Proceedings of the Institute of Acoustics

APPLICATION OF CHAOTIC THEORY TO NONLINEAR NOISE & VIBRATION MEASUREMENT & ANALYSIS

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1 INTRODUCTION

During the last decade, there has been much activities on chaotic theory as chaos has been observed in various physical phenomena, in turbulene, in laser power output, in chemical reactions,in condensed matter physics, astrophysics, vibration etc. The study of chaos can throw insights into nonlinear phenomena and help to solve nonlinear problems. During the past few years, there has been much activities to study the relationship between chaos and 1/f noise, a type of stochastic noise.. Recently, it has been extended to random noise in general, not just limited to this special type of noise. Papers have been published now on the application of chaotic theory to signal processing such as the analysis of industrial noise [1] and the filtering of a background random noise from chaotic signal [2,3]. The applications are to turbulence, to helicopter noise, communication etc. In view of the potential application of chaotic theory to signal processing, in this paper we propose the application of chaotic theory to nonlinear noise and vibration measurement and analysis.

2. IDENTIFICATION OF CHAOS

It is necessary for a noise and vibration signal to be tested first for the existence of chaos. Chaos means exponential sensitivity to initial conditions and therefore occurs, by definition, if there is a positive Lyapunov characteristic exponent (LCE). The LCE associated with a trajectory gives the average rates at which nearby trajectories diverge. Another tool in testing for chaos is to compute power spectra. If the motion is quasiperiodic the spectrum of any coordinate is discrete, whereas chaotic motion will exhibit broadband power spectra. The temporal behaviour of a function y(t) is quasiperiodic if its Fourier transform consists of sharp spikes, i.e. if

$$y(t) = \sum_{j=1}^{n} C_j e^{i\omega_j t} \qquad (1)$$

Quasiperiodic motion is regular. That is, quasi periodicity, like periodicity, is associated with a negative or zero Lyapunov exponent. Quasiperiodic motion can certainly look very complicated and seemingly irregular, but it cannot be truly chaotic in the sense of exponential sensitivity to initial conditions. In particular, the difference between two quasiperiodic trajectories is itself quasi periodic and so we cannot have the exponential separation of initially close trajectories that is the hallmark of chaos.

Since quasiperiodicity implies order, it follows that chaos implies non-quasiperiodic motion. Thus chaotic motion does not have a purely discrete Fourier spectrum as in (1) but must have a broadband, continuous component in its spectrum as in Fig. 1

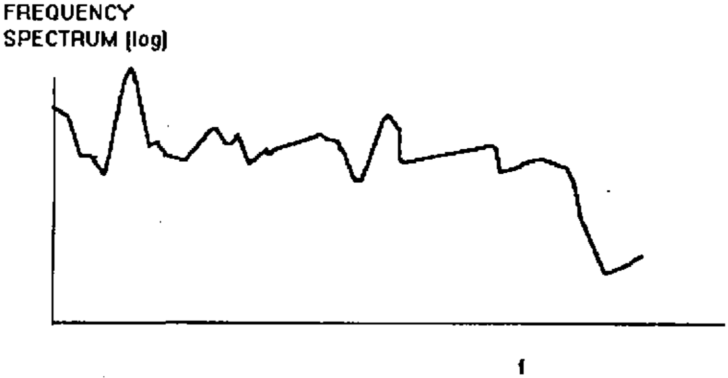
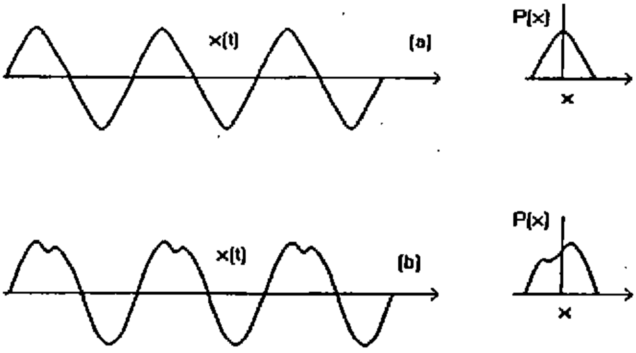


Fig. 1 Typical frequency spectrum of some coordinate of a chaotic system.

Fourier analysis is therefore a very useful tool in distinguishing regular from chaotic motion and furthermore it is generally much cheaper computationally than Lyapunov exponents.

Another test for chaos is to plot the probability density function of the signal to find the existence of multi-maxima which is a characteristic for chaotic behaviour. The examples of amplitude probability density functions and waveforms connected with them are shown in Fig. 2 below:



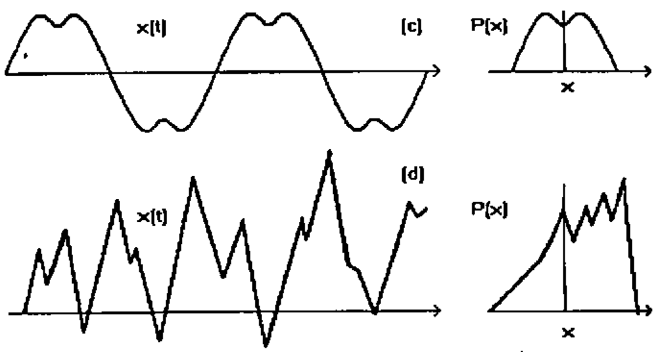


Fig. 2

But it has to be emphasized that the sure way of identifying chaotic behaviour is to compute the LCE.

3. FILTERING OF CHAOTIC SIGNAL EMBEDDED IN A RANDOM NOISE

In the analysis of nonlinear industrial noise using chaotic theory, one has to separate the noise into two portions, the chaotic signal and the random noise. Various methods can be used as shown below:

3.1 Maximum Likehood Processing

In the signal separation problem, we observe the state of a chaotic system through an observation function,h, and in the presence of another signal, i.e.,

$$
\begin{aligned}
y_n &= h(s_n) \\
O_n &= y_n + \omega_n
\end{aligned}
\tag{2}
$$

where $y_n \in R^k$ is the output from the chaotic system and ω_n is the other signal. The signal separation problem is to estimate both y_n and ω_n given the observations $O_{0:N-1} = \{O_0, O_1, \cdots, O_{N-1}\}$. Three categories of signal separation problems:

1. When both the state update function,f, and the observation function,h, are known.

2. When neither the state update nor the observation function is known but we have available a "reference observation". This observation is the result of observing the nonlinear dynamical system without the presence of another signal but in a case in which the initial conditions of the nonlinear dynamical system are different from those for the case in which we observe the signal plus interference.

3. When neither the state update nor the observaiton function is known. When both the state update and the observation functions are known and the other signal is white Gaussion noise, with a noise

correlation matrix $\sigma_\omega^2 I$, then it is possible to determine bounds on the performance of signal separation algorithms. In this case the maximum likelihood solution is given by a trajectory $\{\hat{S}_0, \hat{S}_1, \cdots, \hat{S}_{N-1}\}$ that obeys the constraints of the known dynamics, i.e. $\hat{S}_{n+1} = f(\hat{S}_n)$ and minimizes the different between the observed signal and the predicted observations, i.e. $\sum |h(\hat{S}_n) - O_n|^2$ This is equivalent to estimation of the initial condition $\hat{S}_0$ which maximizes

$$\log(P_r(O_{0:N-1} | \hat{S}_0)) = -\sum_{n=0}^{N-1} \frac{|h(f^n(\hat{S}_0)) - O_n|^2}{2\sigma_w^2} + C \tag{3}$$

Analysis of the likelihood function, Eq. (3), for a chaotic system shows some interesting properties, for the logistic map. The likelihood function contains multiple narrow maxima. The narrowness of the maxima is related to the positive Lyapunov exponents of the system. The presence of multiple maxima occurs because the nonlinear dynamics folds state space trajectories together.

3.2 Signal Separation using Markov Model

The signal processing method used is hidden Markov models. We assume that a sequence of observations are given: $O_{0:N} = \{O_0, O_1, \cdots, O_N\}$, where each observation is the sum of the output of the chaotic system and another signal, i.e. $O_n = y_n + w_n$ and we wish to generate the best estimates for y_n and w_n given the observations. For our initial work, we assume that the other signal, w_n can be modeled as white Gaussian noise with variance σ_w^2. We define two signal estimation algorithms - one based on a maximum likelihood state sequence estimation approach and one based on a maximum aposteriori approach.

1. The maximum likelihood signal estimation approach first estimates the most likely state sequence given the observation, i.e.

$$\hat{q}_{1:N} = \arg\max_{q_{1:N}} P_r(q_{1:N} | O_{1:N}) \tag{4}$$

This is computed using the Viterbi algorithm. The signal y_n is then estimated as the expected value of y_n given the observations and the most likely state sequence, i.e;

$$\hat{y}_n = E[y_n | O_n, \hat{q}_n]$$

$$= m(\hat{q}_n) + \frac{\sigma^2(\hat{q}_n)}{\sigma^2(\hat{q}_n) + \sigma_w^2}(O_n - m(\hat{q}_n)) \tag{5}$$

where $m(\hat{q}_n)$ and $\sigma^2(\hat{q}_n)$ are the mean and variance of the most likely state at time index n.

2. In the maximum aposteriori approach we attempt to estimate the signal y_n as the expected value of y_n given the observations, i.e.,

$$\hat{y}_n = E[y_n | O_{1:N}]$$

$$= \sum_{q_{1:N}} E[y_n | O_n, q_n] P_r(q_{1:N} | O_{1:N}) \quad (6)$$

where the summation is performed over all possible state sequence, $q_{1:N}$, and where $E[y_n | O_n, q_n]$ is computed according to Eq. (5)

We note that in the case of a linear dynamical system, driven by white Gaussian noise, both the maximum likelihood and the maximum aposteriori approaches converge to Kalman smoothing as the number of states goes to infinity. An application of above method is to helicopter noise buried in chaotic noise.

3.3 Application of Convolution to Signal Separation

Convolution can be applied to signal separation. Convolution is filtering and we will consider for the effects of convolution on the two parameters commonly used in the description of chaotic signals- the Lyapunov exponents and the fractal dimension of the attractor. In order to dertermine the effects of filtering on Lyapunov exponents, we represent the time series $Z[n]$ as the scalar observation of a composite system of the original nonlinear dynamics and the filtering dynamics:

$$\begin{aligned} x[n+1] &= F(x[n]) \\ w[n+1] &= Aw[n] + bG(x[n]) \\ Z[n] &= c^T w(n) \end{aligned} \quad (7)$$

The matrices A,b and c are chosen to represent a minimal realization of the system and w[n] the state of the filter at time n. We also assume that the overall composite system is minimal in the sense that there is no pole zero cancellations between any linear component of the original nonlinear system and the cascaded linear system. The invariance of the Lyapunov exponents under smoothing invertible coordinate changes allows us examine certain properties of the filtered signal in this augmented state space with the assurance that the results carry over to the embedded state space.

Convolution also affects the capacity dimension. The filtering of noisy chaotic data to reduce noise will cause errors in fractal dimension estimates. The effect of convolution on the capacity dimension can be examined using the time delay construction. The time delay construction defines a transformation of IR^N, the state space of the original nonlinear system, to IR^N, the space consisting of the reconstructed vectors. The effect of filtering on the capacity dimension of the observed signal Z[n] depends upon the nature fo this transformation.

4.COMPUTATION OF FRACTIONAL HARMONICS

It has been noted by Wei Rongjue et al (4) that chaos is caused by the presence of fractional subharmonics during their experiment on nonpropagating solitons and their transition to chaos. Hence to perform spectrum analysis of chaotic signal, it would be necessary to compute the fractional subharmonics. This can be done by using the theory of multiple scale expansion valid for the solitons. This requires a fractal analysis by applying a multiscale second-order statisitical method. For a fractal surface, there are relationships among fractal dimension, scaling, power spectrum, area size and intensity difference. Fractional subharmonics has fractal nature and one needs to estimate the fractal dimension and transform the scale of frequency spectrum to scale of fractal dimension. The fractal dimension can be determined following the work of Pentland [5] who computed the Fourier transforms of an image, determined the power spectrum and used a linear regression technique on the log of the power spectrum as a function of frequency to estimate the fractal dimension.

5. REFERENCE

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