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A COMPARISON OF PRONY AND NYQUIST SAMPLING FOR THE MEASUREMENT OF SHORT PULSES IN ACOUSTIC CALIBRATION

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SYNOPSIS

The need to simulate free field conditions in tanks and other confined spaces is introduced and the technique of using short pulses to achieve this is briefly discussed. The problems inherent in making actual measurements of these pulses are described.

The principles of the Prony (Coherent Sampling) system are explained and the techniques of oversampling, undersampling and compound sampling are described. The Prony system developed by Brown & Luckey of USRD is briefly described and the figures for the accuracy of the system, as measured by them, are quoted.

The limitations of the Prony System are explored, in particular the difficulties in measuring pulses from integrated systems, due to the need to synchronise the sampling device with the transmitted signal.

The principles of the Nyquist (Random Sampling) technique are explained and the relationships between sampling frequency, signal frequency, pulse length and accuracy are explored.

Some commercially available equipment is described with the relevant manufacturers' specifications.

A possible calibration system based on one of these devices is briefly described.

The conclusion is drawn that the very fast random sampling devices now available make the increased complexity of the Prony system largely redundant, except for certain specialised applications.

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INTRODUCTION

The need to simulate free field conditions in a confined space is a problem common to many branches of acoustics. It is particularly difficult in underwater measurement as large areas of open water, even when available, have problems of access. We are thus constrained to perform the majority of our measurements in test tanks of strictly limited size.

A number of measurement techniques are used to combat the problem, and anechoic linings, white noise systems and time-delay spectrometry all have their place in the field of underwater measurement.

One of the most popular and effective techniques is, however, the short pulse system. A short pulse is emitted by a projector and received on a hydrophone before echoes from the tank sides etc. reach the hydrophone. Time is then allowed for all echoes to die away before the next pulse is sent.

The length of pulse which can be used depends on the difference between the length of the direct path from projector to hydrophone, and the shortest reflected path. This is a function of the tank size and the distance which must be left between the transducers, and is largely independant of frequency. The effective length of the pulse may be reduced by 'Q' in the system, which tends to give the pulse a long rise and fall time, reducing the time for which the pulse amplitude is typical of the steady state condition.

Various methods have been used to measure the amplitude of this short pulse and most rely on some kind of sampling technique.

This paper is concerned with two such techniques:

The Prony or Coherent Sampling method and the Nyquist or Random Sampling method.

The two techniques are not unrelated, as the Prony method relies on Nyquist's theorem, and the Nyquist method uses a constant sampling interval, one of the essential features of the Prony system. The two methods do, however, represent different approaches to the measurement of a waveform by sampling.

THE PRONY SYSTEM

In 1982 C.K. Brown and R.W. Luckey [1] published a description of the Prony system built by them at USRD in Florida.

The essence of the Prony system is that the test signal and the sampling frequency have a precisely known mathematical relationship. At USRD this is achieved by the use of modern frequency synthesising techniques. The two signals are derived from the same master clock using different divisors.

The basic relationship between the test frequency and the sampling frequency is that the sampling 'window' must contain an integral number of periods of each frequency, thus:

$$\frac{N_T}{f_T} = \frac{N_S}{f_s}$$

where:

f_T = Frequency of test signal.

f_s = Sampling Frequency.

N_T = Number of periods of f_T in sampling 'window'.

N_S = Number of periods of f_s in sampling 'window'.

We may idendify several sampling systems which satisfy the above relationship.

Where the test frequency is low, oversampling is used, i.e.

$$N_S \gg N_T$$

This is shown in Figure 1. The actual number of samples has been reduced in the interests of clarity, but the USRD system uses $N_T = 2$, $N_S = 200$.

Figure 2 shows undersampling which is used when the test frequency approaches the sample frequency. The figure shows 9 samples taken over 10 cycles while the figures used by Brown and Luckey were 100 and 101 respectively, though, as in the case of oversampling, two identical sets of samples were taken.

The relationship

$$N_S = N_T - 1$$

is a special case of the general rule that N_S and N_T must have no common factor. Provided this rule is obeyed, any integral number of samples and any (other) integral number of cycles may be used. Figure 3 shows compound sampling in which a signal whose frequency is intermediate between those of Figures 1 and 2 is sampled at the rate of 9 samples in 4 cycles. It is seen that if the samples are mapped onto a single cycle, they are equally spaced through the cycle. Figure 4 shows what happens if the numbers of of cycles and samples have a common factor. When 9 samples are taken of three cycles it is seen that we have only taken three different samples of the waveform, and repeated the readings three times.

Figure 5 is included in the interests of completeness to show that the principle of coherent sampling can be extended to test frequencies higher than the sampling frequency.

ACCURACY OF THE PRONY SYSTEM

A comparison of undersampling and oversampling was made in 1983 by R.E. Scott and J.D. George [2] using a relatively small number of samples (52) in each sampling window. The small number of samples should have the effect of exaggerating any difference between the two measurement techniques.

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FIGURES 1 TO 5 : PRONY SAMPLING

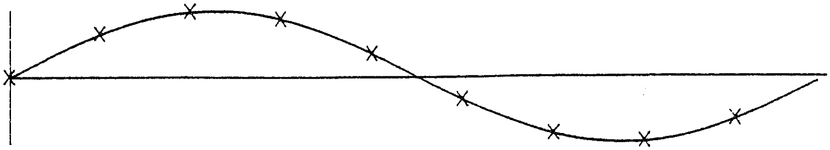


FIGURE 1 : Oversampling, 9 samples in 1 cycle.

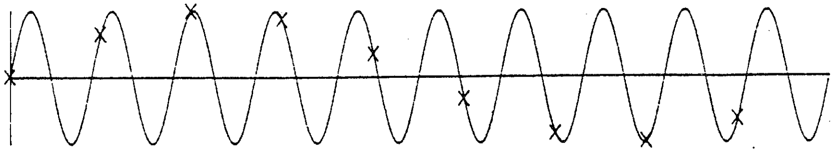


FIGURE 2 : Undersampling, 9 samples in 10 cycles.

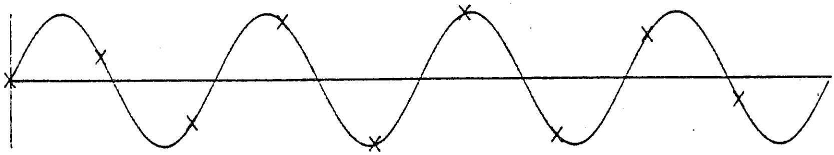


FIGURE 3 : Compound Sampling, 9 samples in 4 cycles.

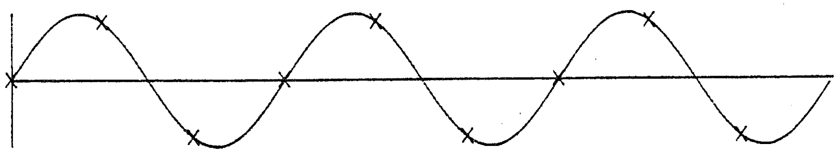


FIGURE 4 : Compound Sampling, 9 samples in 3 cycles.
Readings are repeated due to common factor.

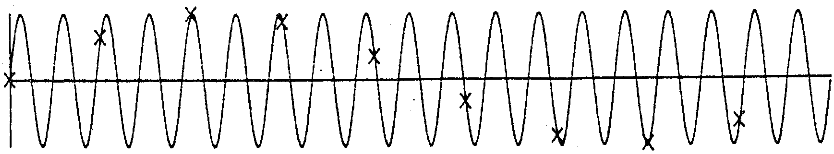


FIGURE 5 : Extended undersampling,
9 samples in 19 cycles.

Their results are summarised in the following table:-

COMPUTED PARAMETER	OVERSAMPLING	UNDERSAMPLING
RMS VOLTAGE (STD DEV)	1.047	1.047
PEAK VOLTAGE	1.485	1.492
DC VOLTAGE (MEAN)	-0.013	-0.013
TOTAL POWER	1.102	1.102
RMS VOLTAGE (DFT)	1.047	1.047
RELATIVE PHASE	0	0
HARMONIC DISTORTION	0.100	0.095

It is seen that apart from a small difference in peak voltage and harmonic distortion, the results are identical.

It is a corrolary of the Nyquist Theorem that if three or more evenly spaced samples of a sinusoid are taken, including one at the zero crossing, the standard deviation of the samples is equal to the RMS value of the sinusoid. The solid curve of Figure 7 shows the expected value of the error if the mean square is calculated from n equally spaced samples. If we accept the undersampling algorithm, then all forms of Prony sampling agree with this criterion and there is no error inherent in the sampling system.

There are three sources of error in the system:

(1) Alignment errors and errors introduced by the analogue circuitry. These are outside the scope of this paper.

(2) Quantising errors due to the fact that a digital system cannot resolve to better than $\pm \frac{1}{2}$ LSB. The effect of these is well known and easily calculated.

(3) Aperture Uncertainty Errors

Because the time at which a sample is taken can only be controlled within certain limits, an error in the sample amplitude will occur.

The higher the rate of change of voltage with time, the greater the error will be. It may be seen with reference to Figures 1 and 5 that a small error in the timing of a sample will only slightly affect its value in the low frequency case (Figure 1); but will cause a substantial error in the high frequency case (Figure 5).

The effect of aperture uncertainty in a system is usually quoted as the frequency above which degradation of the LSB occurs. The subject is treated more fully in [3] from which Figure 6 is taken. The maximum frequency is given by:

$$f_{MAX} = (2\pi \times 2^N \times t)^{-1}$$

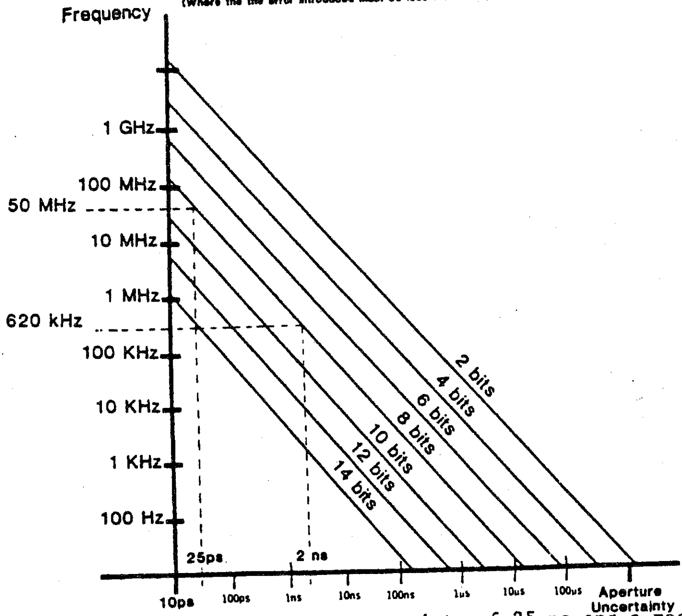
where:

N = The number of bits unaffected.

t = The maximum aperture uncertainty (seconds).

FIGURE 6 MAXIMUM SAMPLE FREQUENCY Versus APERTURE UNCERTAINTY FOR DIFFERENT ADC RESOLUTIONS

(Where the the error introduced must be less than or equal to 1/2 the least significant bit).



The USRD system has an aperture uncertainty of 25 ps and a resolution of 12 bits, giving a Figure for f_{MAX} of 1.55 MHz. Test results obtain by Brown and Luckey do not, however, show any significant loss of accuracy at frequencies higher than that.

This may be attributed to the fact that aperture error tends to be random and its effect is reduced by taking many more samples than the minimum required.

It should also be noted that however bad the aperture uncertainty errors may become, the result cannot be worse than random - sampling the waveform. The results of random sampling are considered later.

The accuracy of the USRD system was measured by Brown and Luckey as RMS scatter. The figures obtained varied from .004% at 1Hz to .014% at 2 MHz. These excellent results are fully in keeping with such a well designed system.

LIMITATIONS OF THE PRONY SYSTEM

The Prony system is suitable for the calibration of transducers, which may be connected directly to the system's signal generator.

It cannot be used, however, to measure pulses emitted by complete systems, as the sampling frequency cannot be synchronised with the signal.

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NYQUIST OR RANDOM SAMPLING METHODS

While the Prony system was being developed at USRD, developments in other branches of physics led to a requirement for very fast transient recorders.

As the waveforms being observed were non-repetitive, coherent sampling techniques were inappropriate, and the only solution was to build waveform recorders with very high sampling rates. Such devices are available with sampling speeds up to 200 MHz. The sampling clock or time base free-runs, and is independent of the signal being received.

Although I call this method a Nyquist or random sampling technique, neither term is completely appropriate. All waveform sampling systems, including the Prony system rely on Nyquist's theorem. In this case, 'Nyquist sampling' is used to mean measurements in which no component of the measured signal exceeds half the sampling frequency. Also, while waveform recorders sample at regular intervals - not randomly, the points on the waveform at which the samples occur are not known, and the sample points are, to that extent, random.

The principles of Information Theory and the Nyquist theorem are well known. They are, however, of fundamental importance to what follows.

In 1924 H. Nyquist [4] showed that bandwidth was necessary for the transmission of information, and the principles of information theory were subsequently developed.

Most of this work was done by R.V.L. Hartley [5] and C.E. Shannon [6] and it is to the latter that we owe the clearest statement of the Nyquist theorem:

'If a function of time f(t) is limited to the band from 0 to W cycles per second it is completely determined by giving its ordinates at a series of discrete points spaced $\frac{1}{2W}$ seconds apart'. If f(t) contains no frequency over W

$$f(t) = \sum_{-\infty}^{\infty} X_n \frac{\sin \quad (2Wt - n)}{\pi \, (2Wt - n)}$$

where:

$$X_n = f\left(\frac{n}{2W}\right)$$

i.e. X_n is the amplitude of the nth sample

It should be realised, however, that a signal of frequency W Hz cannot be determined by ordinates spaced at $\frac{1}{2W}$ seconds in a non-infinite time.

In order to measure a pulse of length t seconds a modulating signal with frequency 1/t Hz must be admitted. When the test frequency f_T is thus modulated, the highest frequency produced is given by:-

$$f_T + \frac{1}{t} \text{ Hz}$$

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For the Nyquist theorem to be satisfied, the sampling frequency must be given by:-

$$f_S = 2\left(f_T + \frac{1}{t}\right) \text{ Hz}$$

It is no coincidence that this is the frequency at which the sample points will sweep through one cycle of the test signal during the currency of the pulse.

If the Nyquist conditions are satisfied as above, the accuracy of the system will depend on the same factors as in the case of coherent sampling. The limiting accuracy will depend on the resolution of the system, possibly degraded by aperture uncertainty error.

EQUIPMENT

The acknowledged leaders in the field of very fast transient recorders are LeCroy Research Systems of California. They produce a range of transient recorders with sampling frequencies from 32 to 200 MHz. Their equipment, which is principally designed for the nuclear physics field, is all built to the CAMAC standard. Unfortunately all their devices are limited to 8-bits resolution. Although this gives sufficient accuracy in the case of full-scale signals, resolution is rapidly lost when the signal level falls.

A low cost solution is offered by digital storage oscilloscopes such as the Gould 4030 series. Sample sets can be transferred to a computer via the G.P.I.B. for processing, but resolution and memory length are limited.

The Hewlett-Packard 5180A waveform recorder seems to offer the best compromise for underwater acoustic work. The 20 MHz sampling speed is sufficient for all predictable requirements while the 10-bit resolution should give adequate accuracy and dynamic range. The 16K word memory enables longer pulses to be sampled at high speed and the manufacturers produce a wide range of compatible computers and peripherals.

A WAVEFORM RECORDER BASED CALIBRATION SYSTEM

The system is based on the Hewlett-Packard 5180A waveform recorder, supported by the 9836S computer and appropriate peripherals. The principal components of the system are connected via a GP1B for control purposes. A separate direct memory access connection is required between the waveform recorder and computer to provide the necessary high speed data transfer.

An attenuated sample of the transmitted pulse is applied to one channel of the waveform recorder and an amplified version of the received pulse is applied to the other. The memory record is divided between the two channels and 8K samples are taken of each pulse. The entire record is then transferred to the computer for processing.

The two pulses are displayed on the computer VDU and the operator is able to select which part of each pulse he wishes to measure. Typically the central half of each pulse is selected. This means that each pulse is represented by about 4,000 samples equally spaced through its central portion.

For a straightforward calibration, the RMS value of each of these sets of samples is taken, corrected for the applied attenuation and amplification and the 'A' value calculated.

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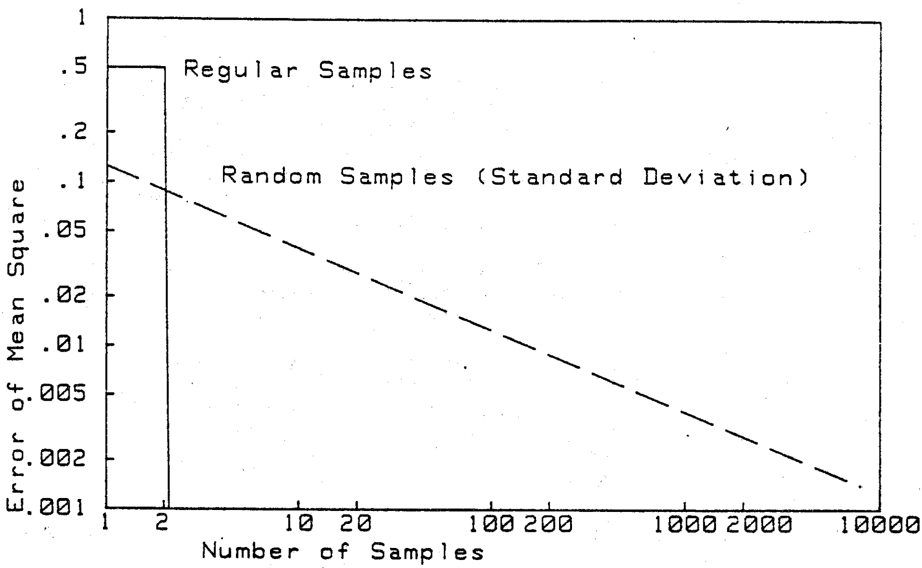
The sensitivity of the device under test may then be calculated from knowledge of the standard transducer sensitivity, and the distance between the two.

The pulse generating device, (a Hewlett-Packard 3314A function generator) and the stepping motors which manipulate the transducer mountings, are controlled via the GPIB as well as the waveform recorder. This enables frequency response and beam pattern information to be generated automatically. These data are then output to a printer and plotter.

FIGURE 7 : SAMPLING ERROR FOR A SINE WAVE



CONCLUSION

Figure 7 shows the error which may be expected when the mean square of a sinusoidal function is calculated from n samples. The solid line shows the ideal result when the samples are spaced at precisely equal intervals. The error falls to zero as soon as the number of samples exceeds two.

The broken line shows the 'worst case' condition where the samples as assumed to occur at completely random times. The error reduces proportionately to $1/\sqrt{n}$ where n is the number of samples taken.

Both the systems described will give results much nearer to the ideal than to the random, but even in the worst case the error obtained with 4000 samples is unlikely to exceed 0.3%.

With a sampling rate of 20 MHz, a pulse length of 200 s is required to obtain 4000 samples. This requires a path difference of 0.3m between the direct and reflected signals, which can be obained in all but the smallest test tanks.

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It is thus seen that with the high sampling speeds obtainable with modern equipment, the additional cost and complexity of coherent sampling systems cannot be justified for ordinary calibration applications.

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