INVESTIGATION OF A PARAMETRIC ARRAY FIELD IN SHALLOW SEA: THEORY AND EXPERIMENT

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1.Introduction

Parametric acoustic arrays (PA) due to the very principle of their operation have such unique properties as wide frequency band, low level of sidelobes, high directivity diagram at relatively small sizes of the pump (primary wave) source. However, due to their inherent drawback - low efficiency - they were used in rather narrow areas,in particular, for calibration purposes, in small-distance hydrosonars with high resolution, etc.

During the last decade a number of rather powerful PA were constructed with low frequency (below 1 KHz) sound pressure in the diagram maximum equal to the one of a monopole source with acoustic power up to hundreds watts. Such level allows one to receive PA radiation at distances up to hundreds kilometers. Thus, due to their high directivity and wide frequency band, PAs seem to be very useful instrument for studying the propagation of a relatively low-frequency sound over large distances in ocean waveguide channels, high directivity probing, etc.

At the same time the application of PA in ocean waveguides involves new theoretical problems because the process of PA field formation in waveguides differs essentially from the one in free space owing to the medium stratification and reflections of interacting waves from the bottom and the surface. This paper considers briefly the theoretical models of the PA field formation in plane acoustic waveguides, the results of laboratory and sea experiments being also discussed.

2. Theoretical description of a PA field in waveguides.

The process of PA field formation is usually described in the frames of the given primary wave (pump) approximation. For example, for the biharmonic pump, the pressure of secondary field p_s at the difference frequency Ω satisfies the inhomogeneous Helmholtz equation:

$$\Delta p_s + K^2 n^2(z) p_s = -Q(x,y,z), \qquad (1)$$

where $K=\Omega/c_0$, $n(z)=c_0/c(z)$ is the medium refraction index, c_0 is equal to the sound velocity at the waveguide axis. The function

$Q(x,y,z)$ describes the spatial distribution of virtual nonlinear sources, produced by the interacting pump waves $p_{1,2}(x,y,z)$ having frequencies $\omega_{1,2}$, $\Omega=|\omega_1-\omega_2|$:

$$Q(x,y,z)=\beta(z)p_1(x,y,z)p_2^*(x,y,z), \qquad (2)$$

where $\beta(z)=\varepsilon\Omega^2/\rho c^4(z)$, ε is the nonlinearity parameter equal to 3.5 for water, ρ is the medium density. The pressure in each of the pump waves $p_{1,2}(x,y,z)$ satisfies the homogeneous Helmholtz equation with the corresponding boundary condition at $x=0$, $*$ denotes the complex conjugation. Using the Green function $G(\vec{r}',\vec{r})$, $\vec{r}=(x,y,z)$ for Eq.(1) we present the secondary wave field of the difference frequency Ω in the form:

$$p_s(\vec{r})=\frac{1}{4\pi}\int d\vec{r}'\,Q(\vec{r}')G(\vec{r}',\vec{r}) \qquad (3)$$

Integration in (3) is made over the whole volume occupied with virtual sources. If the geometry of the pump beam is arbitrary, computation of such a three-fold integral even for the case of the free space is a complicated problem. At the same time it is well known that at certain conditions this integral can be considerably simplified and reduced actually to a one-fold one. This may occur in two extreme cases which are used to be called the Westerwelt and the Berktay models [5].

In the first case (the Westerwelt model) the simplification is associated with the fact that for highly-directed pump waves with sufficient damping, the secondary sources may be considered as being in phase within every cross-section of the initial beam in the whole region of interaction. If the pump beam projector zone has the length L_F, then the validity criterion of the Westerwelt approximation can be formulated in the following way: $L_a \le L_F(\omega/\Omega)$ where L_a is the characteristic length of the travelling wave antenna. Then the integral (3) is reduced to a one-fold one along the axial ray of the pump beam and can be easily calculated [5].

In the opposite case of high divergency pump (the Berktay model) the influence of nonlinear sources front curvature is essential. Thus the effective contribution to radiation is given by the secondary sources, whose transverse cross-section is limited by the first Fresnel zone (determined for the low frequency wave) formed around the ray trajectory connecting the emitter centre with the point of observation. Herewith the expression (3) is also reduced to the one-fold integral along this trajectory. The validity criterion of this approximation is opposite to that of the preceding case: $L_a \gg L_F(\omega/\Omega)$.

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In the intermediate case $L_a \sim L_F(\omega/\Omega)$ to estimate the field characteristics one can use the interpolation of the results for the extreme regimes mentioned above [5].

One may distinguish the analogous regimes also for the case of PA in inhomogeneous medium, in particular, in the waveguide. However, here appears also a complicating factor associated with the emergence of beam bending and the phase shift between the interacting waves due to their reflection from the waveguide boundaries [4]. For the case of highly-diverging pump waves (Berktay regime) an algorithm of calculation of PA field in a refractive waveguide is described in [4], the corresponding solution (for the pump beam which does not touch either bottom or surface) takes the form:

$$p_s(l,\theta,\beta) \approx \left[\frac{\Sigma(0)c(l)}{\Sigma(l)c(0)}\right]^{1/2} \frac{\Omega P_1(\theta,\beta)P_2(\theta,\beta)}{4\rho_0} \int_{R_0}^{l} \frac{\varepsilon}{c^3(\xi)} \left[\frac{\Sigma(0)c(\xi)}{\Sigma(\xi)c(0)}\right]^{1/2} \times \exp(-2\alpha\xi)d\xi, \qquad (4)$$

where l is the distance to the observation point, measured along the ray specified in the vicinity of the pump source by the angles θ and β in the vertical and horizontal planes correspondingly; $\Sigma(l)$ is the dependence of the ray tube cross-section on the ray coordinate; the distance $R_0 = L_a\omega/\Omega$.

In the opposite case of the Westerwelt model let us assume the pump beam being of the following form:

$$p_\nu = P_\nu(\vec{r})\exp\{-i \int \vec{k}_\nu d\vec{\xi} - \alpha\xi\}, \quad \nu=1,2, \qquad (5)$$

where ξ is the ray coordinate measured along the axial ray of the beam, $P_\nu(\vec{r})$ characterizes the distribution of the wave complex amplitude across the beam, $\int \vec{k}_\nu d\vec{\xi}$ is the phase accumulated along the axial ray ($k_\nu = \omega_\nu / c$, $|k_1 - k_2| = K$) , α is the damping coefficient of the primary wave; in view of the condition $|\omega_1 - \omega_2| \ll \omega_{1,2}$ we approximately assume that $\alpha_{1,2} \approx \alpha$. Using the expression for the Green function in the modal representation, the solution for PA field in the far zone may be presented in the the following form (for analytical simplicity an isovelocity waveguide is supposed in this example):

$$p_s \approx \frac{i\varepsilon\Omega^2}{4\ c^3}\left(\frac{2}{\pi}\right)^{1/2} \exp\{i\pi/4\} \frac{W}{H} \sum_m \psi_m(z) \frac{\exp\{-i\kappa_m x\}}{(\kappa_m x)^{1/2}} M_m(\beta,\theta), \qquad (6)$$

where $W = \int (P_1 P_2^* / \rho c) dS_\perp$ has the sense of the pump beam power when

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$P_1 = P_2$, β is the angle measured from the emitter axis in the horizontal plane, $\kappa_m = K\cos\theta_m$ is the horizontal wave number of the m-th mode with vertical structure $\psi_m(z)$. The factor $M_m(\beta,\theta)$ describes the angular distribution (in the horizontal plane) of the parametric radiation at m-th mode depending on the angle of inclination θ of PA axis in the vertical plane. For example, for PA located near the surface and directed to the bottom

$$M_m = F_1(\sigma_m)F_2(\sigma_m) - F_1(-\sigma_m)\, F_2(-\sigma_m), \qquad (7)$$

where $\sigma_m = K\sin\theta_m$ is the vertical wave number of the m-th mode.

When the PA axis is oriented, for example, to the surface:

$$F_1(\sigma_m) = \frac{\exp\{-i\Delta(\sigma_m)x_0 - 2\alpha x_0/\cos(\theta)\} - 1}{i\Delta(\sigma_m) - 2\alpha\ /\cos(\theta)},$$

$$F_2(\sigma_m) = \frac{1 - V\exp\{-i\Delta(\sigma_m)x_0 - 2\alpha\ _0/\cos(\theta\ - i\sigma_m 2H\}}{1 - V\exp\{-i\Delta(\sigma_m)2x_0 - 4\alpha x_0/\cos(\theta) - i\sigma_m 2H\}} \qquad (8)$$

$\Delta_m(\sigma_m) = K\cos(\theta) - \kappa_m\cos(\beta) + (K\sin\ (\theta) - \sigma_m)\mathrm{tg}(\theta)$,

$x_1 = z_0/\mathrm{tg}(\theta)$, $x_0 = H/\mathrm{tg}(\theta)$, V is the reflection coefficient of pump beam from the bottom.

Although the expressions (6)...(8) are somewhat cumbersome, their physical meaning is rather clear. The factor F_1 in (7),(8) characterizes the radiation field of each separate region with length x_0, lying between the points of primary beam reflection, and the factor F_2 is determined by the interference of the fields of all such pieces. Symmetric components with $\pm\sigma_m$ in (7) are associated with the fact that the pump interacts with each of two Brillouin waves [6] corresponding to the given mode. The most considerable distinctions in comparison with the case of the homogeneous space take place at large tilt angles when multiple reflections of the interacting waves by the waveguide boundaries are essential, and the diagram may become a multilobe one unlike the case of uniform space.

3. Laboratory experiments in a waveguide.

First experiments on the PA field formation in the acoustic waveguide were made in laboratory conditions [7]. We also performed model experiments [8], in which the pulse regime was used to provide the mode selection using the difference of the pulses travel time at different modes. The measurements were made in a waveguide with the depth of the water layer varying within the range 35...200 mm. The central frequency of the biharmonic pump beam was equal to 3.1 MHz, and the difference frequency was about 200 kHz. The emitter parameters conformed to the conditions of the applicability of Westerwelt model.

Figure 1, for example, gives theoretical and experimental dependencies of the first mode field level upon the vertical orientation of PA axis. They are in a good agreement. It should be noted that when the mode excitation is optimal, the orientation of the emitter axis in the vertical plane differs from both the direction of the waveguide axis and the direction of the wave vector of the mode Brillouin wave.

As it was shown above, the angular structure of the horizontal plane has multilobe character and should depend on the vertical angle of PA inclination. To illustrate this, the amplitude distribution in the horizontal plane of the first and the second mode pulses was measured. The similar measurements were performed in the regime of long pulses when both modes took part in the signal formation simultaneously (Fig. 2). As it follows from the theory a multilobe diagram forms, and the calculated curves here agree well with the experimental ones.

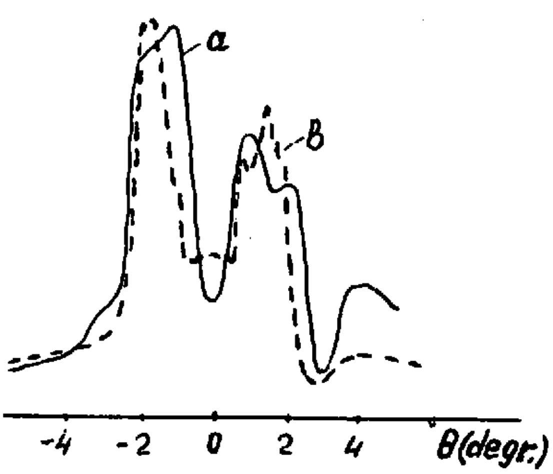

Fig.1.Dependece of 1st mode amplitude upon PA axis tilt angle in the vertical plane
a - experiment, b - theory

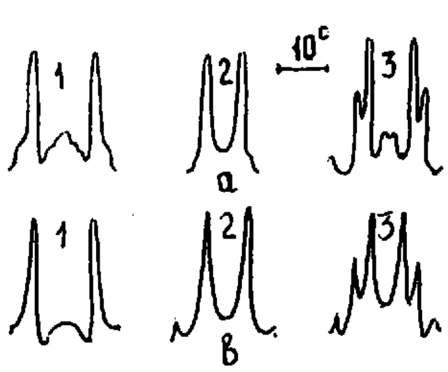

Fig.2.Example of horizontal mode structure
1 - 1st mode, 2 - 2nd mode, 3 -both modes superposition
a - experiment, b - theory

Another experiment [9], which can be described in the frames of the approximation of high directivity PA, was made in the lake with depth 30 m. The biharmonic pump had the mid-band frequency 100 kHz, at 0.5...10 kHz secondary signal. The main field characteristics, such as the mean signal level, the number of extremes and their magnitude in the vertical field section, the characteristic diagram width in the horizontal plane - agree well with the theory.

4. Experiments with a powerful PA in shallow sea.

In this section the results of the measurements of a powerful PA field in the shallow sea are described and interpreted on the basis of a simplified approach. The parameters of the PA in question correspond to the conditions of the applicability of Berktay approximation, and the LF field forms basically before the first reflection of the pump beam from the waveguide boundary, therefore the boundaries influence on the PA field formation is not as significant as in the case considered above.

The ship hydroacoustic emitter with effective aperture 2 x 3 M^2 (the lesser side being oriented vertically) was used in the experiments. The emitter was placed under the ship keel at depth about 5 m. The pump signals frequencies were in the range 2.9 - 3.9 kHz. The total acoustic power of the pump waves radiation was approximately 24 kW. Both cw monochromatic (in the band of 200..1000 Hz) and linearly frequency-modulated (LFM) secondary signals were used. The maximal distance between the emitter and the receiver was 22 km in this experiment.

The measurements of the parametric signal level performed at the distances from 500 m to 2 km from the emitter (where the waveguide properties of the channel do not influence yet), as well as the ones in the region, where the sound propagation is already essentially of the waveguide character, correspond well with theoretical estimates of the field level and horizontal diagram bandwidth. To interpret the measurements data we employed the approach mentioned in the first section based on the assumption that the formation of PA field is of local ("quasiray") character which in the homogeneous space corresponds to Berktay model [1,5].

Experiments in shallow sea were made along the path with sea depth 320 + 350 M. LFM signal with the frequency deviation in the range 200 ÷ 1000 Hz was used. The analyzer frequency bandwidth was usually 50 or 100 Hz. The convolution of the signal coming to the hydrophone with the reference one was performed yielding a number of short pulses passed along various rays or mode groups. For example, figs. 3 and 4 show the convolution of the received signal with the mid-band frequency 350 Hz and the deviation ± 50 Hz, correspondingly for PA and the conventional monopole emitter at the observation distance of

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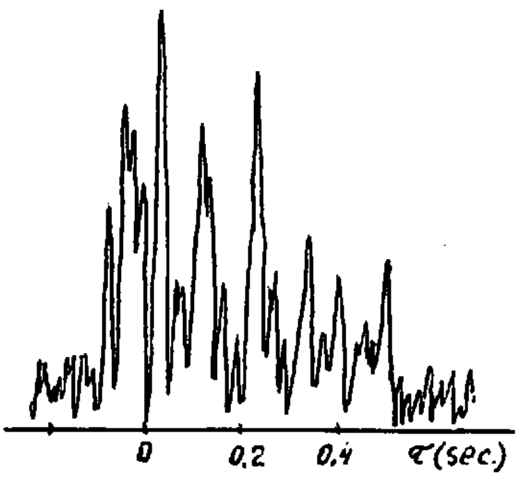

Fig.3

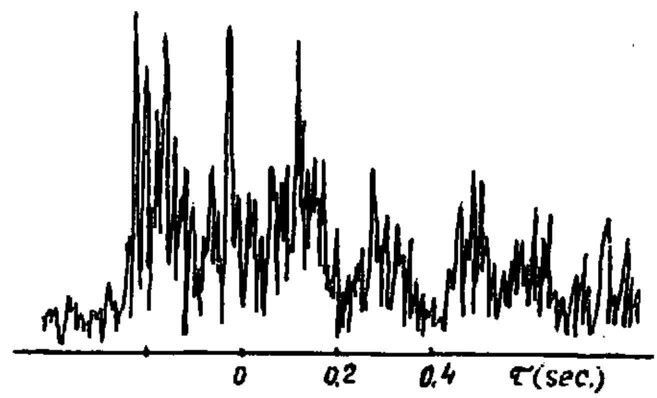

Fig.4

22.2 km. Comparing the figures one may see that the number of PA radiation pulses and the total length of sending pulses is much less than those for the monopole emitter. This is due to the fact that PA excites much less waveguide modes than the monopole emitter.

According to the above theoretical consideration the range of the excited modes may be defined from the directivity diagram in the free space, thus to estimate the number of the excited modes we adopt that their Brilluoin waves are excited only within the directivity diagram width in the vertical plane, and the excited mode with the lowest number should have the turn horizon of Brillouin waves coinciding with the horizon of the emitter depth z_0, for this wave $\theta_n(z_0) = 0$. The calculations (based on the mode program [10]) yield that for the frequency 350 Hz and $z_0 = 5$ M, the lowest excited mode has the number 30, the highest one is about 52-53, and the mode number corresponding to diagram maximum equals to $35 \div 36$ (for the inclination angle of acoustic axis 6° to the bottom). The calculated length of pulse sending herewith appears to be $0.573 \div 0.608$ s, and 0.1 s maximum delay from pulse beginning, coinciding well with the experimental values (see fig. 3). Theoretical and experimental values of the received pulse train duration coincide well for other distances too.

PA proved to be employed also for studying the sound propagation in the ocean over long distances. In particular, the stable reception occurred at distances of several hundreds up to 1000 km [11] in deep sea for signal frequencies 230..700 Hz.

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5.Conclusion

Now we see, what peculiarities do appear in PA operation due to the waveguide sound propagation. The theoretical analysis revealed some interesting physical features of the PA - waveguide systems. It has been shown that the extended "virtual" antenna makes it possible to form the field consisting of few given modes. Herewith the field structure in the shallow sea turns to be especially complex if the pump beam reflects from the bottom and the surface. In such a case the directivity diagram differs essentially from the PA diagram in free space; it is multilobe and has various forms for different modes.

Laboratory modeling and the sea experiments (with scales up to dozens and hundreds kilometers) have confirmed the theoretical conclusions. The experiments have demonstrated the possibility of the selective excitation of the given mode groups and the formation of the narrow directivity diagram in the horizontal plane which could hardly be achieved by other methods, but are rather important for some applications, for example, for studying reverberation from the selected directions, for defining the damping of separate modes, in the schemes of the ocean mode tomography, etc.

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