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IMPROVED GENERALIZED CROSS-CORRELATION ALGORITHM FOR TIME DELAY ESTIMATION BASED ON SECONDARY CORRELATION AND DATA SEGMENTATION

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In order to improve the performance of the generalized cross correlation (GCC) algorithm for time delay estimation (TDE) in a low SNR environment and when the signals are non-stationary, an improved GCC algorithm based on secondary correlation and data segmentation (GCCSS) is proposed. First, the received signals are divided into several segments, and two selection criteria based on correlation coefficient and stationarity of signals are provided to select credible data for following calculation. Then the second correlation was used in the algorithm to suppress interference noise. The result of the second correlation is multiplied by a weighting function to get the generalized cross correlation function. Finally, an exponent transform and multiply processing based on segment were proposed to de-noise. This algorithm is verified by some practical data. The experiment results demonstrate the superior TDE performance of GCCSS under a low SNR circumstance and its feasibility and practicality.

Keywords: time delay estimation, secondary correlation, segmentation

1. Introduction

Time delay is the time difference of arrival signals acquired by several sensors, and has important applications in many areas such as acoustic source localization with microphones, speech signal processing and mechanical fault diagnosis.

Typical TDE methods include the GCC method [1] and the least mean square (LMS) adaptive method [2]. The GCC method has less computation and is easy to apply in the real-time systems and thus it becomes a commonly used method in TDE. The GCC method calculates the weighed cross-correlation function between two received signals and later searches for the maximum peak. The cross-correlation function is based on the second order statistics theory, thus the performance of GCC declines when the signals are non-stationary. In addition, environmental noise has significant influence on the accuracy of GCC method, especially in a low SNR environment.

This paper proposes an improved GCC algorithm based on secondary correlation and data segmentation (GCCSS). This new algorithm divides received signals into several segments, and then selects credible data segments (credible data segments are the signal segments, applying which into TDE method can lead to a correct time delay) with two selection criteria based on correlation coefficient and stationarity of signals for following calculation. Then the second correlation multiplied by a weighting function was used in the algorithm to process the selected signal segments. Finally, an exponent transform and multiply processing based on segment were proposed to de-noise. The GCCSS algorithm improves the performance and accuracy of TDE method, leading to a higher accuracy of localization.

2. GCC algorithm for TDE

Correlation analysis is the basic method to compare the similarity of time domain between two signals. Assuming the discrete event signal model of two received signals is:

$$x_1(n) = s(n) + v_1(n) \tag{1}$$

$$x_2(n) = \alpha s(n-D) + v_2(n) \tag{2}$$

Where $s(n)$ represents the original signal, $v_1(n)$ and $v_2(n)$ represent the environmental noise, D is the time delay between two signal channels. Assuming that original signal and noise are unrelated, the cross-correlation function between $x_1(n)$ and $x_2(n)$ can be represented as:

$$R_{12}(n) = R_{ss}(n-D) \tag{3}$$

Where $R_{ss}(n-D)$ is the auto-correlation function of the original signal. By searching the maximum peak of it, time delay can be obtained. Environmental noise has significant influence on the accuracy of this method, leading to the peak of $R_{ss}(n-D)$ being unobvious, especially in a low SNR environment.

In order to improve the performance of TDE, Knapp and Carter proposed the GCC algorithm [1]. This method pre-filters the signals before calculate the correlation. This step whitens the signals and noise, accentuates the original signal and suppresses the noise power to improve the TDE accuracy. In practice, the weighting cross-correlation function is usually obtained by giving a certain weight on cross-spectrum in the frequency domain, and then inverse transforms to the time domain to get the generalized cross-correlation function between the two signals, expressed as:

$$R_{12}(n) = F^{-1}\left[H(f)G_{12}(f)\right] \tag{4}$$

Where $G_{12}(f)$ is the cross-spectrum between signals, $H(f)$ is the weighting function. The principle of the GCC algorithm is shown in Fig. 1.

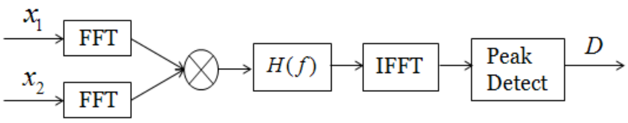

Figure 1: Principle diagram of the GCC based time delay estimation

The key step in GCC algorithm is to choose an appropriate weighting function. The reference [1] described six commonly used function and their characteristics, in which, the Eckart filter and ML estimator can achieve the Cramer-Rao lower bound [3].

3. GCC algorithm based on second correlation (GCCS)

The second correlation algorithm for TDE is an improvement of cross-correlation algorithm [4]. This method firstly calculates the auto-correlation function and cross-correlation function between two received signals $x_1(n)$ and $x_2(n)$:

$$R_{11}(m) = E\left[x_1(n)x_1(n-m)\right] \tag{5}$$

$$R_{12}(m) = E\left[x_1(n)x_2(n-m)\right] \tag{6}$$

Then consider above two correlation functions as new signals, and calculate the cross-correlation between them:

$$R_{RR}(m) = E\left[R_{11}(n)R_{12}(n-m)\right] = R_{RS}(m-D) + R_{RN}(m) \tag{7}$$

Where R_{RS} is the second correlation of original signal $s(n)$, R_{RN} is the second correlation of noise. Same with the cross-correlation algorithm, by detecting the maximum peak of $R_{RR}(m)$, time delay can be obtained.

Both GCC algorithm and second correlation algorithm can improve the performance of TDE when external noise exists, however, they also have some disadvantages respectively. So some papers [5, 6] combine the GCC algorithms and second correlation algorithm, proposing an improved GCC algorithms based on second correlation (GCCS). In GCCS, after obtaining the results of auto-correlation function $R_{11}(m)$ and cross-correlation function $R_{12}(m)$, the cross-spectrum $G_{R_{11}R_{12}}(\omega)$ between them can be calculated. Same as Eq. (4), choosing a certain weighting function on cross-spectrum in the frequency domain to get weighted cross-correlation function and then inverse transforms to the time domain to get the generalized cross-correlation function between the two $R_{11}(m)$ and $R_{12}(m)$.

4. GCC algorithm based on second correlation and data segmentation (GCCSS)

4.1 GCCSS for TDE

In a low SNR environment, the correlation of signals would weaken with the decline of SNR. When putting the whole received signal into the above algorithms, there would be several peaks or no obvious peak in cross-correlation function. So this paper proposes a GCCSS method. This algorithm divides the received signals into several segments, selects credible segments (describe in 4.2), uses GCCS algorithm to get the generalized cross correlation function of credible segments, and then proposes an exponent transform and multiply processing based on segmentation to de-noise.

The cross-correlation function between $x_1(n)$ and $x_2(n)$ is [7]:

$$\begin{aligned} R_{12}(m) &= \sum_{n=0}^{N-1} x_1(n)\, x_2(n+m) \\ &= \sum_{n=0}^{P} x_1(n)\, x_2(n+m) + \sum_{n=P+1}^{N-1} x_1(n)\, x_2(n+m) \\ &\le \sum_{n=0}^{P} x_1(n)\, x_2(n+m) \sum_{n=P+1}^{N-1} x_1(n)\, x_2(n+m) \\ &= R_{12}(m,1:P) R_{12}(m,P+1:N-1) \end{aligned} \tag{8}$$

From Eq. (8) we can get that cross-correlation between two whole signals is less than the product of the cross-correlations between several signal segments. So dividing the received signals into K segments with length of N, then calculating cross-correlation functions of these segments:

$$R_i(x_1,x_2) = E\left\{x_1\left[(i-1)N+1:iN\right]\cdot x_2\left[(i-1)N+1:iN\right]\right\}, i=1,2,...,K \tag{9}$$

Exponent transform can significantly increase the distance between high cross-correlation and low cross-correlation, so we apply exponent transform into $R_i(x_1,x_2)$:

$$f\left(R_i(x_1,x_2)\right) = \exp\left(R_i(x_1,x_2)\right) \tag{10}$$

Then operate with multiplication:

$$R(x_1,x_2) = \prod_{i=1}^{K} \exp\left(R_i(x_1,x_2)\right) \tag{11}$$

The peak of $R(x_1,x_2)$ is the time delay between $x_1(n)$ and $x_2(n)$.

4.2 Selection criteria for credible data

This paper provides two selection criteria based on correlation coefficient and stationarity of signals to select credible data for the following TDE. This step can weaken the impact of noise, leading to a sharper peak of correlation function. Meanwhile, it can be viewed as a preprocessing for non-stationary signals, improving performance of TDE when the signals are non-stationary.

4.2.1 Selection criteria based on correlation coefficient

The key point of GCC is to calculate the cross-correlation function correctly, which is a basic parameter to compare the similarity of time domain between two signals. The correlation of signals would weaken with the decline of SNR, so generally, signal segments with large correlation coefficient have higher possibility to be credible data [7]. Selection criteria based on correlation coefficient is described as Eq. (12):

$$R_i\left(x_1, x_2\right) > 0.5 \tag{12}$$

Where $R_i\left(x_1, x_2\right)$ is the correlation coefficient of segment i. The segments with correlation coefficient lager than 0.5 would be judged to be credible data.

4.2.2 Selection criteria based on stationarity of signals

GCC algorithm is based on the second order statistics theory, thus the performance of GCC declines when the signals are non-stationary. Based on this fact, we test stationarity of signal segments, and which with good stationarity would be judged to be credible data.

The stationarity testing method is based on surrogate data [8, 9]. The method first specifies some stationary process as a null hypothesis, then generates a family of stationary surrogate data which are consistent with this null hypothesis. A process can be viewed as stationary relatively to this observation scale if its time-varying spectrum undergoes no evolution. So computes a discriminating statistic related to spectrum for the original and for each of the surrogate data sets, and compares them to see whether the null hypothesis is rejected.

Given a signal $x(t)$, Fourier transform it to $X\left(f\right) = \int e^{-i2\pi ft} x(t)\,\mathrm{d}t$,then the surrogate data can be written as:

$$s\left(t\right) = \int e^{i2\pi ft} \left|X\left(f\right)\right| \mathrm{e}^{i\varphi_f}\,\mathrm{d}f \tag{13}$$

Where φ_f is a random phase, uniformly distributed over $\left[-\pi, \pi\right]$.

The multitaper spectrogram estimate of $x(t)$ WVS can be expressed as [10]:

$$S_{x,K}\left(t, f\right) = \frac{1}{K}\sum_{k=1}^{K}\left|\int_{-\infty}^{\infty} x\left(s\right) h_k\left(s-t\right) e^{-i2\pi fs} ds\right|^2 \tag{14}$$

Where $h_k(t)$ is the k first Hermite functions.

According to reference [9], given a set of spectral “slices” $S_{x,K}\left(t_n, f\right)$, an average spectrum via the marginalization in time can be computed as:

$$\left\langle S_{x,K}\left(t_n, f\right)\right\rangle_{n=1,2,\ldots,\mathrm{N}} = \frac{1}{N}\sum_{n=1}^{N} S_{x,K}\left(t_n, f\right) \tag{15}$$

Then compare it to each spectral slice according to some dissimilarity measure $\kappa\left(G, H\right)$ [11], leading to the series of distances:

$$c_n^{(x)} = \kappa\left(S_{x,K}\left(t_n, \cdot\right), \left\langle S_{x,K}\left(t_n, \cdot\right)\right\rangle_{n=1,2,\ldots,\mathrm{N}}\right) \tag{16}$$

Choosing the variances of original signal and surrogate data to test stationarity:

$$\Theta_1 = \frac{1}{N}\sum_{n=1}^{N}\left(c_n^{(x)} - \left\langle c_n^{(x)}\right\rangle\right)^2 \tag{17}$$

$$\Theta_0 = \frac{1}{N}\sum_{n=1}^{N}\left(c_n^{(s_j)} - \left\langle c_n^{(s_j)}\right\rangle\right)^2,\ j = 1, 2, \ldots, J \tag{18}$$

From Eq. (18), this distribution allows for the determination of a threshold γ above which the null hypothesis is rejected, in other words, when $\Theta_1 > \gamma$, the original signal is nonstationary, when $\Theta_1 < \gamma$, the original signal is stationary.

Summarizing the analyses above, we can obtain the principle of the GCCSS algorithm, shown in Fig. 2.

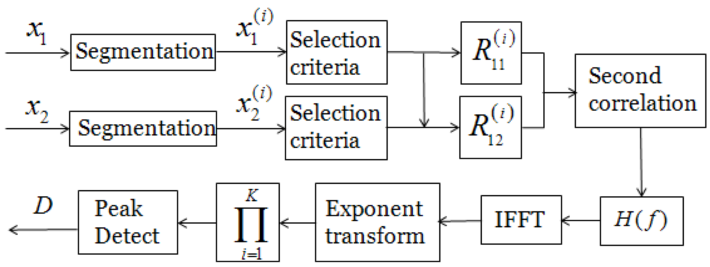

Figure 2: Principle diagram of the GCCSS based time delay estimation

5. Simulation and analysis

The anti-noise performance of the GCCSS algorithm is further tested by conducting a computer simulation using different SNR. Selecting a vibration signal of an engine to be original signal S_1, another original signal S_2 is obtained after a delay of 20 sampling points. The length of the signal was 2000 sampling points. By adding Gaussian white noise to original signals, we can get signal x_1 and x_2 .SCOT was chosen as the weighting coefficient.

(1)The standard deviation of time delay can be defined as $\sigma = \sqrt{\sum_{i=1}^{N}(\hat{\tau}-\tau)^2 \Big/ N}$. Setting the signal x_1 with 5dB SNR, and signal x_2 with SNR varying from -15dB to 15dB.The GCC algorithm, GCCS algorithm, and GCCSS algorithm are implemented in MATLAB, and the time delay estimation performance analysis is operated by computing time delay 100 times and then comparing the standard deviations. The result is shown in Fig. 3.

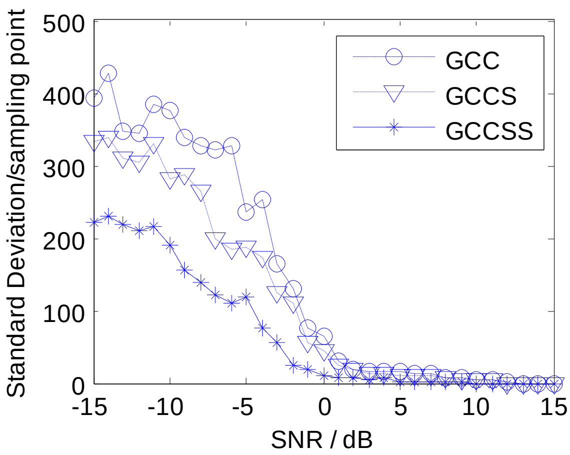

Figure 3: Standard deviations of three algorithms

Fig. 3 shows that the accuracy of three algorithms decreases with the decline of SNR. Especially when the SNR is lower than 0dB, standard deviations of three algorithms significantly increase with the decline of SNR, but the gradient of GCCSS algorithms is lower. Obviously, the anti-noise performance of GCCSS algorithm is better than other two algorithms.

(2)Setting the signal x_1 with 5dB SNR, and signal x_2 with 10dB, 5dB, and 0dB SNR respectively. The time delay estimation performance analysis is operated by comparing the cross-correlation functions of three algorithms. The result is shown in Fig. 4-6.

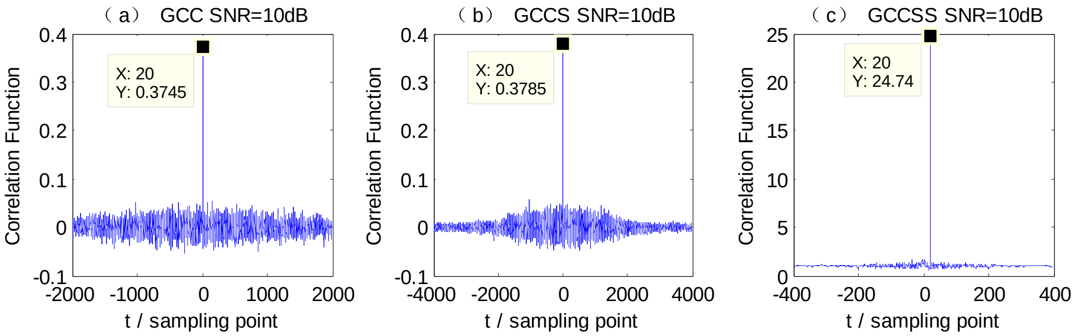

Figure 4: Cross-correlation functions when SNR=10dB

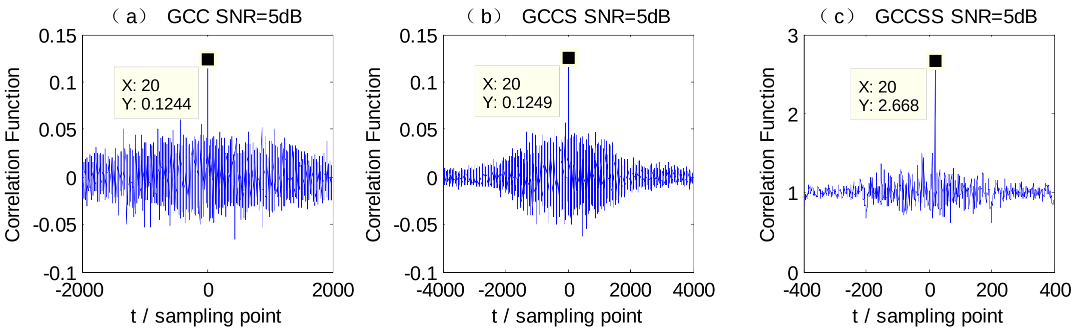

Figure 5: Cross-correlation functions when SNR=5dB

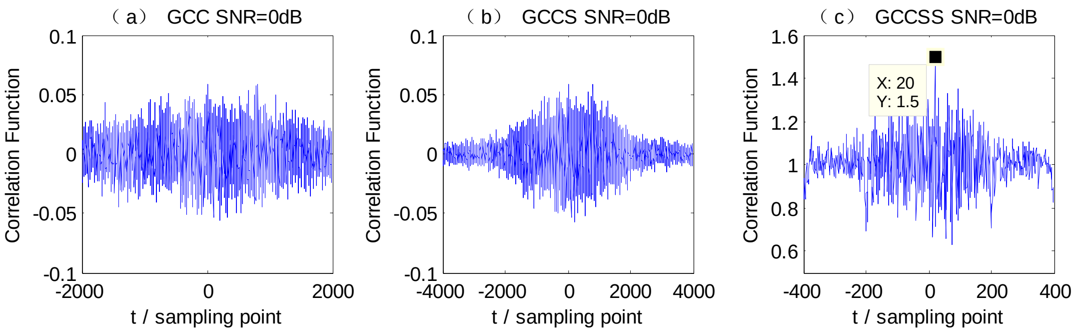

Figure 6: Cross-correlation functions when SNR=0dB

The sharpness of the cross-correlation function peak reflects the precision of the algorithm. As shown in Fig. 4-6, the GCCSS algorithm can reduce the side lobe and sharpen the main peak of the correlation function, improving the TDE algorithm accuracy. As shown in Fig. 4, when SNR=10dB, the peaks of the cross-correlation function are all sharp, and three algorithms TDE suggest 20 sampling points, demonstrating high precisions. As shown in Fig. 5, when SNR=5dB, the cross-correlation function peaks of the GCC method and GCCS method begin to expand with the decline of SNR. However, the GCCSS method peak remains sharp. As shown in Fig. 6, when SNR=0dB, the peaks of the GCC method and GCCS method are submerged in the interference, while the peak of GCCSS method expands but a correct time delay still can be get by peak detection. Obviously, the anti-noise performance of GCCSS method increases with the processing of signal segmentation, second correlation and exponent transform.

6. The application of GCCSS method in acoustic source localization experiment

The application effect of GCCSS method is verified by an acoustic source localization experiment in a factory. In the acoustic source localization method based on the time difference of arrival, the time delay significantly impacts the localization accuracy. The sound is generated by a loudspeaker, and acquired by three microphones. Choosing the Gaussian white noise and successive pulse signal as two kinds of acoustic source, the engines in the factory can be used to generate environment noise. The coordinates of the microphones are (2, 0, 0), (-1, 1.73, 0), and (-1, -1.73, 0), the coordinate of the loudspeaker is (4, 3, 1), as shown in Fig. 7. The results of time delay estimation based on GCCSS algorithm are shown in Table 1.

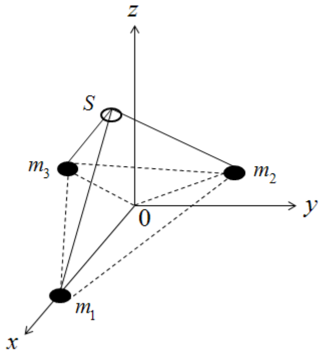

Figure 7: Location model of three microphones and loudspeaker

Table 1: Time delay estimation based on GCCSS algorithm in the localization experiment

Source	SNR	microphone 1& 2		microphone 1& 3	
		time delay/s	error	time delay/s	error
Gaussian white noise	high	0.0043	4.4%	0.0094	1.1%
Gaussian white noise	low	0.0041	8.9%	0.0091	4.2%
successive pulse signal	high	0.0043	4.4%	0.0093	2.1%
successive pulse signal	low	0.0038	15.6%	0.0087	8.4%

From the result in Table 1, the GCCSS method is suitable for time delay estimation in acoustic source localization experiment. The results are accurate in a high SNR environment, and the estimation accuracy decrease slightly with the decline of SNR. The GCCSS method can be applied to both non-stationary signals and stationary signals.

Choosing the Gaussian white noise to conduct the experiment 20 times in different SNR environment (10 times in low SNR environment and 10 times in high SNR environment), then computes the time delay by GCC method and GCCSS method. The localization can be realized by the method in reference [12], and the result of localization is shown in Fig. 8.

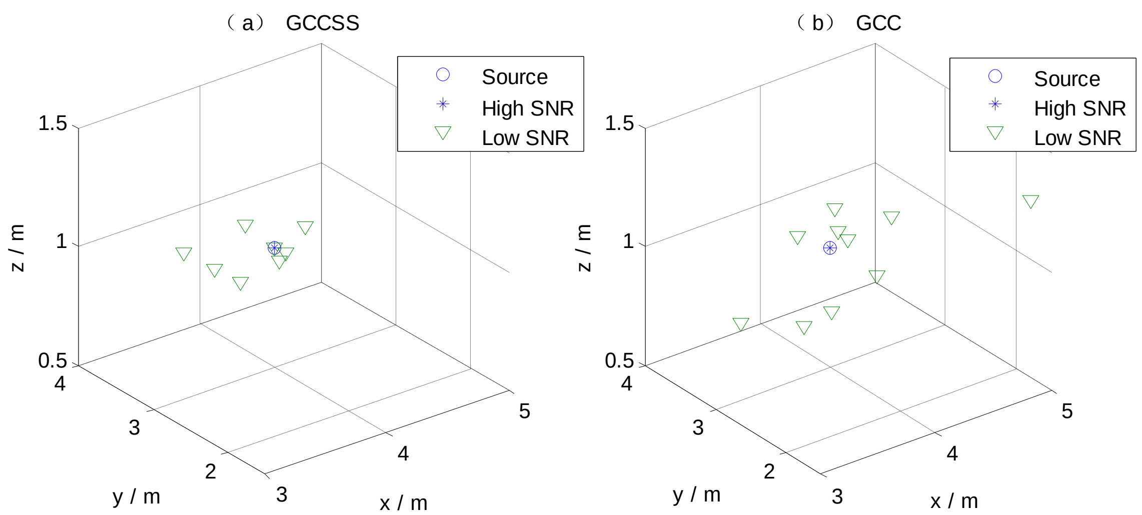

Figure 7: Experiment result of acoustic source localization

As shown in Fig. 8, the localization accuracy of GCC method is 60%, while the localization accuracy of GCCSS method is 75%. When SNR is high, TDE accuracy of GCC algorithm and GCCSS algorithm is both high, the 10 localization results are all correct. When SNR is low, for GCCSS method, three localization results are correct, two localization results take small errors; for

GCC method, two localization results take small errors, other results deviate from the correct location. Obviously, GCCSS exhibits a stronger anti-noise performance than GCC.

7. Conclusions

GCC algorithm is a typical TDE method, however, the accuracy of GCC algorithm declines in a low SNR environment. This paper improves the performance of GCC algorithm by processing signals with segmentation, second correlation and exponent transform, leading to a sharper correlation function peak and improving the TDE accuracy. The experiment results suggest that the GCCSS method is superior to the GCC method and GCCS method, and can have an effective application in acoustic source localization.

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