WAVENUMBER-FREQUENCY SPECTRUM OF TURBULENT FLUCTUATING PRESSURE ON AIRFOIL SURFACE

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To research the wavenumber-frequency spectrum of the fluctuating pressure is helpful to understand the statistical characteristics of turbulent structure and provides input conditions for accurate prediction of the flow noise. Based on the wavenumber-frequency spectrum of the fluctuating pressure on a flat plate, which is proposed according to Corcos theory and experimental measurements, this paper used a large eddy simulation (LES) method to calculate the flow field of NACA0012 airfoil, and try to give a numerical calculation method of calculating the wavenumber-frequency spectrum of the fluctuating pressure on three dimensional surface by means of Fourier transform. Firstly, the wing surface is discretized, and the wavenumber-frequency spectrum on the plate is applied to the local wing surface to obtain the analysis results of the wavenumber-frequency spectrum of the turbulent fluctuating pressure on wing surface. Then, the results are compared with the numerical results by normalization processing. The results show that the numerical results are in good agreement with the analytical ones and the calculation method is reasonable and reliable. Finally, the wavenumber-frequency spectrum model on the plate is extended to the three-dimensional surface, which provides the reference and establishes the foundation to analyze wavenumber-frequency spectrum of the turbulent fluctuating pressure on more complex structure.

Keywords: Turbulent fluctuation pressure; Wavenumber-frequency spectrum; Three-dimensional surface; Fourier transform; Numerical calculation method.

1. Introduction

Ship Fluctuation analysis method is widely used in structural vibration, which considers the structural vibration as the vibration field and analyses the wavenumber-frequency spectrum so that can obtained information on the propagation of structural vibration wave. On the basis of Lighthill's acoustic analogy equation, the wavenumber-frequency spectrum of turbulent fluctuating pressure is introduced as the input condition to predict the flow noise of object motion. The wavenumber-frequency spectrum is an analysis of the structural vibration/ turbulent fluctuating pressure in the frequency and wavenumber domains, which reflects the energy distribution of vibration/pressure and includes not only the frequency domain information but also the wavenumber domain information, so it is of great importance to the study of the propagation and control of vibration and the generation and control of flow noise.

Turbulent fluctuating pressure is an important characterization of unsteady flow, and it is also an important source of fluid induced structural vibration and noise. Corcos is one of the first to propose a wavenumber-frequency spectrum model on plat. The main purpose is to understand the temporal and spatial characteristics of the turbulence structure and provide the input conditions for predicting the acoustic radiation of flow noise. At present, the study on wavenumber-frequency spectrum of turbulent fluctuating pressure is mainly based on the experimental measurement and Fourier transform. However, with the rapid development of computational fluid dynamics, using CFD simulation and calculation to describe the flow field information of the moving objects have been able to meet most of the actual needs in engineering and no longer need to carry out a time-consuming and laborious experimental study, which provides the possibility for the numerical calculation of turbulent fluctuating pressure and its wavenumber-frequency spectrum.

2. Wavenumber-Frequency Spectrum Model

2.1 On Plate

For the fully developed turbulent layer on an infinite plate, we assume here that the turbulent pressure field is spatially homogeneous and stationary with respect to time, in other words, the space-time correlations of the boundary layer pressure field are dependent only on the separation distance and temporal interval. Under this assumption, Corcos [1] proposed the following wavenumber-frequency spectral density model $S_{qq}\left(k_s, k_t, \omega_0\right)$ of wall pressure fluctuation:

$$S_{qq}(\mathbf{k},\omega_0)=S_0(\omega_0)\left(\frac{U_c}{\omega_0}\right)^2 \hat{A}\left(1-\frac{k_s U_c}{\omega_0}\right)\hat{B}\left(\frac{k_t U_c}{\omega_0}\right) \quad (1)$$

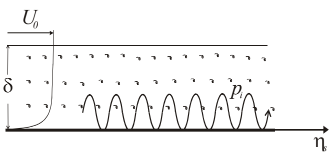


Fig.1 Boundary layer turbulence over a flat plate

Where $S_0(\omega_0)$ is the wall point pressure frequency spectrum, A and B are non-dimensional functions determined from experimental data. $\hat{A}$ and $\hat{B}$ are the Fourier transforms of A and B, k_s is the wavenumber in the η_s -direction (stream-wise) and k_t is the wavenumber in the η_t -direction (span-wise), and U_c is the convection speed.

Brooks and Hodgson（1981）[2] show that the functions A and B given by:

$$A(\beta)=e^{-\varsigma_1|\beta|}, \quad B(\beta)=e^{-\varsigma_2|\beta|} \quad (2)$$

This equation provide a good fit to their experimental data, where ς_1 and ς_2 are adjustable coefficients. From experiment:

$$\zeta_1=0.11, \quad \zeta_2=0.6 \quad (3)$$

Substituting Eqs. (2) and (3) into Eqs. (1), one obtains the Brook version of Corcos's spectrum, of the form:

$$S_{qq}\left(\mathbf{k},\omega_0\right)=S_0\left(\omega_0\right)S_1\left(k_s\right)S_2\left(k_t\right) \tag{4}$$

where

$$S_1\left(k_s\right)=\frac{l_1}{\pi}\left[\frac{1}{1+l_1^2\left(\omega_0/U_c-k_s\right)^2}\right] \tag{5}$$

$$S_2\left(k_t\right)=\frac{l_2}{\pi}\left[\frac{1}{1+l_2^2k_t^2}\right] \tag{6}$$

The integral scales l_1 and l_2 in the stream wise and span wise directions are defined by （Brooks and Hodgson 1981）[2]

$$l_1=\frac{\int_0^\infty \xi_1 A\left(\omega_0\,\xi_1/U_c\right)d\xi_1}{\int_0^\infty A\left(\omega_0\,\xi_1/U_c\right)d\xi_1},\; l_2=\frac{\int_0^\infty \xi_2 B\left(\omega_0\,\xi_2/U_c\right)d\xi_2}{\int_0^\infty B\left(\omega_0\,\xi_2/U_c\right)d\xi_2} \tag{7}$$

If the convection speed U_c is assumed to be constant, substituting Eqs. (2) and (3) into Eqs. (7) gives

$$l_1=U_c/\zeta_1\omega_0\;,\quad l_2=U_c/\zeta_2\omega_0 \tag{8}$$

A point pressure spectrum $S_0(\omega_0)$ based on data collated by Chase (1980) for an infinite flat plate can be found in the book by Howe (1998)[3]. In non-dimensional form, it is given by

$$\overline{\overline{S}}_0\left(\overline{\overline{\omega}}_0\right)=\frac{6.1409\times10^{-6}\,\overline{\overline{\omega}}_0^2}{\left(\overline{\overline{\omega}}_0^2+0.0144\right)^{1.5}} \tag{9}$$

where：

$\overline{\overline{S}}_0\left(\overline{\overline{\omega}}_0\right)=S_0\left(\omega_0\right)\left(U_0/\delta^*\right)/\left(0.5\rho_0U_0^2\right)^2$ ， $\overline{\overline{\omega}}_0$ is the non-dimensional frequency defined by $\overline{\overline{\omega}}_0=\omega_0\delta^*/U_0$, U_0 is the airfoil velocity or mean-flow velocity, ω_0 is the source angular frequency, and δ^* is the displacement thickness of the turbulent boundary layer.

Brooks, Pope and Marcolini (1989)[4] have measured the boundary layer thickness for a NACA0012 airfoil section with chords ranging between 3.54cm to 30.48cm, with a range of Mach numbers between 0.115 to 0.213, and an angle of attack between 0° to 20° . Based on this data, the following empirical expressions for the natural transition boundary layers were obtained for the boundary layer displacement thickness δ^* versus distance η_e at zero angle of attack

$$\delta^* / \eta_e = 10^{[3.0187 - 1.5397 \log R_e + 0.1059 (\log R_e)^2]} \quad (10)$$

The effect of attack angle α on the boundary layer turbulence for the pressure and suction sides, compared with that at $\alpha = 0$, was found to vary as followings:

on the pressure side:

$$\delta_p^* / \delta_0^* = 10^{[-0.0432\alpha + 0.00113\alpha^2]} \quad (11)$$

on the suction side:

$$\frac{\delta_s^*}{\delta_0^*} = \begin{cases} 10^{0.0679\alpha} & 0^\circ \le \alpha \le 7.5^\circ \\ 0.0162(10^{0.3066\alpha}) & 7.5^\circ \le \alpha \le 12.5^\circ \\ 52.42(10^{0.0258\alpha}) & 12.5^\circ \le \alpha \le 25^\circ \end{cases} \quad (12)$$

Where the zero subscripts indicate zero angle of attack and the angle of attack α is measured in degrees. The subscript " p " expresses the boundary thickness for the pressure side while " s " is for the suction side.

2.2 On NACA0012 Airfoil

Figure 2 shows schematically boundary layer turbulence over one side of an airfoil. Due to the curvature of the airfoil surface and the non-zero angle of attack, the features of the turbulence over the airfoil differ from those over a flat plate in three important respects: (1) boundary layer thickness varies along the streamwise direction; (2) local incoming velocity U_0 is non-uniform due to potential flow effects; (3) there is a pressure gradient in the streamwise direction within the boundary layer. We assume here that the boundary layer thickness, the incoming velocity and the pressure gradient change very little over a small facet of the airfoil surface so that the Corcos pressure spectrum remains locally valid. We further assume that an airfoil with the same local inflow velocity $U_0(\mathbf{y})$ and boundary layer thickness $\delta(\mathbf{y})$ (as shown in Fig. 2) develops the same pressure spectrum as a flat plate under the same conditions. Corcos' model of pressure spectrum will therefore be extended to a realistic airfoil by applying it locally to a small region on the airfoil surface, which is small compared with an acoustic and hydrodynamic wavelength.

By curve fitting the experimental data of both Yu & Joshi (1979) and Brooks & Hodgson (1980) measured on an airfoil, Chou & George (1984)[5] present an empirical expression for $\bar{\bar{S}}_0\left(\bar{\bar{\omega}}_0\right)$ covering two frequency ranges. For $\bar{\bar{\omega}}_0$ <0.06,

$$\bar{\bar{S}}_0\left(\bar{\bar{\omega}}_0\right) = \frac{1.732 \times 10^{-3} \bar{\bar{\omega}}_0}{\left(1 - 5.489\bar{\bar{\omega}}_0 + 36.74\bar{\bar{\omega}}_0^2 + 0.1505\bar{\bar{\omega}}_0^5\right)} \quad (13)$$

and for $0.06<\bar{\bar{\omega}}_0<20$,

$$\bar{\bar{S}}_0\left(\bar{\bar{\omega}}_0\right)=1.4216\times10^{-3}\bar{\bar{\omega}}_0\Big/(0.3261+4.1837\bar{\bar{\omega}}_0+22.818\bar{\bar{\omega}}_0^2+0.0013\bar{\bar{\omega}}_0^3+0.0028\bar{\bar{\omega}}_0^5) \qquad (14)$$

3. Comparison Of Numerical Solution And Analytical Solution

Here we take NACA0012 airfoil as an example, which has a chord length of 0.3048 m and a span length of 0.4752 m, and moves in the $-y_1$ direction in the air with Mach number $M=0.208$, attack angle $\alpha=4°$ and the convective velocity coefficient $c_u=0.8$. The computational domain is divided by O structured grid, and the total number of units is 1805814. As shown in the fig.3.

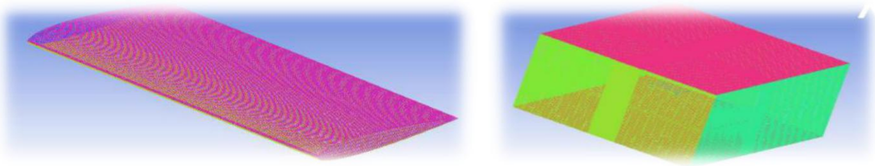


Fig.3 Partition of computing grid

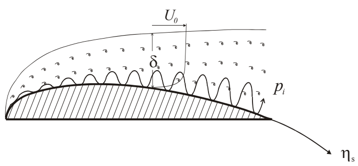


Fig.2 Boundary layer turbulence over an airfoil

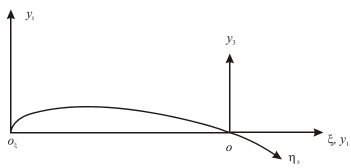


Fig.4 Airfoil coordinates

Fig.4 shows the curvilinear coordinate system, (ξ, y_t) used to express the airfoil profile, where ξ is the abscissa normalized on chord and y_t is the ordinate of the airfoil thickness distribution, also normalized on the chord. The streamwise coordinate η_s may be obtained from [6]

$$\eta_s=c\int_1^{1-|y_1|/c}\sqrt{1+\left(dy_t/d\xi\right)^2}\,d\xi \qquad (15)$$

where $c=2b$ is airfoil chord length. For a NACA0012 airfoil, the thickness distribution is given by

$$y_t=0.6(0.2969\sqrt{\xi}-0.126\xi-0.3516\xi^2+0.2843\xi^3-0.1015\xi^4) \qquad (16)$$

where $0\le\xi\le1$. The derivative of y_t required by Eq. (15) is

$$dy_t/d\xi=0.6(0.14845/\sqrt{\xi}-0.126-0.7032\xi+0.8529\xi^2-0.406\xi^3) \qquad (17)$$

Large eddy simulation (LES) was used to calculate the flow field of the above model, with the analysis frequency is 3000Hz, the step is taken as 1.667e-4s and the total 3000 step is calculated, eventually obtained the time domain data $p(\eta,\zeta,t)$ of the fluctuating pressure of the suction surface and pressure surface which relates to time and location, where η and ζ are respectively the streamwise and spanwise coordinates. Note that the numerical calculation can only get discrete data, so we can take $p(\eta,\zeta,t)$ as a three-dimensional matrix $(M \times N \times K)$. Then, according to the discrete Fourier transform, the time domain data are transformed into frequency domain and wavenumber domain. And finally the wavenumber-frequency spectrum is obtained. This process can be achieved through the Matlab programming.

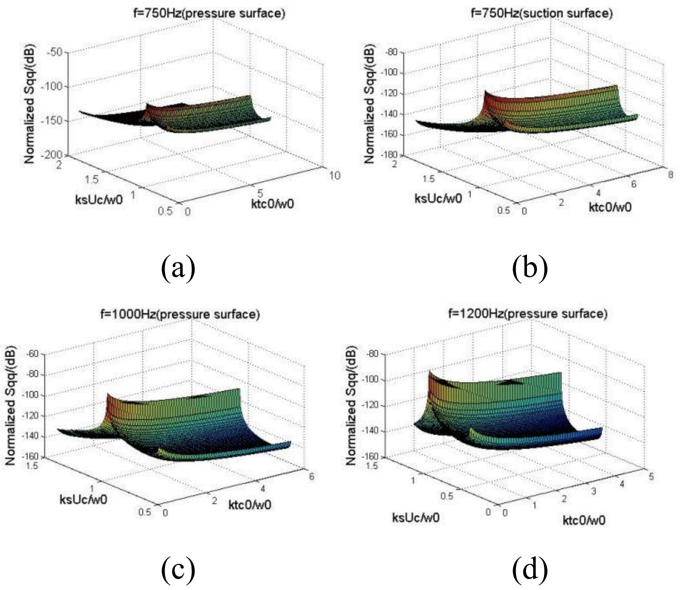
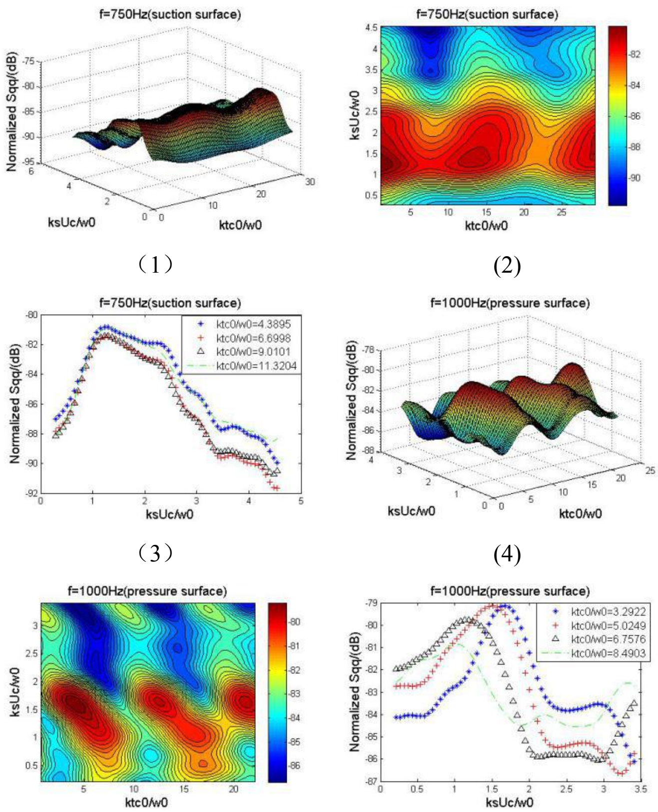


Fig.5 Analytical solution of wavenumber spectrum for pressure and suction surfaces at different frequencies



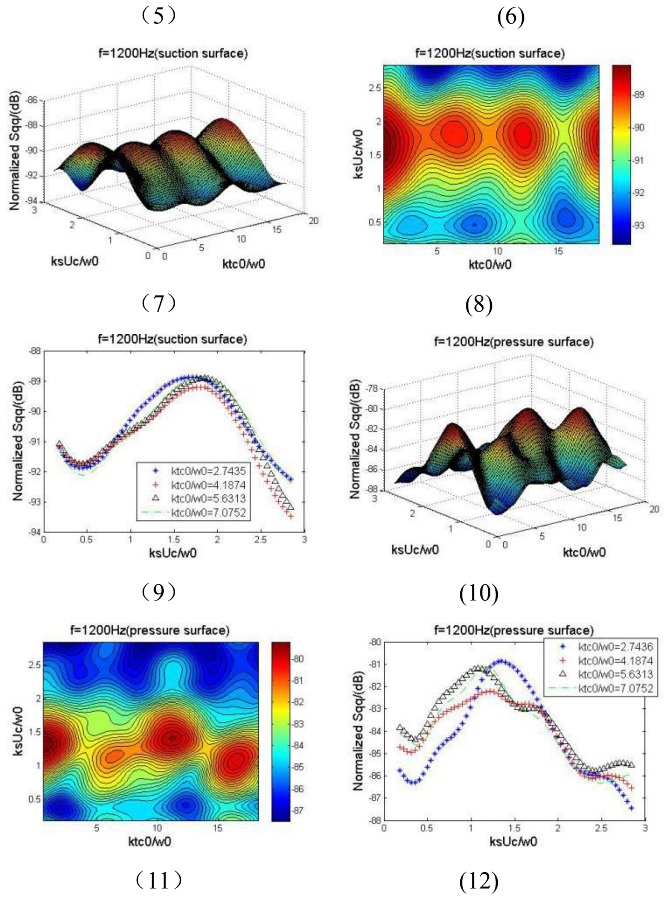


Fig.6 Numerical solution of wavenumber spectrum for pressure and suction surfaces at different frequencies

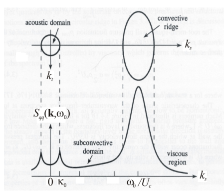


Fig.7 Characteristic of the wall pressure spectrum, see Howe (1998)

Figure 5 are the analytical solution of wavenumber-frequency spectrum on surface of NACA0012 airfoil which is obtained from the Corcos's plate wavenumber-frequency spectrum model. Where（a）、（c）and（d）are respectively the streamwise-spanwise wavenumber spectrum in frequency of 750Hz、1000Hz and 1200Hz on the pressure surface of NA0012 airfoil, while（b）is the streamwise-spanwise wavenumber spectrum in frequency of 750Hz on the suction surface of NA0012 airfoil. Figure 6 are the numerical solution of wavenumber-frequency spectrum on surface of NACA0012 airfoil by CFD calculation, where（1）、（2）and（3）are respectively the wavenumber spectrum of suction surface corresponding to the frequency of 750Hz,（4）、（5）and（6）are respectively the wavenumber spectrum of pressure surface corresponding to the frequency of 1000Hz,（7）、（8）、（9）and（10）、（11）、（12）are respectively the wavenumber spectrum of suction surface and pressure surface corresponding to the frequency of 1200Hz. All the variables in figure 5 and figure 6 are normalized, where the dimensionless streamwise wavenumber is $k_s U_c / \omega_0$, the dimensionless spanwise wavenumber is $k_t c_0 / \omega_0$, and the non-dimensional form of wavenumber-frequency spectrum amplitude is $S_{qq}(k,\omega) / (\rho_0^2 U_0^3 \delta^{*3})$. It can be found that the peak value $S_{qq}(k,\omega) / (\rho_0^2 U_0^3 \delta^{*3})$ of wavenumber-frequency spectrum appears in the vicinity of dimensionless streamwise wavenumber $k_s U_c / \omega_0$ equal to 1 through comparative analysis, which is consistent with the general characteristics of Howe（1998）[3] that the wavenumber-frequency spectrum $S_{qq}(k,\omega)$ of wall fluctuating pressure at a fixed frequency varies with streamwise wavenumber. As shown in fig.7, there are two main peaks. The largest peak occurs in the ‘convective domain’, where the turbulent eddies convect at speeds slower than the speed of sound c_0 . Most of the energy convects at, or close to, the characteristic eddy convection velocity U_c . Turbulent energy in this region is said to be in the convective ridge. The second peak is in the vicinity of the acoustic wavenumber κ_0 where $\kappa_0 = \omega_0 / c_0$ .The range $k < |\kappa_0|$ corresponds to the ‘acoustic domain’. However, it can be seen from the figure（3）、（6）、（9）、（12）that the maximum peak value appears in the vicinity of streamwise wavenumber $k_s U_c / \omega_0$ slightly larger than 1, which is due to the existence of the wall shear stress so that the spectrum occurs migration. In contrast figure（7） and figure（10）, it is found that the wavenumber spectrum of pressure surface and suction surface is asymmetric due to the presence of attack angle, and the energy of the pressure surface is higher than that of the suction surface.

4. Concluding Remarks

For predicting the flow noise of moving object accurately, it is feasible to analyze the wavenumber-frequency spectrum of wall turbulent fluctuating pressure and then make it as an input condition according to the Lighthill equation and the development of acoustic analogy theory. Therefore, this paper analyzes the turbulent fluctuation pressure spectrum model and obtains analytical solution by applying it to real NACA0012 airfoil. At the same time, CFD is used to calculate the flow field and then obtains the numerical solution of the wavenumber-frequency spectrum of turbulent fluctuating pressure on NACA0012 airfoil surface. The plate turbulent pressure spectrum model has limitations, which is proposed for the infinite plate, without considering the effects of scale effect and turbulent shear stress. By comparative analysis it shows that the CFD calculation results can satisfy the general law of the theory, and can reflect the specific information of real flow, so we consider it is accurate and reliable. It provides a method for numerical calculation of the wavenumber-frequency spectrum of wall turbulent fluctuating pressure on arbitrary more complex body surface, and also provides a possibility to accurately predict turbulent noise of arbitrary shape in frequency domain.

5. REFERENCES

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