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PREDICTION OF THE ABSORPTION COEFFICIENT OF STRATIFIED POROUS MATERIALS WITH PERFORATED FACINGS.

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1. Introduction

One of the techniques to obtain a large absorption coefficient at low frequencies is to cover a porous material with a perforated facing. This type of material has been used for many years and models predicting the surface impedance and the absorption coefficient at normal incidence have been carried out [1]. This paper describes a model for the calculation of the surface impedance of layered porous materials with perforated facings at oblique incidence. Comparisons are made with experimental results obtained in an anechoic room with a two microphone technique.

2. Theoretical model - facing with cillindrical apartures [2,3].

Consider N_0 layers of porous material, stuck on a hard surface and covered with a perforated plate with circular apartures with radius r_0 and at a distance d of each other (see figure 1). A plane sound wave is incident in the xz plane with an angle of incidence θ . If the spatial period $\lambda/\sin\theta$ of the sound wave in the ox direction is equal to an integer multiple of d, the sound field is periodic in this direction with a period : $Nd=\lambda/\sin\theta$. The period in the direction perpendicular to ox is d. As a consequence, a two dimensional development in Fourier series of the acoustic field in the stratified material an be worked out. The (m,n) mode of the sound pressure in the upper layer can be written as :

$$P_{m,n}(x,y,z)=\cos(\frac{2\pi my}{d})e^{j2\pi\frac{n}{Nd}x}(A_{m,n}e^{jk_{m,n}(N_0)z}+B_{m,n}e^{-jk_{m,n}(N_0)z}) \quad (1)$$

where :

$$k_{m,n}(N_0)=(k^2(N_0)-(\frac{2\pi m}{d})^2-(\frac{2\pi n}{Nd})^2)^{1/2} \quad (2)$$

and :

$$B_{m,n}(N_0)=\beta_{m,n}A_{m,n}(N_0) \tag{3}$$

$$\beta_{m,n}=\frac{Z_{m,n}(B)k_{m,n}(N_0)-Z_c(N_0)k(N_0)}{Z_{m,n}(B)k_{m,n}(N_0)+Z_c(N_0)k(N_0)} \tag{4}$$

$Z_c(N_0)$ and $k(N_0)$ are the characteristic impedance resp. the propagation constant in layer N_0. $Z_{m,n}(B)$ is the impedance at B related to the (m,n) mode which can be calculated in the following way. The impedance at M_1 related to the (m,n) mode is given by :

$$Z_{m,n}(M_1)=-jZ_c(1)\frac{k(1)}{k_{m,n}(1)}cotg(k_{m,n}(1)l_1) \tag{5}$$

where $Z_c(1)$ is the characteristic impedance and l_1 the thickness of layer 1.
If h(1) and h(2) are the porosities of the first and the second layer, the impedance in M_1 is related to the impedance in M_1' by :

$$Z_{m,n}(M'_1)=\frac{h(2)}{h(1)}Z_{m,n}(M_1) \tag{6}$$

The impedance at M_2 related to the (m,n) mode can be written as :

$$Z_{m,n}(M_2)=\frac{Z_c(2)\frac{k(2)}{k_{m,n}(2)}[-jZ_{m,n}(M'_1)cotg(k_{m,n}l_2)+Z_c(2)\frac{k(2)}{k_{m,n}(2)}]}{Z_{m,n}(M'_1)-jZ_c\frac{k(2)}{K_{m,n}(2)}cotg(k_{m,n}(2)l_2)} \tag{7}$$

The impedances at the boundaries of the layers related to each mode can be obtained by this way up to B in the N_0th layer in contact with the facing.
The pressure field can be written at the contact surface with the facing :

$$p(x,y,0)=\sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty}\cos\frac{2\pi mx}{d}e^{j\frac{2\pi ny}{Nd}}A_{m,n}(N_0)(1+\beta_{m,n}) \tag{8}$$

the z component of the velocty field is given by :

$$v_z(x,y,0)=\sum_{m=0}^{\infty}\sum_{n=-\infty}^{\infty}\cos(\frac{2\pi mx}{d})e^{j\frac{2\pi ny}{Nd}}A_{m,n}(N_0)(1-\beta_{m,n})\frac{k_{m,n}(N_0)}{Z_c(N_0)k(N_0)} \tag{9}$$

The velocity amplitude is assumed to be uniform in each aparture. Let U_0 be the velocity on the hole at x=0 and let U_l be the velocity at x=2dl than :

$$U_l=U_0e^{-j\frac{2\pi l}{N}} \tag{10}$$

Integrating over each hole area yealds an equation for $A_{m,n}(N_0)$. The impedance at B is than given by :

$$Z(B)=\frac{2}{\pi h(N_0)}\sum_{m=0}^{\infty}\sum_{n=-1,-1\pm N}\nu_{m,n}Z_{m,n}(B)\frac{J_1^2(2\pi r_0(\frac{m^2}{d^2}+\frac{N^2}{N^2d^2})^{1/2})}{(m^2+\frac{n^2}{N^2})} \tag{11}$$

The impedance in the free air close to the facing becomes :

$$Z=\frac{Z(B)}{s}+j(e+d_0)\rho_0\omega \tag{12}$$

where s is the perforated area and e is the external added mass that can be calculated with equation (11) for a semi infinite air space when the (1,0) mode is excluded.

3. Theoretical model - facings with sits.

Let us now consider the case where the facing has parallel slits of width a, separated by a distance d. A plane wave is incident on the material with angle Θ with the z axes and angle ϕ with the x axes. The sound field is periodic in the presence of the facing if :

$$\frac{\lambda}{\sin\theta\,\cos\phi}=Nd \tag{13}$$

The sound field can be developped in a Fourier series like in the case of circular apartures giving :

$$p(x,y,0)=\sum_{n=-\infty}^{\infty} e^{-j2\pi\frac{n}{Nd}y}A_n(N_0)(1+\beta_n)e^{-jk(1)x} \tag{14}$$

$$v_z(x,y,0)=\sum_{n=-\infty}^{\infty} e^{-j2\pi\frac{n}{Nd}y}A_n(N_0)(1-\beta_n)e^{-jk(1)x}\frac{k_n(N_0)}{Z_c(N_0)k(N_0)} \tag{15}$$

where :

$$k_n(i)=(k^2(i)-k^2\sin^2\theta\sin^2\phi-\frac{4\pi^2n^2}{N^2d^2})^{1/2} \tag{16}$$

making the same hypotheses for the amplitude of the z component of the velocity as in the preveous paragraph and integrating over the spacial period leads to an expression for the coefficient $A_n(N_0)$. This results in an equation for the impedance at B :

$$Z(B)=\frac{1}{ad}\sum_{n=-1,-1\pm N}\frac{\sin^2(\frac{\pi na}{Nd})}{(\frac{\pi n}{Nd})}Z_n(B) \qquad (17)$$

the impedance Z in free air close to the facing is :

$$Z=\frac{Z(B)}{s}+j\omega\rho_0(e+d_0) \qquad (18)$$

where e can be calculated in the same way as in the previous paragraph.

4. Experimental results and calculations.

Systematic calculations of the influence of the perforated plate on the absorption coefficient can be performed. The characteristic impedance and the propagation constant can be calculated with the model published in reference 3 if the flow resistivity and the porosity of the material are known.

Measurements have been performed in an anechoic room using a two microphone technique described in reference 3. The figure 2 shows the measurement and the prediction of the absorption coefficient of a porous material with a perforated facing with circular apartures. The parameters are indicated in the figure captions. The figure 3 shows the absorption coefficient of the same porous material, covered with a perforated plate with slits. The parameters are indicated in the figure caption. A good areement between the theoretical model and the experiments is observed and the model can be used for the development of new sound absorbing materials.

5. References

1. U.Ingard,R.H.Bolt "Absorption characteristics of acoustic material with perforated facings" J.Acoust.Soc.Am 23, 533-540 (1951)

2. P.Guignouard,M.Meisser,J.F.Allard,P.Rebillard,C.Depollier "Prediction and measurement of the acoustical impedance and absortion coefficient at oblique incidence of porous layers with perforated facings." Noise Con.Eng.J. Vol 36 nr 3 p 129-135

3. J.F.Allard "Sound propagation in porous media - modelling of sound absorbing materials." to be published. Elsevier

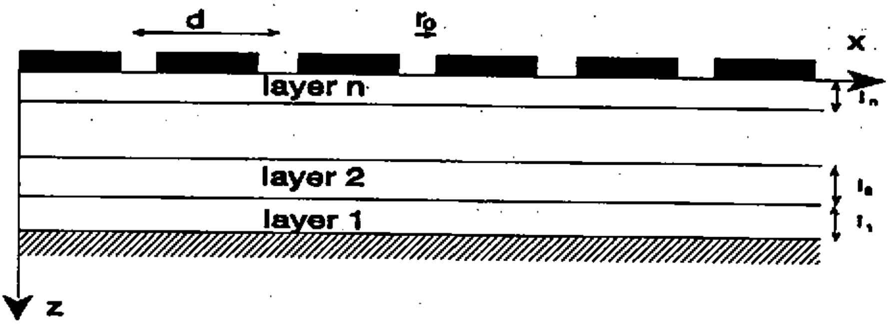

Figure 1. N layers of porous material,covered with a perforated plate.

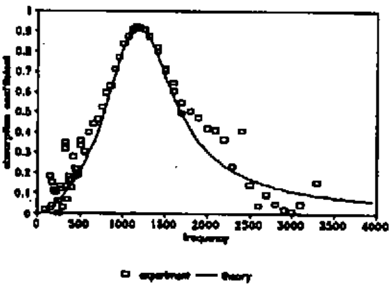

Figure 2. Absorption coefficient of a 2 cm thick foam layer, covered with a plate with circular apartures.s=0.06,normal incidence.

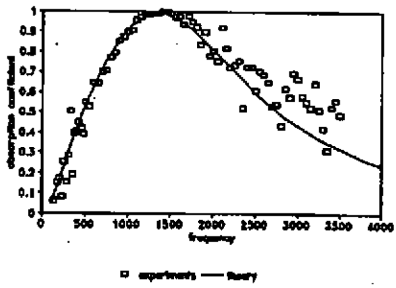

Figure 3. Absorption coefficient of the same foam material as in figure 2, covered with a plate with slits. s = 0.10, Theta=60, Phi=0.