OPTIMAL TIME-DOMAIN BEAMFORMING WITH SIMULATED ANNEALING

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1. INTRODUCTION

This paper contains simulated and experimental results for an optimal time-domain beamformer that determines multiple source bearings and time series by parameter optimization [1,2]. The development of this signal processing method was motivated by an analogous frequency-domain method [3]. Simulated annealing [4-7] is used to search for the best least-squares fit to the data over all possible source bearings and time series. Due to improvements in the optimization procedure, the current version of the optimal beamforming algorithm converges faster than the version described in [1,2].

2. OPTIMAL BEAMFORMING

A linear array of N equally-spaced hydrophones receives M plane-wave signals with time series $p_m(t)$. The signal measured at the nth hydrophone is

$$P_n(t) = \sum_{m=1}^{M} p_m(t - n\tau_m) + \Psi_n(t) , \tag{1}$$

where τ_m is the delay corresponding to the hydrophone spacing and the bearing of the mth source. The noise time series $\Psi_n(t)$ may consist of a combination of white noise, ambient noise, signals from other sources, and other types of noise. The replica pressure at the nth hydrophone is defined in terms of the test time series $q_m(t)$ and test delays σ_m by

$$Q_n(t) = \sum_{m=1}^{M} q_m(t - n\sigma_m) . \tag{2}$$

An optimal estimate of the bearings and time series is obtained by minimizing the following energy function over all possible sets of test parameters:

$$E(q_1, \sigma_1, q_2, \sigma_2, \ldots, q_M, \sigma_M) = \sum_{n=1}^{N} \int \left[P_n(t) - Q_n(t)\right]^2 dt . \tag{3}$$

We solve this optimization problem, which may involve thousands of unknowns after discretizing in time, using simulated annealing. Optimal beamforming is practical with this Monte Carlo method, which is analogous to the cooling of a liquid to form a perfect crystal. An artificial control parameter that is referred to as the temperature is lowered slowly throughout the search process.

The solution for the bearings and time series that minimizes the energy is not unique. In the noise-free case with two sources, for example, the energy vanishes for

$$q_1(t) = p_1(t) + f(t)\,, \tag{4}$$

$$q_2(t) = p_2(t) - f(t)\,, \tag{5}$$

where $f(t)$ is an arbitrary function (including arbitrary amplitude) of period $\tau_1 - \tau_2$. This ambiguity may be removed by imposing an additional constraint. In simulations, it is possible to suppress the ambiguity by using signal time series that have compact support so that $f(t)$ vanishes [1]. Other approaches are required for processing real data.

Since the ambiguous function represents a correlation between the source time series, it may be suppressed for long time series by minimizing the source correlations [2]. Due to the high

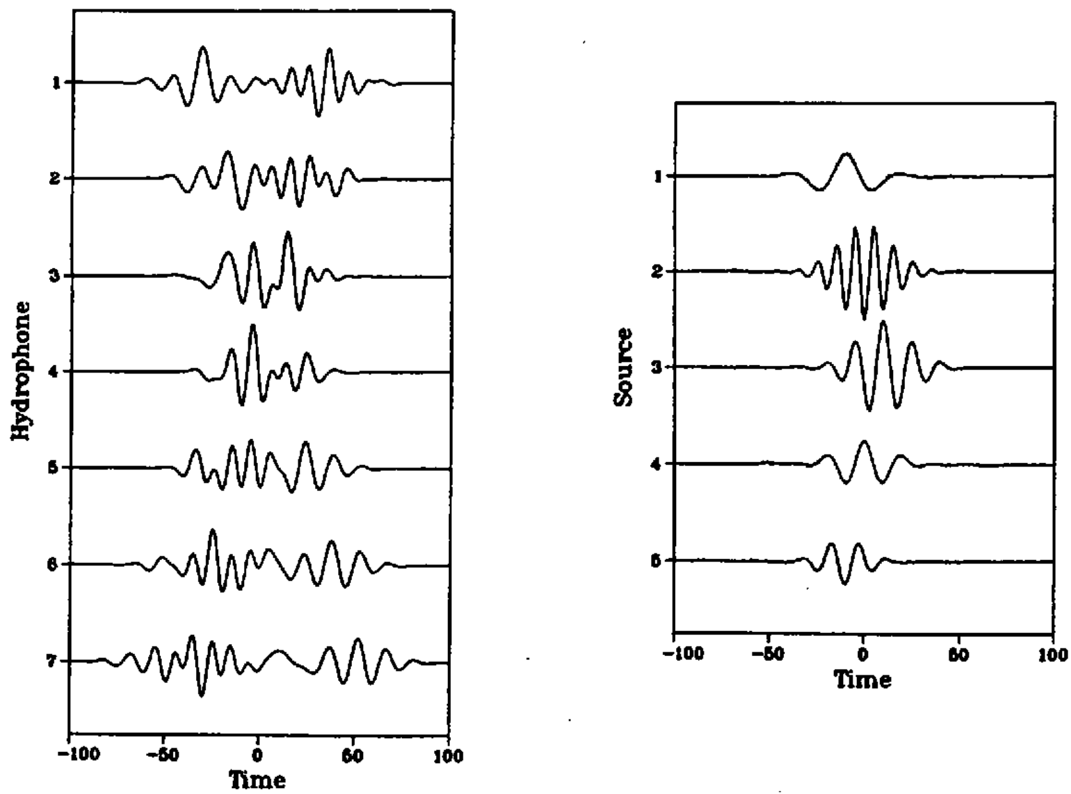

FIG 1. Left: The hydrophone data for the example involving five sources. Right: The recovered time series (solid curves) and the true time series (dashed curves).

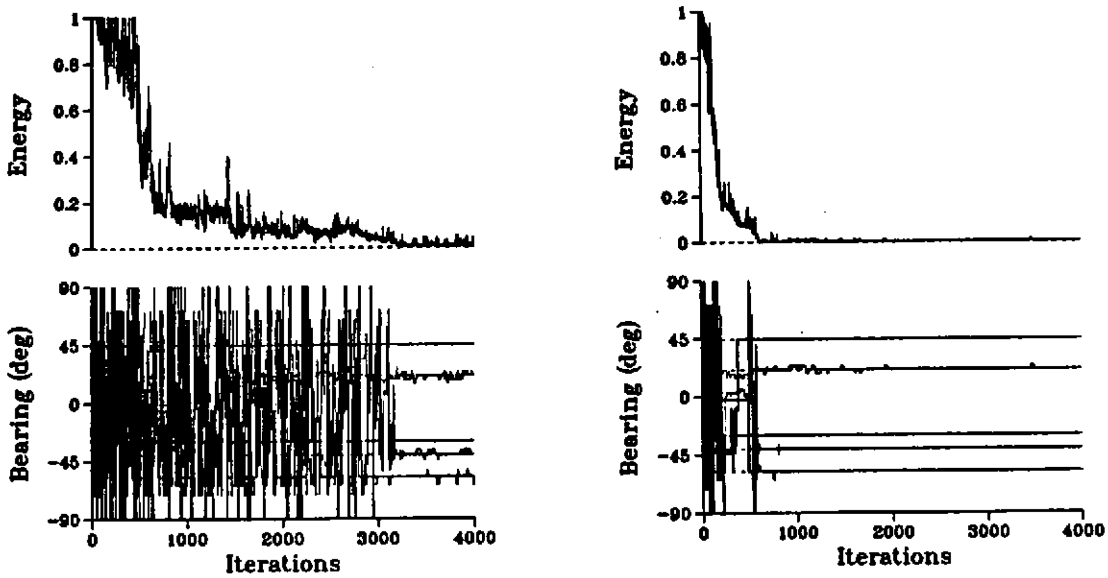

FIG 2. The bearings and energy of the simulated annealing search for the example involving five sources. Left: The original search algorithm. Right: The improved search algorithm.

dimension of the parameter space, however, it is not practical to implement this approach with Lagrange multipliers. Furthermore, it is often desirable to apply the optimal beamformer to short time series. Fortunately, there is a simple way to minimize the source correlations that works even for short time series. With the simulated annealing algorithm described in [1,2], the source correlations may be approximately minimized by terminating the search process when the source bearings lock in and the energy stabilizes at a low value. With the improved search algorithm described in the following section, it is not necessary to terminate the search process to prevent the ambiguity from growing.

3. AN IMPROVED SEARCH ALGORITHM

Simulated annealing searches for the global minimum of the energy through a sequence of iterations in which the unknown parameters are perturbed. In the standard form of simulated annealing, perturbations that decrease the energy are always accepted and perturbations that increase the energy are accepted according to a Boltzmann probability distribution to allow escape from local minima [4-7]. The performance of the optimal beamformer has been improved by applying the standard acceptance criteria for the source bearings while accepting time series perturbations only if the cost function decreases. This search algorithm can escape from local minima in the bearings. There are no local minima in the time series because the energy function is a parabola in each of the time series parameters.

With this search procedure, the periodic ambiguity does not grow after the bearings have locked in. This can be a significant advantage, especially if several sources are present. We illustrate the improved search algorithm with an example from [1,2] that involves five simulated Gaussian signals and seven hydrophones. The hydrophone data and recovered time series appear in Figure 1. The Markov processes of bearing estimates appear in Figure 2 for the original search algorithm of [1,2] and for the improved search algorithm. The improved algorithm converges much faster. The time series recovered using the improved algorithm and the true time series appearing in Figure 1 are in excellent agreement.

Like all signal processing methods, the optimal beamformer has difficulty detecting weak signals that are hidden by strong signals. To reduce this limitation, we first apply the optimal beamformer to determine the parameters of the strong signals and subtract these time series from the hydrophone data. The optimal beamformer is then applied to search for signals in the reduced data. We refer to this approach as fractional annealing. Time series extracted by fractional annealing appear in Figure 3. The optimal beamformer was first applied to extract the two loud sources. Note that a small periodic ambiguity is visible in the region where the true signals vanish. The quiet source was then extracted from the reduced data. All three of the sources are accurately recovered. This approach has also been applied successfully to real data.

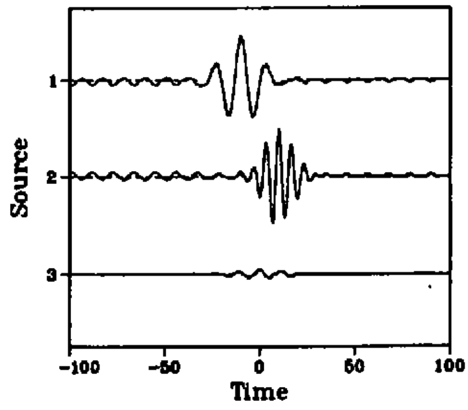

FIG 3. Recovered time series for the fractional annealing example.

4. TOWED ARRAY DATA

The optimal beamformer has been used to process data from an array of hydrophones towed behind a ship in the Atlantic Ocean. To eliminate 180 Hz noise from the data, we used the following generalization of Eq. (2):

$$Q_n(t) = A_n \sin(\omega t + \phi) + \sum_{m=1}^{M} q_m(t - n\sigma_m) \, , \tag{6}$$

where $\omega = 360\pi$ s^{-1}. The phase ϕ and the amplitudes A_n are added to the parameter search space. Since electromagnetic waves travel much faster than sound waves, ϕ is assumed to be independent of n.

An example of some processed data appears in Figure 4. It was assumed that the data contains signals from the tow ship and from two other ships. A large contribution from the 180 Hz noise is evident in hydrophone 6. The algorithm converged to the tow ship direction and to bearings that were confirmed to correspond to two other ships. The replica and true hydrophone data are in excellent agreement. The tow ship signal is the top source time series in Figure 4.

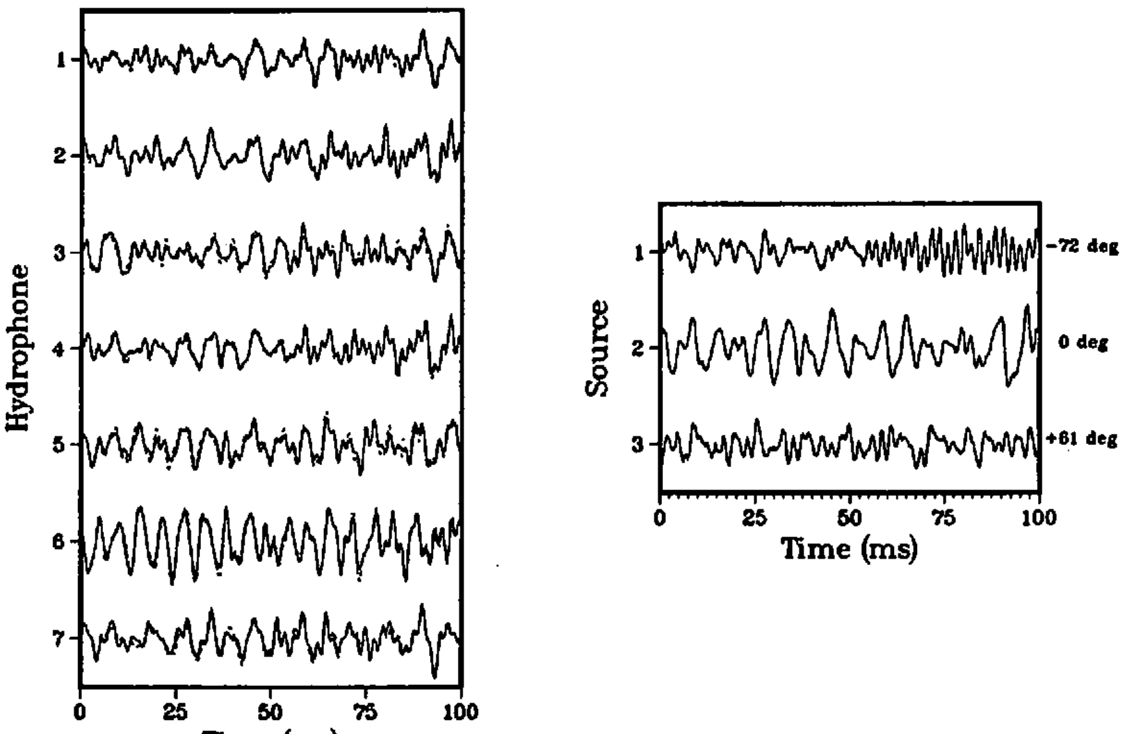

FIG 4. Results for the example involving real data. Left: The best replica hydrophone signals (solid curves) and the data (dashed curves). Right: The recovered source time series.

5. CONCLUSION

The simulated annealing search algorithm for the optimal beamformer has been improved. The improved algorithm converges faster than the original algorithm. Fractional annealing is a useful generalization of the optimal beamforming algorithm for detecting weak signals. The optimal beamformer also performs very well for real data.

6. REFERENCES

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