

CALCULATIONS OF GROUND VIBRATIONS FROM HEAVY-FREIGHT TRAINS

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1. INTRODUCTION

Railway-induced ground vibrations may cause noticeable movements of nearby buildings that result in damage or disamenity, both directly and by generating structure-borne noise [1-3]. Both effects depend strongly on the spectra of railway-induced ground vibrations that, in turn, are determined by the contributions of different excitation mechanisms and by soil properties.

Spectra of railway-induced ground vibrations, including train-speed dependent components, have been studied experimentally [4,5], and qualitative analysis of the quasi-static excitation mechanism has been attempted [6]. However, no rigorous theoretical investigations of railway-generated ground vibrations have been carried out and no quantitatively calculated spectra exist in the literature.

In this paper we consider theoretically the effect of quasi-static pressure of wheel axles onto the track-soil system. In the case of welded rails and perfect wheels, this mechanism is the major contributor to train-speed-dependent components of the low-frequency vibration spectra (up to 50 Hz), including the so called passage frequency $f_p = v/d$, where v is train speed and d is distance between sleepers.

2. STATEMENT OF THE PROBLEM

We consider a train having N carriages and moving with speed v on welded track with sleeper periodicity d . The excitation being considered results from load forces applied to the track from each wheel axle causing downward deflection of the track. For the low-frequency components of the spectra most relevant for ground vibration, these deflections can be considered as quasi-static, producing a wave-like motion along the track with speed v and resulting in a distribution of the axle load over all the sleepers involved in the deflection distance. Thus, each sleeper acts as a vertical force applied to the ground during the time necessary for a deflection curve to pass through the sleeper. This should result in generation of elastic ground vibrations. Since, in the relevant frequency band, the characteristic wave-lengths of generated elastic waves are much larger than the sleeper dimensions, each sleeper can be considered as a point-source vertical force. The problem then requires superposition of the elastic fields radiated by all sleepers caused by the passage of all axles.

CALCULATIONS OF GROUND VIBRATIONS FROM HEAVY-FREIGHT TRAINS

An important aspect of the above is calculation of the track deflection curve as a function of the elastic properties of track and soil and of the magnitude of the axle load. The form of the deflection curve determines the ground vibration frequency spectrum generated by each sleeper. In turn, these spectra strongly affect the total vibration spectrum generated by a passing train.

3. DETERMINATION OF THE TRACK DEFLECTION CURVE

Since the track deflection distance is greater than the distance between sleepers, one can ignore the influence of rail periodic support by sleepers in the quasi-static problem of track deflection under the impact of a moving load. Instead we treat a track (i.e. two parallel rails with periodically fastened sleepers) as an Euler-Bernoulli elastic beam of uniform weight p lying on an elastic or viscoelastic foundation occupying the semispace $z > 0$. Deflection of such a beam subjected to a vertical point load has been considered by many authors. The classical solution [7,8] starts with the static beam equation that takes into account a force reaction in the elastic foundation proportional to the deflection magnitude w . If E and I are Young's modulus and the cross-sectional momentum of the beam, α is the proportionality coefficient of the elastic foundation, x is the distance along the beam and F_m is a vertical point force applied at $x=0$, then the solution for w has the form

$$w = (F_m/8EI\beta^3) \exp(-\beta|x|) [\cos(\beta x) + \sin(\beta|x|)] + p/\alpha, \quad (1)$$

where $\beta = (\alpha/4EI)^{1/4}$. According to eqn (1), one can take $x_0 = \pi/\beta$ as the total deflection distance.

The constant α in eqn (1) depends particularly on the stiffness of the ground and of the rubber pads inserted between rail and sleepers. Calculation of α for typical British Rail tracks [9,10] gives the values $\alpha = 61.8 \text{ MN/m}^2$ and $\beta = 1.28 \text{ m}^{-1}$. For the typical distance between sleepers, $d = 0.7 \text{ m}$, this implies that about seven sleepers are involved in the deflection curve associated with each axle.

A more recent approach to an analogous problem in mechanics [8] acknowledges that for this type of loading tensile stresses cannot be transmitted between the beam and the elastic foundation. This model is more appropriate for track-soil contacts which can respond only to compressive stresses. Thus the contact nonlinearity of a real boundary between track and ground is taken into account.

Analysis of this model shows [8] that for values of the axle load $F_m \leq F_{cr} = (2p/\beta)\exp(\pi)$ the simple classical solution (1) which describes a continuous contact between track and foundation remains valid. However, for $F_m > F_{cr}$ the solution becomes more complicated and involves peripheral bulges of the track with loss of contact between track and soil. In this case the

CALCULATIONS OF GROUND VIBRATIONS FROM HEAVY-FREIGHT TRAINS

problem is solved numerically for coordinates x_0 and x_1 of the deformed track where it intersects ground level ($z = 0$), and for the five coefficients describing the shape of the deflection curve as a function of applied load.

For our purposes it is sufficient to use a rather rough analytical approximation of the deflection function for $F_m > F_{cr}$, where only one parameter x_0 as a function of F_m is taken into account:

$$w = \begin{cases} p/\alpha + (F_m/8EI\beta^3)\cos(\pi x/2x_0) & \text{for } |x| < x_0, \\ 0 & \text{for } |x| > x_0. \end{cases} \quad (2)$$

Numerical data for x_0 as a function of F_m calculated in the paper [8] over the range $1.6/\beta < x_0 < \pi/\beta$ can be approximated by the equation

$$x_0 = (1/\beta)\{\pi - [0.4 \log(\beta F_m/2e\pi p)]^{0.3}\} \quad (3)$$

which describes the decrease in x_0 with increasing applied load F_m . Note that this approximation is invalid for very large loads when x_0 approaches the sleeper period d . In this case, the effect of periodic sleeper support should be taken into account [10].

4. GENERATION OF GROUND VIBRATIONS BY INDIVIDUAL SLEEPERS

To calculate ground vibrations generated by individual sleepers let us consider each sleeper as a point source of vertical force applied to the surface $z=0$ at $x = 0$ and $y = 0$, with time dependence determined by the passage of the deflection curve through the sleeper:

$$P(t) = F_m[2w(vt)/w_{\max}](d/x_0), \quad (4)$$

where w_{\max} is the maximum value of $w(vt)$. Terms on the right of F_m take into account the distribution of axle load between sleepers within the deflection curve. We shall now make use of results from the well-known axisymmetric Lamb problem for the excitation of an elastic semispace by a vertical point force applied to the surface. The solution of this problem describes the corresponding components of the dynamic Green's tensor G_{zj} (or, for simplicity, the components of the Green's function) for the elastic semispace. This function satisfies the dynamic equations of elasticity for a semispace, assumed isotropic and homogeneous, and the appropriate boundary conditions.

CALCULATIONS OF GROUND VIBRATIONS FROM HEAVY-FREIGHT TRAINS

In what follows only Rayleigh surface waves (the Rayleigh part of the Green's function) are considered because they transfer most of the vibrational energy. For these waves the spectral density of the vertical vibrational velocity at $z=0$ may be written in the form [11]

$$v_z(\rho, \omega) = P(\omega) G_{zz}(\rho, \omega) = V(\omega) (1/\sqrt{\rho}) \exp(ik\rho - i3\pi/4), \quad (5)$$

where

$$V(\omega) = (\pi/2)^{1/2} P(\omega) (-i\omega) q(k_R)^{1/2} k_t^2 / \mu F'(k_R). \quad (6)$$

Here $\rho = [(x-x')^2 + (y-y')^2]^{1/2}$ is the distance between the source (with current coordinates x', y') and the point of observation (with coordinates x, y), $\omega = 2\pi f$ is a circular frequency, $k_R = \omega/c_R$ is the wavenumber of a Rayleigh surface wave where c_R is the Rayleigh wave propagation velocity, $k_l = \omega/c_l$ and $k_t = \omega/c_t$ are the wavenumbers of longitudinal and shear bulk elastic waves, where $c_l = [(\lambda + 2\mu)/\rho_0]^{1/2}$ and $c_t = (\mu/\rho_0)^{1/2}$ are longitudinal and shear propagation velocities, λ and μ are Lamé constants, ρ_0 is the ground mass density, and $q = (k_R^2 - k_t^2)^{1/2}$. The factor $F'(k_R)$ is a derivative of the Rayleigh determinant

$$F(k) = (2k^2 - k_t^2)^2 - 4k^2(k^2 - k_t^2)^{1/2}(k^2 - k_l^2)^{1/2} \quad (7)$$

taken for $k = k_R$, and $P(\omega) = (1/2\pi) \int_{-\infty}^{\infty} P(t) \exp(i\omega t) dt$ is a Fourier transform of $P(t)$. The factor $1/\sqrt{\rho}$ in eqn (5) describes the cylindrical spreading of Rayleigh waves with propagation distance.

For a viscoelastic semispace the elastic constants λ and μ should be considered as complex numbers taking into account frequency dependent attenuation and dispersion of elastic waves [12]. This would make c_R , c_t and c_l complex also, resulting in a decrease in the amplitudes of all waves with distance and a broadening of their time-forms because of velocity dispersion. In the calculations below we will not take velocity dispersion into account since it is rather small for the homogeneous semispace being considered.

It is seen from (4) and (5) that the Fourier transform $P(\omega)$ plays a very important role in determining the spectra of radiated waves. In the case under consideration, $P(\omega)$ should be determined separately for $F_m \leq F_{cr}$ and for $F_m > F_{cr}$ for which the deflection function $w(x)$ is described by eqns (1) and (2), (3) respectively. Taking the Fourier transforms, one can easily obtain the corresponding analytical expressions for $P(\omega)$:

CALCULATIONS OF GROUND VIBRATIONS FROM HEAVY-FREIGHT TRAINS

$$P(\omega) = (F_m d / \pi x_0) \{ (2\beta v + \omega) / [(\beta v)^2 + (\beta v + \omega)^2] + (2\beta v - \omega) / [(\beta v)^2 + (\beta v - \omega)^2] \}, \quad \text{for } F_m \leq F_{cr}; \quad (8)$$

$$P(\omega) = (F_m d / \pi x_0) (2\pi v / x_0) \cos(\omega x_0 / v) / [\omega^2 - (\pi v / 2 x_0)^2], \quad \text{for } F_m > F_{cr}, \quad (9)$$

where x_0 is determined by eqn (3).

One can generalise these results to describe the action of two axle loads separated by the distance a (the case of a bogie):

$$P_b(\omega) = 2P(\omega) \cos(\omega a / 2v). \quad (10)$$

5. CONSIDERATION OF ALL SLEEPERS AND AXLES

To calculate the vibration field radiated by a complete moving train requires the superposition of fields generated by each sleeper activated by all axles of all carriages, with the time and space differences between sources (sleepers) being taken into account.

Using the Green's function formalism this may be written in the form

$$v_z(x, y, \omega) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(x', y', \omega) G_{zz}(\rho, \omega) dx' dy', \quad (11)$$

where $P(x', y', \omega)$ describes the total distribution of forces along the track. This distribution is found by taking a Fourier transform of the time and space dependent track deflection function.

It is useful firstly to consider a single axle load moving with speed v along the track lying on perfectly elastic ground. Then the load force which makes a wave-like motion along the track may be written in the form

$$P(t, x', y'=0) = \sum_{m=-\infty}^{\infty} P(t - x'/v) \delta(x' - md) \delta(y'), \quad (12)$$

with the Fourier transform

CALCULATIONS OF GROUND VIBRATIONS FROM HEAVY-FREIGHT TRAINS

$$P(x', y', \omega) = (1/2\pi) \int_{-\infty}^{\infty} \sum_{m=-\infty}^{\infty} P(t-x'/v) \exp(i\omega t) \delta(x'-md) \delta(y') dt, \quad (13)$$

where $\delta(x'-md)$ takes the periodic distribution of sleepers into account. Integration yields

$$P(x', y', \omega) = P(\omega) \exp(i\omega x'/v) \sum_{m=-\infty}^{\infty} \delta(x'-md) \delta(y'). \quad (14)$$

Substituting eqn (14) into eqn (11) and using the properties of integrating delta-functions we have, after taking eqns (5),(6) into account, the following expression for the vertical vibration velocity of Rayleigh waves generated at $x=0$, $y=y_0$ by a single axle load moving along the track with speed v :

$$v_z(x=0, y=y_0, \omega) = -iV(\omega) \sum_{m=-\infty}^{\infty} \exp[i(\omega/v)md + i(\omega/c_R)\rho_m] / \sqrt{\rho_m}, \quad (15)$$

where $\rho_m = [y_0^2 + (md)^2]^{1/2}$. Formula (15) shows that a single moving load generates a quasi-discrete spectrum with frequency peaks close to f_{ps} , where $f_p = v/d$ is the so called passage frequency, and $s = 1, 2, 3, \dots$. Deviation from perfect discreteness results from the $i(\omega/c_R)\rho_m$ term in eqn (15) which takes into account path-length differences of waves propagated from each sleeper to the point of observation.

To take account of all axles and carriages one needs a more complicated load function:

$$P(t, x', y=0) = \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} A_n [P(t - (x'+nL)/v) + P(t - (x' + M + nL)/v)] \delta(x'-md) \delta(y'). \quad (16)$$

Here N is the number of carriages, M is the distance between bogies in each carriage and L is the total carriage length. Dimensionless quantity A_n is an amplitude weight-factor to account for different carriage masses, but for simplicity we will suppose all carriage masses to be equal ($A_n=1$).

Substituting eqn (16) into eqn (14) and then into eqn (11), and making simple transformations similar to the above, one obtains the following expression for the frequency spectra of vertical vibrations at $z=0$ generated by a moving train:

$$v_z(x=0, y=y_0, \omega) = -iV(\omega) \sum_{m=-\infty}^{\infty} \sum_{n=0}^{N-1} [\exp(-\gamma\omega\rho_m/c_R) / \sqrt{\rho_m}] [1 + \exp(iM\omega/v)] \cdot \exp(i(\omega/v)(md + nL) + i(\omega/c_R)\rho_m). \quad (17)$$

CALCULATIONS OF GROUND VIBRATIONS FROM HEAVY-FREIGHT TRAINS

In writing eqn (17) we account for attenuation in soil by replacing $1/c_R$ in the exponentials by the complex value $1/c_R + i\gamma/c_R$, where $\gamma \ll 1$ is a constant describing the "strength" of dissipation of Rayleigh waves in soil (eqn (17) implies a linear frequency dependence of soil attenuation, in agreement with experimental data [13,14]).

The summation over m in eqn (17) considers an infinite number of sleepers. However, the contribution of remote sleepers is small because of soil attenuation and cylindrical spreading, and a few hundred sleepers are adequate for practical calculations.

It follows from eqn (17) that the spectrum of train-induced vibrations is quasi-discrete, with the maxima at frequencies determined by the condition $(\omega/v)(md + nL) = 2\pi l$, where $l = 1, 2, 3, \dots$. Obviously, $n=0$ corresponds to the passage frequencies f_{ps} determined by the sleeper period d . Other more frequent maxima are determined either by the carriage length L ($m=0$) or by a combination of both parameters (for $n \neq 0, m \neq 0$).

There are many zeros present in the train vibration spectra. These zeros may be used in practice for suppressing vibrations at chosen frequencies. The most important zeros are those which do not depend on a number of sleepers or carriages and are determined only by the geometrical parameters of a carriage. One of these zeros is determined by the distance a between the wheel axles in a bogie (see eqn (10) for the spectrum P_b). Setting P_b to zero, one can obtain $f_z = (v/a)(n + 1/2)$ for zero-frequencies. If, for instance, we want to use this condition to suppress one of the train passage frequencies f_{ps} , we should choose f_z to be equal to f_{ps} . It follows from this that the value of a should be determined by

$$a = (d/s)(n + 1/2). \quad (18)$$

It is sensible to choose a value of a close to existing values. For British Rail heavy-freight carriages $a = 2.2$ m usually. Therefore, to suppress the main passage frequency ($s=1$) one can choose $a = 2.45$ m corresponding to $n=3$ in eqn (18).

Other important zero frequencies reflect the distance M between bogies in a carriage. Condition (18) is also valid for this case if a in eqn (18) is replaced by M . The value of M providing suppression of the main passage frequency which is closest to the British Rail standard ($M=4.88$ m) is 4.55 m, corresponding to $n=6$.

6. NUMERICAL CALCULATIONS AND DISCUSSION

Numerical calculations of train-induced ground vibrations described by equations (17), (6)-(10) and (3) have been carried out for different values of applied load F_m , train speed v , soil attenuation coefficient γ , and for different geometrical parameters of both track and train: d , a , L , M . The elastic parameters of the soil considered were $c_R = 250$ m/s, $c_t = 272$ m/s, $c_l = 471$ m/s (corresponding to a Poisson's ratio of $\sigma=0.25$). The mass density of soil ρ_0 was set at 2000 kg/m³.

CALCULATIONS OF GROUND VIBRATIONS FROM HEAVY-FREIGHT TRAINS

Figure 1 shows the theoretical frequency spectra for a 5-carriage train moving with speed $v = 50$ km/h. Calculations have been carried out for an axle load of 100 kN (below the critical value $F_{cr} = 108.3$ kN determined for a typical track weight with concrete sleepers, $p = 3$ kN/m) and for a load of 200 kN (above the critical value). Geometrical parameters of the train were chosen typical of British Rail heavy-freight trains [6]: $L = 8.3$ m, $M = 4.88$ m and $a = 2.2$ m. The soil attenuation parameter γ was chosen to be 0.00478, y_0 was 30 m. According to the figure, both spectra have maxima at the train passage frequencies (the main passage frequency for $v = 50$ km/h is 20 Hz) and at frequencies determined by train geometrical parameters. Generation is more efficient at 200 kN axle load, especially at higher frequencies; this results mainly from the sharper form of the deflection curve $w(t-x/v)$ and hence the wider spectrum $P(\omega)$. Note that the shapes and intensities (in dB) of the calculated spectra are in good agreement with experimental data [4,5].

The vibration spectra calculated for different train speeds, $v = 50$ km/h and $v = 40$ km/h, are shown in Fig.2 for an axle load of 100 kN. The parameters of track, train and soil are the same as before. As expected, changing the train speed displaces the spectral maxima.

The effect of soil attenuation γ on the vibration spectra is shown in Fig.3 for the fixed distance $y_0 = 30$ m and axle load of 200 kN; all other conditions being the same as in Fig.1. It is seen that with increasing γ all spectral components are damped, the damping being stronger at higher frequencies. Note that spectral components at the passage frequencies are less affected by soil attenuation than those at combined frequencies determined by train geometrical parameters. This is because the passage frequency response is determined mainly by sleepers close to the point of observation, whereas the response at combined frequencies is determined by signals from remote carriages which are strongly attenuated.

Fig. 4 shows how choice of the distance a between axles in a bogie can suppress ground vibration intensity at the main passage frequency $f_p = v/d$. The corresponding distance $a = 2.45$ m was calculated using eqn (18) for $d = 0.7$ m. The axle load was 100 kN, the train speed $v = 50$ km/h, and the distance $y_0 = 30$ m, other parameters being same as in previous figures. According to the figure, the vibration level at the lowest passage frequency is suppressed by about 20 dB, i.e. a factor of 10 relative to that for $a = 2.2$ m. Of course, the same effect could be achieved by changing the sleeper period d .

7. CONCLUSIONS

Generation of ground vibrations by moving trains has been considered theoretically using the Green's function formalism. Expressions for the ground vibration spectra have been obtained as functions of track, train and soil parameters.

Numerical calculations have been carried out for different physical and geometric properties of track, train and soil. The shapes and intensities of the spectra are in good agreement with experimental data.

CALCULATIONS OF GROUND VIBRATIONS FROM HEAVY-FREIGHT TRAINS

Vibration spectra depend strongly on the axle loads of the carriages: if the axle load exceeds a critical value beyond which peripheral bulges appear in the track, the vibration level increases significantly, especially at higher frequencies.

By proper selection of the distance between wheel axles in a bogie, and between bogies in a carriage (or between sleepers in a track) it is possible effectively to suppress vibration levels at the train passage frequencies.

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CALCULATIONS OF GROUND VIBRATIONS FROM HEAVY-FREIGHT TRAINS

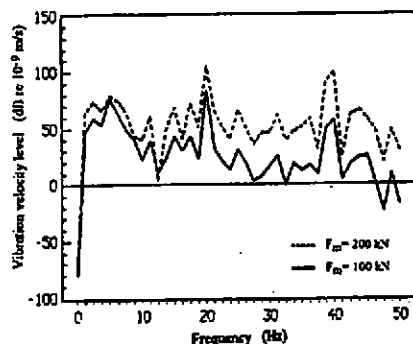


Fig. 1. Ground vibration spectra for different axle loads, F_m

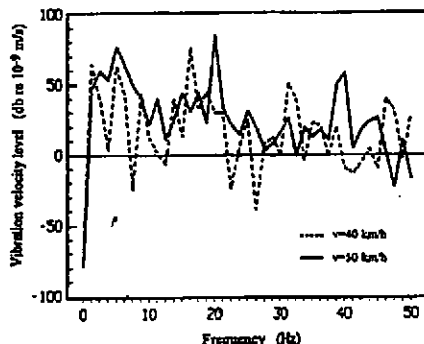


Fig. 2. Influence of a train speed v on ground vibration spectra

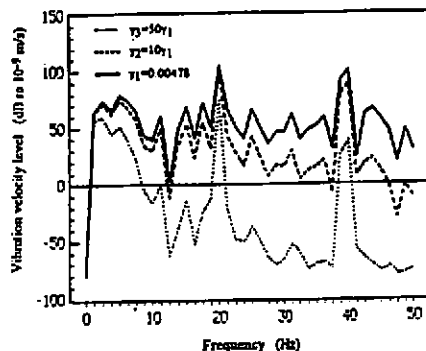


Fig. 3. Ground vibration spectra for different soil attenuation, γ : $\gamma_0 = 30$ m

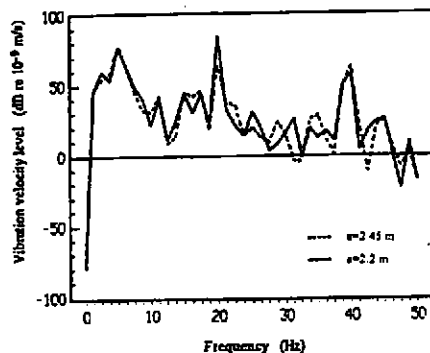


Fig. 4. Suppression of ground vibrations at the main passage frequency by selecting the appropriate distance a between the axes in a bogie