A PI SHAPE PIEZOELECTRIC ACTUATOR FOR VIBRATION CONTROL APPLICATION

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A Π (Pi) shape piezoelectric actuator (PIPA) is proposed in order to make piezoelectric stack can be more conveniently mounted on beam or plate structure for vibration control applications. PIPA has flat bottom provides convenient mounting surface. It consists of two main components, a piezoelectric stack and a Pi shape pedestal. They are assembled together by a bolt which applies pre-compression load to the piezoelectric stack and makes the stack can be removed and replaced conveniently in practice. Equation about the relation between actuating moment of PIPA and driving voltage was derived. It was applied to the vibration control of a cantilevered beam. Thermal elasticity analogy method was used in the modeling of the cantilevered beam with PIPA mounted on its root surface. The state space model of this piezoelectric control system was established for vibration control system simulation. Linear quadric regulator (LQR) and neural network model predictive (NNP) control strategies were adopted respectively. Simulation results demonstrate that using PIPA with those control strategies the first bending mode amplitude of the beam can be suppressed by about 79% and 95%.
Keywords: Pi shape pedestal, piezoelectric actuator, vibration control simulation

1. Introduction

The output actuation force of piezoelectric stack is much bigger than that of piezoelectric patch actuator. But compared with piezoelectric patch actuators, piezoelectric stack actuators are not very convenient to be incorporated with beam or plate type of structures. In practical applications various external components have to be developed to facilitate the combination of piezoelectric stack actuator and the host structure to be controlled [1~4].

In the present study, a Π (pi) shape piezoelectric stack actuator (PIPA) is proposed to overcome the mounting inconvenience of piezoelectric stack to beam or plate structure. The actuating moment equation of PIPA is derived. Then it is applied in the vibration control of a cantilevered beam. Thermal-elasticity analogy method is used in the FEM modelling of the cantilevered beam bonded with PIPA and state space model is established for the vibration control system. Two different control strategies, linear quadric regulator (LQR) control and neural network predictive (NNP) control are adopted in the active vibration control systems, respectively.

2. PIPA's working principle and actuation moment analysis

PIPA is assembled by three components, a piezoelectric stack, a Π shape metal pedestal and a bolt. The flat bottom surface of the pedestal can sever as the bonding surface for its mounting.

Photograph of an in-house made PIPA is shown in Fig.1, the material of pedestal is duralumin. Left sub-figure shows the three components of PIPA and the right sub-figure shows an assembled PIPA, a 5-cent euro coin is presented in this figure for comparison of their size.

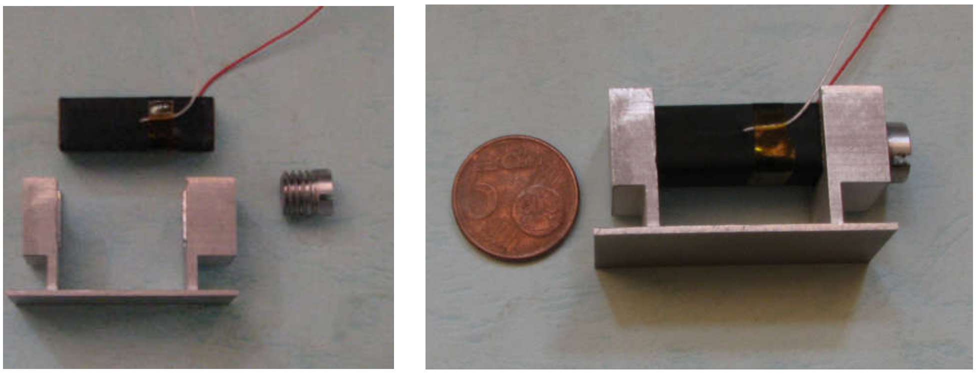

Figure 1: In-house made PIPA.

When driving voltage is applied to a piezoelectric stack, expansion along the axial direction of the stack will be induced due to the inverse piezoelectric effect. If the stack is constrained at its two ends, actuating power in the manner of reaction moments will be generated. In this study, an inverted Π shape pedestal is designed as an installation platform and an external lever for the piezoelectric stack.

The two ends of the stack are constrained by the two arms of this inverted Π shape pedestal with a bolt. Thus the longitudinal deformation of the piezoelectric stack will drive the bottom plate of PIPA to bend when driving voltage V is applied to the stack. If the bottom plate of PIPA is bonded on the surface of beam or plate structure, actuating moment will subject to them.

Assume p is the pre-compression load subjected to the stack. The induced displacement along the thickness direction of each piezoelectric wafer of the stack can be obtained by the following equation[5]

$$\delta_3 = S_{33}^E t\, p / A_s + d_{33} V \tag{1}$$

where S_{33}^E is the flexible coefficient of piezoelectric material under a constant driving voltage, d_{33} is the piezoelectric strain constant, t is the thickness of a single wafer. A_s is the area of the wafer which is also the cross section area of the stack.

Then the deformation of the entire stack can be described as

$$\delta_s = n\delta_3 = n(S_{33}^E t\, p / A_s + d_{33} V) \tag{2}$$

where n is the total number of piezoelectric wafers of the stack. And the axial strain on the cross section of the stack can be obtained by

$$\begin{aligned} \varepsilon_s &= \frac{\delta_s}{l_s} = \frac{n}{n \cdot t}(S_{33}^E tp / A_s + d_{33} V) \\ &= S_{33}^E p / A_s + \frac{d_{33}}{t} V \end{aligned} \tag{3}$$

where $l_s = n \cdot t$ is the total thickness of all the piezoelectric wafers of the stack and the stack is considered as a bar element, which means only the axial force is considered.

According to the Hooke's law, the axial stress σ_s and the axial strain has the relationship which can be described as

$$\sigma_s = E_{ps} \varepsilon_s \tag{4}$$

where E_{ps} is the elastic modulus of piezoelectric material. Define F_a as the internal axial force of the stack we have

$$F_a = \sigma_s A_s \tag{5}$$

From Eqs. (3)、 (4) and (5) we can get

$$F_a = E_{ps}\varepsilon_s A_s = E_{ps}(S_{33}^E p + \frac{d_{33}}{t} A_s V) \tag{6}$$

As aforementioned the piezoelectric stack is constrained by the Π shape pedestal with a bolt, thus an output axial force will be generated because the deformation is constrained, eventually the actuating moment will be given by

$$M_a(t) = F_a h = E_{ps}(S_{33}^E p + \frac{d_{33}}{t} A_s V(t))h \tag{7}$$

where h is the height of PIPA, it is defined as the distance from the axis of the stack to the bottom surface of PIPA.

Eq. (7) indicates the proportional relationship between the output moment and the height of the stack, for specific pre-compression force and driving voltage. According to this equation the output moment of PIPA can be designed to meet different requirements in practical applications.

3. Vibration control simulation of cantilever beam with PIPA

In order to demonstrate the feasibility and ability of PIPA for structural vibration control application, it is used in the active vibration control of a cantilever beam.

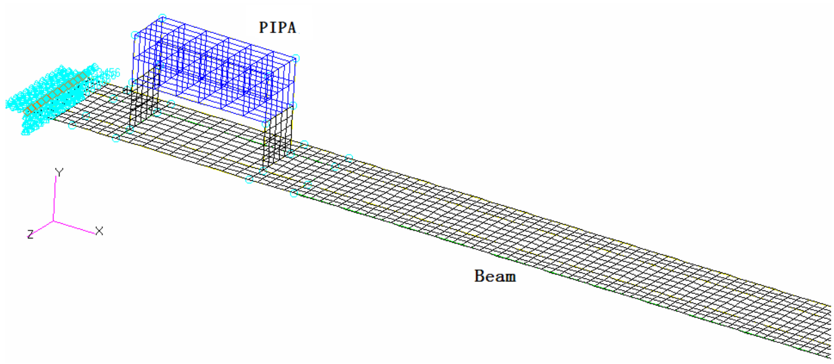

Figure 2: Finite element model of cantilever beam bonded with PIPA.

The dimension and the material properties of the beam and the piezoelectric stack used in PIPA are listed in Table 1. PIPA is bonded on the root surface of the beam as shown in Fig. 2. The height from the beam surface to the axis of the stack is 10mm.

Table 1 Dimensions of the beam and the piezoelectric stack.

Dimension	Beam	Stack
Length(mm)	300	30
Width(mm)	20	10
Thickness(mm)	1.5	10
Young's Modulus (GPa)	70	43
Poisson Ratio	0.33	0.35
Density(Kg/m^3)	2550	7600
d_{33}(m/V)	0	750x10^{-12}

3.1 Thermal elasticity analogy modelling

The analogy relationship between inverse piezoelectric effect and thermal elastic effect was first presented by Freed in 1997[6]. Using this analogy theory the inverse piezoelectric effect can be simulated by thermal elastic effect.

The governing equation of an un-damped vibration system can be written as

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = \mathbf{f} \tag{8}$$

where $\mathbf{x} \in \mathbf{R}^n$ represents the displacement vector of structure, $\mathbf{f}$ is the actuating force vector of actuators, $\mathbf{M}$ and $\mathbf{K}$ are the mass and stiffness matrices of the structure respectively, in which the mass and stiffness contributions of piezoelectric material have already been considered. Using the theory of thermal-elastic analysis, and defining an artificial thermal expansion coefficient $\alpha = d_{33}/t$, a deformation vector $\mathbf{S}_t$ can be obtained when a unit change of temperature load is applied to the structure, where d_{33} and t are the strain constant and thickness of the piezoelectric material. Here t is equal to the thickness of a single wafer in the stack. This vector can be equivalent to the deformation vector induced by a unit driving voltage applied to a piezoelectric actuator. In general, one deformation vector is required for each piezoelectric actuator. Then the actuating force vector can be written as

$$\mathbf{f} = \mathbf{K}\mathbf{S}_t V \tag{9}$$

where V represents the driving voltage applied on piezoelectric actuator.

At this stage, we would define the first time coordinate transformation equation as

$$\mathbf{x} = \begin{bmatrix} \mathbf{\Phi} & \mathbf{S}_t \end{bmatrix} \begin{Bmatrix} \mathbf{z} \\ \mathbf{z}_r \end{Bmatrix} \tag{10}$$

where $\mathbf{\Phi}$ is the normalized eigenvector matrix of Eq.(8) with respect to generalized mass. Substituting Eq.(9) and Eq.(10) into Eq.(8) and multiply both sides of Eq.(8) by $\begin{bmatrix} \mathbf{\Phi} & \mathbf{S}_t \end{bmatrix}^T$ we can obtain

$$\begin{bmatrix} \mathbf{I} & \mathbf{M}_r \\ \mathbf{M}_r^T & \mathbf{M}_{rr} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{z}} \\ \ddot{\mathbf{z}}_\mathbf{r} \end{Bmatrix} + \begin{bmatrix} \mathbf{\Lambda} & \mathbf{K}_r \\ \mathbf{K}_r^T & \mathbf{K}_{rr} \end{bmatrix} \begin{Bmatrix} \mathbf{z} \\ \mathbf{z}_r \end{Bmatrix} = \begin{Bmatrix} \mathbf{K}_r \\ \mathbf{K}_{rr} \end{Bmatrix} V \tag{11a}$$

or

$$\overline{\mathbf{M}} \begin{Bmatrix} \ddot{\mathbf{z}} \\ \ddot{\mathbf{z}}_r \end{Bmatrix} + \overline{\mathbf{K}} \begin{Bmatrix} \mathbf{z} \\ \mathbf{z}_\mathbf{r} \end{Bmatrix} = \begin{Bmatrix} \mathbf{K}_r \\ \mathbf{K}_{rr} \end{Bmatrix} V \tag{11b}$$

where $\mathbf{M}_r = \mathbf{\Phi}^T \mathbf{M}\mathbf{S}_t$, $\mathbf{M}_{rr} = \mathbf{S}_t^T \mathbf{M}\mathbf{S}_t$, $\mathbf{K}_r = \mathbf{\Phi}^T \mathbf{K}\mathbf{S}_t$, $\mathbf{K}_{rr} = \mathbf{S}_t^T \mathbf{K}\mathbf{S}_t$, $\mathbf{\Lambda}$ is the eigenvalue matrix of $\mathbf{M}$ and $\mathbf{K}$, $\mathbf{I}$ is an identity matrix, $\overline{\mathbf{M}} = \begin{bmatrix} \mathbf{I} & \mathbf{M}_r \\ \mathbf{M}_r^T & \mathbf{M}_{rr} \end{bmatrix}$, $\overline{\mathbf{K}} = \begin{bmatrix} \mathbf{\Lambda} & \mathbf{K}_r \\ \mathbf{K}_r^T & \mathbf{K}_{rr} \end{bmatrix}$. Assume that only the first m order of normal modes are extracted, then $\mathbf{\Phi} \in \mathbf{R}^{n \times m}$, $\mathbf{\Lambda} \in \mathbf{R}^{m \times m}$. Note that the deformation vector obtained from thermal-elastic analysis is a static result which makes Eq.(11) a stiff equation. In order to eliminate the ill condition of this stiff equation, second time coordinate transformation has to be made by the following equation

$$\begin{Bmatrix} \mathbf{z} \\ \mathbf{z}_r \end{Bmatrix} = \mathbf{W} \begin{Bmatrix} \mathbf{\eta} \\ \mathbf{\eta}_\mathbf{r} \end{Bmatrix} \tag{12}$$

where $\mathbf{W}$ is the right singularity matrix of $\overline{\mathbf{M}}$, Implementing Eq.(12) we have

$$\begin{bmatrix} \mathbf{\sigma} & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix} \begin{Bmatrix} \ddot{\mathbf{\eta}} \\ \ddot{\mathbf{\eta}}_\mathbf{r} \end{Bmatrix} + \begin{bmatrix} \mathbf{k} & \mathbf{k}_r \\ \mathbf{k}_r^T & \mathbf{k}_{rr} \end{bmatrix} \begin{Bmatrix} \mathbf{\eta} \\ \mathbf{\eta}_\mathbf{r} \end{Bmatrix} = \begin{Bmatrix} \mathbf{b} \\ \mathbf{b}_\mathbf{r} \end{Bmatrix} V \tag{13}$$

where σ is the non-zero singularity value matrix of $\overline{\mathbf{M}}$, U is the left singularity matrix of $\overline{\mathbf{M}}$, and

$$\begin{bmatrix} \mathbf{k} & \mathbf{k}_r \\ \mathbf{k}_r^T & \mathbf{k}_{rr} \end{bmatrix} = \mathbf{U}^T \overline{\mathbf{K}} \mathbf{W} \tag{14}$$

$$\begin{Bmatrix} \mathbf{b} \\ \mathbf{b_r} \end{Bmatrix} = \mathbf{U}^T \begin{Bmatrix} \mathbf{K}_r \\ \mathbf{K}_{rr} \end{Bmatrix} \tag{15}$$

$\boldsymbol{\eta}_r$ can be solved from Eq. (13)

$$\boldsymbol{\eta}_\mathbf{r} = -\mathbf{k}_{rr}^{-1}\mathbf{k}_r^T\boldsymbol{\eta} + \mathbf{k}_{rr}^T\mathbf{b}_\mathbf{r}V \tag{16}$$

Substitute Eq. (16) into Eq. (13) and finally we have

$$\boldsymbol{\sigma}\ddot{\boldsymbol{\eta}} + (\mathbf{k} - \mathbf{k}_r\mathbf{k}_{rr}^{-1}\mathbf{k}_r^T)\boldsymbol{\eta} = (\mathbf{b} - \mathbf{k}_r\mathbf{k}_{rr}^{-1}\mathbf{k}_r^T)V \tag{17}$$

Eq.(17) is the reduced order model for the active vibration control system with piezoelectric actuator. State space model of the active control system can be constructed by Eq.(17) according to Eqs. (18) and (19).

$$\mathbf{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\boldsymbol{\sigma}^{-1}(\mathbf{k} - \mathbf{k_r}\mathbf{k_{rr}^{-1}}\mathbf{k}_r^T) & -\boldsymbol{\sigma}^{-1}\overline{\mathbf{C}} \end{bmatrix} \tag{18}$$

$$\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \boldsymbol{\sigma}^{-1}(\mathbf{b} - \mathbf{k}_r\mathbf{k}_{rr}^{-1}\mathbf{k}_r^T) \end{bmatrix}, \quad \mathbf{C} = \begin{bmatrix} \boldsymbol{\varphi}^T \\ \mathbf{T}_r^T \end{bmatrix}\mathbf{W} \tag{19}$$

where $\overline{\mathbf{C}}$ is the assuming Rayleigh damping matrix of the structure.

With the built state space model the controller can be designed once the control strategy has been chosen, and then numerical simulation of piezoelectric vibration control can be implemented.

3.2 Active vibration control strategies

Linear quadric regulator (LQR) control is a classical control strategy, in the design of LQR controller, the control force at the *ith* step depends on the vibration response of the structure. To determine this control force that gives an optimal linear control. Define the cost function J as

$$J = \frac{1}{2}\sum_{0}^{N}(x_i^T\mathbf{Q}x_i + u_i^T\mathbf{R}u_i) \tag{20}$$

where $\mathbf{Q}$ and $\mathbf{R}$ are weighting matrices. Minimizing the cost function J in Eq. (20) by introducing Hamiltonian function, it gives

$$u_i = -(\mathbf{GPG} + \mathbf{R})^{-1}\mathbf{G}^T F_s x_i \tag{21}$$

$$\mathbf{G} = \mathbf{A}^{-1}(e^{\mathbf{A}\Delta t} - \mathbf{I})\mathbf{B} \tag{22}$$

$$F_s = e^{A\Delta t} \tag{23}$$

where $\mathbf{P}$ is the steady state Riccati matrix, which is given by

$$\mathbf{P} = \mathbf{Q} + F_s^T\mathbf{P}(\mathbf{I} + \mathbf{GRG}^T\mathbf{P})^{-1}F_s \tag{24}$$

With the help of state space model of the structure to be controlled, LQR controller can be designed. The control effectiveness of LQR controller depends a lot on the modelling accuracy of structure to be controlled.

Neural network model predictive (NNP) control is a kind of intelligent control strategy[7,8]. NNP controller can output a control signal, which will force the response of a controlled system to keep the same as a given reference signal. It uses a neural network model to predict future responses of a system to potential control signals. In the controller，an optimization algorithm give control signals to optimize future performance of the controlled system. The neural network model of a control system is trained off-line in batch form using a chosen training algorithm. A block diagram in Fig.3 illustrates the structure of neural network model predictive control process.

The implementation of NNP controller starts from a training process of a neural network model for the controlled system. In the design of NNP controller for vibration control, a constant zero value is adopted as a reference signal. The output of the control system, which is also set to be the displacement response of the beam, will be controlled to follow this reference signal by NNP controller. Using this control strategy the displacement response of the beam will be suppressed.

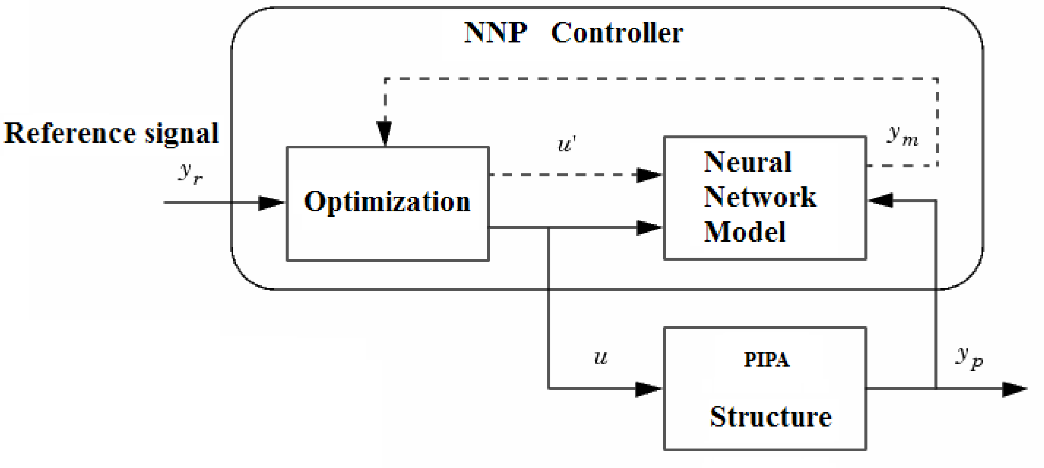

Figure3: Block diagram for NNP control process.

4. Simulation of active vibration control using PIPA

In the simulation of active vibration control, state space model of a cantilevered beam bonded with PIPA has been established beforehand. The cantilevered beam system is excited by sinusoidal signal, frequency value is equal to the first bending mode frequency of the beam. The controller is set to be turned on automatically at time step $t = 4s$. LQR and NNP control strategies described above are adopted in vibration control system simulation, respectively.

Fig. 4 shows the tip point displacement responses of the cantilevered beam bonded with PIPA when LQR control strategy is adopted. The first bending mode response of the beam is suppressed by more than 79%.

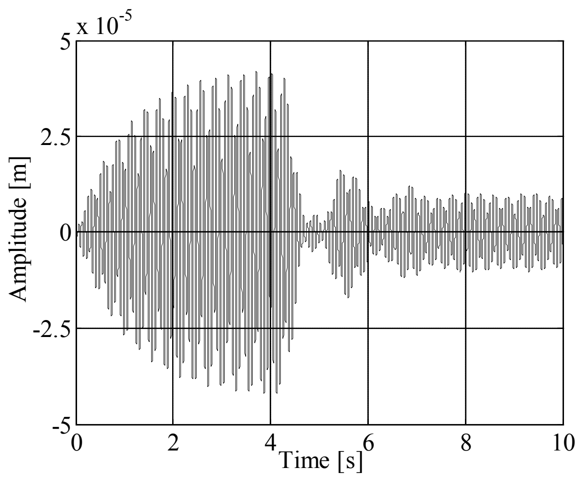

Figure 4: LQR vibration control with PIPA.

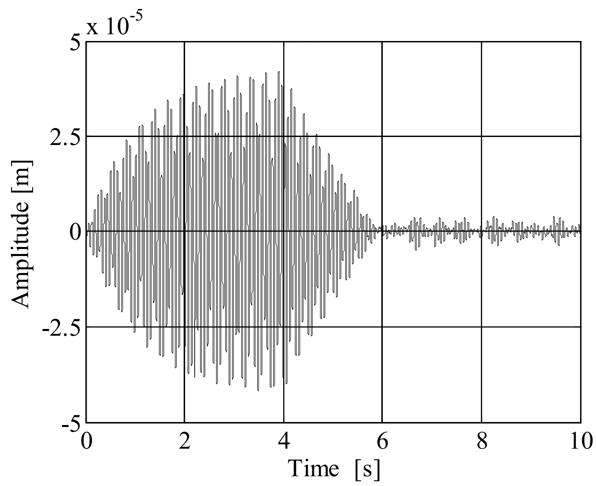

Figure 5: NNP vibration control with PIPA.

Fig. 5 shows the tip point displacement responses of the beam where NNP control strategy was adopted. It can be found that after NNP controller was turned on at time $t = 4s$, the displacement amplitude of the first bending mode response is suppressed by more than 95%. This control results is quite satisfying but the design process of NNP controller would take more time than that of LQR controller.

Simulation results demonstrate that PIPA can sever as an effective actuator in active structural vibration control system with different control strategies.

5. Conclusion

In the present study an in-house made Π shape piezoelectric stack actuator (PIPA) was proposed and demonstrated in the active vibration control of a cantilevered beam. With the design of Π shape pedestal piezoelectric stack is converted to a bending type actuator. PIPA has flat bottom which provides a convenient installation surface. Two main components of PIPA, piezoelectric stack and pedestal, are assembled together by a bolt which provide pre-compression constraint for piezoelectric stack and also makes the piezoelectric stack can be removed and replaced conveniently in practice.

The actuation moment output equation of this new actuator was derived, which indicates that the output moment of the actuator is proportional with the height of pedestal. Active vibration control system simulation results of a cantilevered beam show that with the adoption of PIPA the first bending mode response amplitude of the beam can be reduced by79% and 95% when LQR and NNP control strategies were applied respectively.

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