2 \,[1 + Ck^2a^2(Mn_o)]^2 \qquad (7)$$

Very important result could be seen from the last expression. It is evident, that for moving body it is always necessary to take into account surrounding fluid flow, even if Mach number values are very small. One could see, that it leads to relative field corrections increased by large factor $(ka)^2$ instead of well known small Mach number correction. To be accurate, use of Born approximation demands, that parameter $(ka)^2M$ should be small enough, but practically the result obtained could be successfully used in estimates up to parameter mentioned being of the order of unity.

3. MULTIPLE SCATTERING AND FOCUSING PHENOMENON.

3.1. Multiple wave scattering.

Using shortwave sound scattering on flow inhomogeneities and moving body surface anysotropy properties obtained above, as well as formal multiple scattering theory diagram technics, one could find solution of governing equation in more general form, than it is possible by means of Born approximation. For instance, in

ON SOUND PROPAGATION NEAR MOVING BODY

approximation of smooth perturbations, general solution for calibrated potential ψ could be presented in the form $\psi=\psi_0\exp(ikn_0r+iS)$, where S is additional eikonal defined by expression of the types (5) and (6). From physical point of view S is additional phase gain acquired by wave passing inhomogeneous region of localized flow near moving body. For medium here is taken to be moving, sound velocity in the vicinity of body equals to $c+Un_0$ instead of simply c. Additional phase gain, which usually equals $k\int dl(-\Delta c/c)$, is defined here by integral of the form

$$S = \frac{i}{c}\int^{z} dl n_0(kiUn_0 + n_0\nabla U) \quad , \tag{8}$$

where low integration limit should be taken equal to $-\infty$ for refracted waves and to zero for shadow waves. Expression (8) practically coincides with solution (5), if the latter would be integrated over transverse coordinates $\mathbf{r}'$ with respect to incident wave propagation direction $\mathbf{n}_0$, and condition of slow changes of velocity $U(\mathbf{r})$ and its derivative in comparison to fast oscillations observed due to wave processes is met. On this stage, in fact, transition from wave to ray problem statement takes place, and considering small rays deflections from their initial directions, integration in expression (8) could be simplified and done along $\mathbf{n}_0$ direction instead of rays involved real trajectories. Thus multiple wave scattering problem in the vicinity of moving body for large parameter (ka) values could be reduced to rays refraction problem in fluid localized flow region. Formally it could be explained in such a way - Lighthill equation (2) form in that case could be reduced to the form of conventional homogeneous wave equation with effective space dependent sound velocity defined by velocity of surrounding fluid flow $U(\mathbf{r})$.

3.2 Sound focusing phenomenon.

For Mach number is small, additional phase gain derived from expression (8), is small as well. But conventional dependence of eikonal S on transverse coordinates $r_\perp$ leads to distortion of initially plane propagating wave front. Although this distortion is not large, on great distances from the body it leads to space radial reconstruction of wave field. Due to this process field characteristics in definite space regions could be changed substantially. Expanding expression (8), taken near body shadow border $\mathbf{r}_\perp=\vec{\rho}_s$ (ρ_s=a for sphere), in transverse coordinates Teylor series, one could found out that, with accuracy up to square terms, eikonal (8) to be found could be approximated by following expression

$$S(r_\perp,z) = S(\vec{\rho}_s,z) + (r_\perp-\vec{\rho}_s)_\alpha \frac{\partial S}{\partial r_{\perp\alpha}} + \frac{1}{2}(r_\perp-\vec{\rho}_s)_\alpha(r_\perp-\vec{\rho}_s)_\beta \frac{\partial^2 S}{\partial r_{\perp\alpha}\partial r_{\perp\beta}} \tag{9}$$

Field far from the body could be found here by the method proposed in paper [5]. To begin with, by means of previous section expressions, one could find eikonal S in the vicinity of body for z approaching z_0, solving corresponding equations, including inhomogeneous equation (4), in eikonal approximation. Then it is of use

ON SOUND PROPAGATION NEAR MOVING BODY

to introduce special control surface situated on longitudinal coordinate $z=z_1$ ($z=rn_o$), to the right of the body, at distance large enough, that practically, in the region $z>z_1$, fluid velocity U(r) could be neglected. This distance is also chosen to be comparable to Fresnel zone dimensions, where ray approximation is still valid. Then in that region governing equation (2) could be solved, but in simple homogeneous form, without right-hand side. It could be fulfilled by means of Green relationship, just like in paragraph 2.2.3 above, or simply by Huygens principle, which takes the same form for ka>>1. To proceed, one could take problem solution on the $z=z_1$ as initial distribution, obtained in the first stage. Then, using expression (9) for eikonal S on coordinate $z=z_1$, problem solution far from the body $z>>z_1$ could be written in most general form

$$\psi = \frac{ke^{ikn_o r}}{2\pi i(z-z_1)} \int d^2r' \exp i[\frac{k(r_\perp-\vec{\rho}_s)}{2(z-z_1)} + S(z_1,r_\perp)]. \qquad (10)$$

3.3 Discussion of results obtained.

Further field amplitude calculation according to expression (10), taking into account rays refracted by flow outside body as well as the rays related to shadow wave, shows, that on distance from the center of sphere, equal in the order of magnitude to body transverse dimension divided by Mach number, substantial field amplification is possible. This way localized flow, which takes place in the vicinity of moving sphere, could form a type of long-focus acoustic lens. This lens, depending on changing in time local flow velocity value and sign, provides additional, in comparence to conventional diffraction, divergence or convergence of corresponding propagating rays. In this process, as it was shown in paper [5] for arbitrary body form, total ray deflection angle, which was shown to be proportional to magnitude $[k^{-1}\partial S/\partial r_\perp + a/(z-z_1)]$, could in principle acquire positive or negative value. For the case, where rays are converged, lens focus is formed on the distance $R \approx k(\partial^2 S/\partial r_\perp^2)^{-1}$ from sphere center, in other words, when observation point is approaching this region, field amplitude formally is reaching infinite value, in the same way as expression $[(1/R)-1/(z-z_1)]^{-1}$. In fact, this focus could be formed only for rays corresponding to shadow wave, while refracted rays, travelling outside body, are deflected much weaker, depending on their distance of closest approach to body. That is why we could speak here about acoustical lens only taking into account a lot of reservations mentioned above. Moreover, sound field scattering for the case of asymmetrical flow near asymmetrical body, could form distributed caustic, in the vicinity of which wave field, in fact, always remains finite. It is important to mention, that even in classical case, wave field near caustics behave as Airy function, and its value there is finite as well [10]. Nevertheless, in practice, when caustics are formed, field amplitude in their vicinity could be amplified substantially, and this amplification could be observed experimentally. Almost the same phenomenon of wave field focusing by localized flow in the absence of moving body, was numerically modeled and theoretically analyzed in paper [11] for sound scattering by moving Hill vortex.

4. CONCLUSION

Main result of the work should be underlined once more as follows - when problem of wave diffraction on body moving in the medium is to be solved, it is very important to take into account accompanying fluid flow. Corrections to scattered field related to wave scattering on flow, found in the frames of Lighthill equation, substantially exceed known corrections related to corresponding moving body surface reflection, found in the frames of Helmholtz equation. Moreover, corrections to be found could be of an order comparable with field, scattered by body at rest itself, and in definite fluid regions could even exceed it. If wave focusing phenomenon analyzed in paragraph 3 would be taken into account, it should become necessary, from our point of view, to reexamine in that light a long row of wave diffraction problems, where moving bodies are involved, solved before. It is worth to notice, that problems of such kind are important not only in acoustics, but in various fields of physics using wave process mechanics as well. For instance, related results could be observed in the problems of wave scattering on charged particles moving with substantial velocities; particles oncoming beams mutual scattering; same phenomena could be responsible for scattering amplification due to particles motion in the media as well. It becomes obvious, that lens formation in optics is possible not only due to local change of media parameters (density, charge particles concentration, dielectric constant), but due to local media motion itself as well. Taking into account astronomical problem aspect, moving of optical scatterers (black holes with accretion halo, planet nebulae, halaxies etc.) should lead to special kind of lens formation as well, with focusing ability concurring to that predicted on the basis of other principles known before. For instance, corresponding Mach numbers for very distant astronomic objects moving in accordance with principle of expanding Universe, could be so large, that focusing of light due to their movement would be noticeable enough and could even exceed purely gravitational effects taken into account now for their prediction. In that light, the problem of distant objects (quasars) anomalous radiance could be reexamined, taking into account, that on the way to the earth light is probably passing relatively compact scattering region moving with sublight velocity, where additional focusing is available. In that way, from our point of view, the problem of quasars itself with their enormous radiance which is still not understood now, could be probably explained, at least partly.

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ON SOUND PROPAGATION NEAR MOVING BODY

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