Proceedings of the Institute of Acoustics

SOUND SCATTERING ON SHELLS WITH ARBITRARY CROSS-SECTIONAL CONTOUR

V V Muzychenko

N.N.Andreev Acoustics Institute
4, Shvernik st., Moscow, 117036, USSR.
Tel.: (095) 126-9870, Fax : (095) 126-8411.

1. INTRODUCTION

A scattering problem of sound wave, which falls normally to infinite thin cylindrical shell with arbitrary contour of cross-section is considered. The sistem of shell equations may be obtained from the equations of elastic equilibrium of thin shell by V.Z.Vlasov. The analitical expression for Green function of shell is found with help of simply layer potential. The density of simply sources is defined from the system of boundary integral equations. In partial cases (absolutely rigid and absolutely soft boundary, elastic shell with circular contour) the known expressions are followed.

2. TO THE THEORY OF SOUND SCATTERING

There are many papers devoted to the scattering problem when the cylindrical shell has a circular contour of cross- section (see, for example, [1]) and the definition of scattering amplitude in this case is not a difficult problem. But when the cross-sectional contour is an arbitrary plane curve the equations of shell elastic oscillations are avery complicated system of three differential equations of high order with alternative coefficients, because the radius of curvature $R(\varphi)$ and the coefficient of quadratic form $B(\varphi)$ are arbitrary functions of angle φ . This system may be obtained from the equations of elastic equilibrium of thin shell by V.Z.Vlasov [2]. When normal incidence of plane wave on infinite cylindrical shell occures the oscillations depend only from φ and the vector of elastic deformation has two components: tangential u_1 and normal u_3 . In this case the equations of shell oscillations are obtained in [3] (formulae (8)):

$$\begin{cases} \dfrac{\partial^2 u_1}{\partial x^2} - \dfrac{\partial q u_3}{\partial x} + p^2 u_1 = 0 \ , \\ -q\dfrac{\partial u_1}{\partial x} + \dfrac{h^2}{12}\dfrac{\partial^4 u_3}{\partial x^4} + q^2 u_3 - p^2 u_3 = \dfrac{1}{B}\,(P_o - P_s) \ . \end{cases} \quad (1)$$

Here: $p^2=\rho h\omega^2(1-\nu^2)/Eh$; ρ - the density of material; h - shell thickness; ω -circular frequency; $E=E_o(1-i\eta)$; ν - Young's modulus and Poisson ratio; η - damping coefficient; P_o and P_s - amplitudes of incident and scattering sound waves (Inside the shell there is no medium); $B=Eh/(1-\nu^2)$; $q(\varphi)=1/R(\varphi)$; $dx=Rd\varphi$. We suppose that the time dependence of all quantities is $exp(-i\omega t)$.

In the system (1) the only quantity which depends on x is curvature q .

Last years the problem of sound scattering on cylindrical area with arbitrary contour of cross-section the method of simply sources (monopoles) is used [4], which density σ may be found by means of integral equations solved on computers.

For absolutely soft boundary we have:

$$P = P_o + P_s = 0 \ , \quad (2)$$

and the integral equation is:

$$\frac{1}{i\rho\omega} P_s(\mathbf{r}) = \oint_S \sigma(\mathbf{r}')\mathcal{G}(\mathbf{r},\mathbf{r}')ds' \ . \quad (3)$$

And when the boundary is absolutely rigid:

$$V_n = V_o + V_s = 0 \ , \quad (4)$$

$$V_o = \frac{1}{i\rho\omega}\left[\frac{\partial P_o}{\partial n}\right] \ , \ V_s = \frac{1}{i\rho\omega}\left[\frac{\partial P_s}{\partial n}\right] \ , \quad (5)$$

and the integral equation is:

$$\frac{\sigma}{2} - \int_S \sigma(\mathbf{r}')\frac{\partial \mathcal{G}(\mathbf{r},\mathbf{r}')}{\partial n}ds' = V_o \ , \quad (6)$$

where $\mathcal{G}(\mathbf{r},\mathbf{r}')=\frac{1}{4\pi}exp(ik|\mathbf{r}-\mathbf{r}'|)/|\mathbf{r}-\mathbf{r}'|$ - Green function for Helmgoltz equation in three dimension case; $k=\omega/c$ - wave number ofsound medium. In two dimension case the Green function is

$\mathcal{G}(\mathbf{r},\mathbf{r}')=(i/4)H_0^{(1)}(k|\mathbf{r}-\mathbf{r}'|)$, where $H_0^{(1)}(k|\mathbf{r}-\mathbf{r}'|)$ - is a Hankel function of the first kind.

In papers [4,5] the efficiency of the boundary elements method for culculative definition of $\sigma(\mathbf{r})$ is proved.

In this report the method of boundary integral equations is derived for the case when the contour S is thin elastic shell described by equations (1). It is necessary to find Green function $u_3 \equiv K(x,\xi)$ of system (1). It means their solution when in right part we change (P_0+P_s) by Dirac's function $\delta(x-\xi)$. When ω is arbitrary the problem has no analitical solution. But in the asimptotical (WKB) case $K(x,\xi)$ can be obtained by method described in [3].

Let us take the solution in form:

$$u_{1,3}(x) \sim exp[i\Phi(x)] , \qquad (7)$$

where $\Phi(x)=\int_{\xi}^{x} K(x)dx$; $K(x)=2\pi/\lambda(x)$.

The conditions of applicability are:

$$\frac{\lambda}{R} \ll 1 , \ \lambda \frac{1}{R}\frac{\partial R}{\partial x} \ll 1 . \qquad (8)$$

The substitution of (7) into (1) gives a dispersion equation (see [3], formulae (10)), which has six roots. The point source $\delta(x-\xi)$ generates waves which run in directions $\pm|x-\xi|$ and are of following types: flexible uniform waves ($K_1(x)=\sqrt[4]{p^2/(h^2/12)}$), flexible nonuniform waves ($K_2(x)=iK_1(x)$) and lognitudinal waves ($K_3(x)=\sqrt{p^2-q^2(x)}$). Note, that roots $K_{1,2}$ are functions of x only if the parameters E , ρ , h depend on x. In considered model only curvature $q=q(x)$ and, hence, only root K_3 is a function of x .

The corrections to the roots $K_{1,2,3}(x)$ are being found by WKB - method:

$$\delta K_{1,2}(x) = \frac{3}{2}i\left[\left(\frac{\partial K_{1,2}}{\partial x}\right)/K_{1,2}\right] , \ \delta K_3(x) = \frac{1}{2}i\left[\left(\frac{\partial K_3}{\partial x}\right)/K_3\right] . \qquad (9)$$

By integrating the equation (1) on x in limits $[\xi-\varepsilon,\xi+\varepsilon]$, when $\varepsilon\to 0$ we have:

$$\begin{cases} \left[u'_{1+} - u'_{1-}\right] - \left[qu_{3+} - qu_{3-}\right] = 0 , \\ - q\left[u_{1+} - u_{1-}\right] + \dfrac{h^2}{12}\left[\dfrac{\partial^3 u_{3+}}{\partial x^3} - \dfrac{\partial^3 u_{3-}}{\partial x^3}\right] = \dfrac{1}{B} . \end{cases} \quad (10)$$

In addition, the conditions of continuity are fulfiled in the point $x=\xi$:

$$u_{1+} = u_{1-} , \; u_{3+} = u_{3-} , \quad (11)$$

and the rotation angle of section is zero:

$$\frac{\partial u_{3+}}{\partial x} = 0 , \; \frac{\partial u_{3-}}{\partial x} = 0 . \quad (12)$$

We seek $K(x,\xi)$ in form:
when $x>\xi$:

$$K(x,\xi) = u_{3+} = A_{1+} e^{i\int_\xi^x [K_1(x')+\delta K_1(x')]dx'} + A_{2+} e^{-\int_\xi^x [K_2(x')+\delta K_2(x')]dx'} + A_{3+} e^{i\int_\xi^x [K_3(x')+\delta K_3(x')]dx'} , \quad (13)$$

when $x<\xi$:

$$K(x,\xi) = u_{3-} = A_{1-} e^{-i\int_\xi^x [K_1(x')+\delta K_1(x')]dx'} + A_{2-} e^{+\int_\xi^x [K_2(x')+\delta K_2(x')]dx'} + A_{3-} e^{-i\int_\xi^x [K_3(x')+\delta K_3(x')]dx'} . \quad (14)$$

The wave $u_{1\pm}$ is bound with every wave $u_{3\pm}$ and its amplitude is $B_{j\pm}=\beta(\pm K_j)A_{j\pm}$, $j=1,2,3$, where $\beta(\pm K_j)= =(\pm K_j)q/(K_j^2-p^2)$ is defined by substitution (7) into the first equftion of system (1).

Six quantities $A_{j\pm}$ we fined after substituting expressions (13) and (14) in six conditions (10), (11) and (12). Then we have:

$$K(x,\xi) = \frac{i}{4DK_1^3(x)}\left[\frac{K_1(\xi)}{K_1(x)}\right]^{3/2} e^{i\left|\int_\xi^x K_1(\xi)d\xi\right|} - \frac{1}{4DK_1^3(x)}\left[\frac{K_1(\xi)}{K_1(x)}\right]^{3/2} e^{-\left|\int_\xi^x K_1(\xi)d\xi\right|} +$$

$$+\frac{i q^2(x)}{2Bp^2\sqrt{p^2-q^2(x)}}\left[\frac{K_3(\xi)}{K_3(x)}\right]^{1/2} e^{i\left|\int_\xi^x K_3(\xi)d\xi\right|}, \qquad (15)$$

where $D=Eh^3/[12(1-\nu^2)]=B(h^2/12)$.

Note, the multipliers $[K_1(\xi)/K_1(x)]^{3/2}$ and $[K_3(\xi)/K_3(x)]^{1/2}$ appear from (9), and when $x=\xi$ they turn to one. In the case when only parameter q depends on x in (15) only multiplier $[K_3(\xi)/K_3(x)]^{1/2}$ differs from one when $x\neq\xi$.

With the help of analitical expression for Green function (15)we obtain the relation on the contour S :

$$\dot{u}_3 = V_0(x) + V_s(x) = -(i\omega)\oint K(x,\xi)\Big[P_0(\xi)+P_s(\xi)\Big]d\xi \ . \qquad (16)$$

If now we introduce the density $\sigma(x)$ according to the integral equation:

$$\frac{1}{i\rho\omega}P_s(x) = \oint_S \sigma(\xi)G(x,\xi)d\xi \ , \qquad (17)$$

then using (5) the system of integral equations (16), (17) is sufficient to solve the scattering problem.

The system of equations (16), (17) may be performed to one integral equation on the contour S :

$$-(i\omega)\oint_S K(x,\xi)\Big[P_0+i\rho\omega\oint_S \sigma(\xi')G(\xi,\xi')d\xi'\Big]d\xi \ -$$

$$-\oint_S \sigma(\xi)\frac{\partial G(x,\xi)}{\partial n}d\xi = \frac{1}{i\rho\omega}\Big(\frac{\partial P_0}{\partial n}\Big) \ . \qquad (18)$$

Note, that in the case of absolutely soft boundary $K(x,\xi)\to\infty$ we have the problem (3), and in the case of absolutely rigid boundary $K(x,\xi)\to 0$ we have the problem (6).

When the contour S is a circle of radius R_0 it is convenient to use an expantion for P_0 , v_0 , P_s , v_s : $P_0=\sum_{m=-\infty}^{\infty} P_0^m exp(im\varphi)$ and so on.

Using the relations:

$$v = \frac{\partial\Phi}{\partial n}\ , \ P = i\rho\omega\Phi \ , \ Z_s = -i\rho\omega\frac{\partial\Phi}{(\partial\Phi/\partial n)}$$

(here: Φ and $(\partial\Phi/\partial n)$ - field potential and its normal derivative; Z_s - radiation impedance) we have:

$$V_o^m = -P_o^m/Z_o^m \ , \ V_s^m = -P_s^m/Z_s^m \ , \tag{19}$$

and from (16):

$$V_o^m + V_s^m = \frac{1}{Z_y^m}\left[P_o^m + P_s^m\right] , \tag{20}$$

where $\frac{1}{Z_y^m}=-(i\omega)\oint_S K(x,\xi)exp[i\frac{n}{R}(x-\xi)]d\xi$, and $d\xi=Rd\varphi$.

Note, that expressions Z_o^m and Z_y^m have a sense of impedance of incident wave and mechanical impedance of elastic vibration of shell correspondingly [1].

For the m-component of Fourier transform of P_s we have from (19) and (20):

$$P_s^m = -P_o^m \frac{Z_s^m}{Z_o^m}\left[1 + \frac{Z_o^m-Z_s^m}{Z_y^m-Z_s^m}\right] . \tag{21}$$

The last expression is identical to the formulae (13) from [1]. Note, that for circular contour it is not necessary to find the density of simply sources σ .

3. REFERENCES

[1] V V Muzychenko & S A Rybak, 'Low-frequency resonance scattering of sound by finite cylindrical shells (review)', Sov. Phys. Acoust. 4, p325 (1988).

[2] V Z Vlasov, Selected works (in Russian), Vol.1 (Izd. AN SSSR, Moskva, 1962).

[3] S A Rybak, 'Vibrations of double-curvature shells', Sov. Phys. Acoust. 2, p149 (1973).

[4] S M Rao, P K Raju, 'Application of the method of moments to acoustic scattering from multiple bodies of arbitrary shape', J. Acoust. Soc. Amer. 3, p1143 (1989).

[5] S P Sun, P K Raju, S M Rao, 'Application of the Method of Moments to Acoustic Scattering from Multiple Infinitely Long Fluid Filled Cylinders and Shells', Proc. Int. Congr. on Recent Developments in Air - and Structure - Borne Sound and Vibrations, Auburn Univ., AL, USA, V.2, p577 (1990).