# QUANTUM PHONONIC CRYSTAL

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Artificial metamaterial can be used to raise critical temperature of high temperature superconductor based on the theory of Kirzhnits et al which introduced effective dielectric response function to enhance attractive electron-electron interaction and hence the critical tempera ture of superconductor. It is based on the effective dielectric response function which controls the critical temperature rather than the critical energy gap of the natural high temperature superconductor. The artificial superconductivity shows the power of the metamaterial in control and manipulation of phase transition. The enhancement of the effective dielectric response will take place at the resonance near zero dielectric constant.. There is a type of phononic crystals based on the idea of localized resonant structures that exhibit spectral gaps with a lattice constant two orders of magnitude smaller than the relevant wavelength Disordered composi tion made from such localized resonant structures behaves as a material with effective negative elastic constants and a total wave reflector within certain tunable sonic frequency ranges. In this paper, one introduces the concept of quantum phononic crystal which is to shrink this type of phononic crystal to having unit cell the size of ten nanometres. Here the quantum phononic crystal will enable a negative dielectric response function near the resonance frequency.of the plot of the dielectric response function versus the ultrasound frequency. The negative dielectric response will increase the critical temperature of the superconductor.Towards the end of the paper, it is also pointed out that metamaterial is a form of artificial phase transition. For instance phase transition from the positive phase material to the negative phase material and the artificial high temperature superconductivity is also an artificial phase transition fabricated using metamaterial.Based on transport theory and transport properties, one is able to fabricate other forms of metamaterials such as artificial piezoelectricity and artificial ferromagnetism.

Keywords: dielectric response function, phase manipulation

## 1.Introduction

The concept of artificial high temperature superconductor based on metamaterials was proposed in 2014.[1] Artificial metamaterial can be used to raise critical temperature of high temperature superconductor based on the theory of Kirzhnits et al[2] which introduced effective dielectric response function to enhance attractive electron-electron interaction and hence the critical temperature of superconductor.The theory of the artificial high temperature superconductivity is different from that of the natural high temperature superconductivity. It is based on the effective dielectric response function which controls the critical temperature rather than the critical energy gap of the natural high temperature superconductor. The artificial superconductivity shows the power of the metamaterial in control and manipulation of phase transition. The enhancement of the effective dielectric response will take place at the resonance near zero dielectric constant.. There is a type of phononic crystal based on the idea of localized resonant structures that exhibit spectral gaps with a lattice constant two orders of magnitude smaller than the relevant wavelength Disordered composition made from such localized resonant structures behave as a material with effective negative elastic constants and a total wave reflector within certain tunable sonic frequency ranges. [3]In this paper, we would like to introduce the concept of quantum phononic crystal which is to shrink this type of phononic crystal to having unit cell the size of ten nanometres. Since artificial quantum metamaterial has been successfully fabricated, the 3D printing method can be investigated for the fabrication of the quantum phononic crystal. The concept of quantum phononic crystal can be applied to artificial high temperature superconductor as the metamaterial is able to control and manipulate the geometric structure of the unit cell.

Sofar there has been no successful theory of high temperature superconductivity. But recently there is a proposal based on artificial metamaterial to raise the critical temperature of superconductivity with the introduction of effective dielectric response function to enhance the attractive electron-electron interaction and hence the critical temperature of superconductivity with the basic theoretical framework or the BCS theory remained. It is also found that the high temperature superconductor has similar electronic structure as a metamaterial leading to the possibility to manipulate the effective response function artificially. With the successful fabrication of the world's first quantum metamaterial or superconductor metamaterial showing the possibility of artificial superconductivity,here we propose the fabrication of the high temperature superconductor based on the resonator-based geometric structure of quantum phononic crystal.

## 2. Metamaterial route to high temperature superconductor

Igor Smolyaninov and Vera Smolyaninova [1] at the University of Maryland in College Park, USA proposed superconductor as a special form of metamaterial that steers electrons instead of light. That raises the tentalising possibility that the secret to high temperature superconductivity could lie in the development of a special form of metamaterial that steers electrons instead of light and the development of a new generation of metamaterials that exploit this idea further. The idea of steering electrons is to enhance the power of clearing the way of obstruction for the Cooper pairs by phonons further by manipulating this power further.

Indeed Ugor Smolyaninov and Vera Smolyaninova[1] suggested that high temperature superconductors such as BSCCO (bismuth strontitium calcium copper oxides) which superconducting at around 100K have a formal similarity with the metamaterials that physicists have already built to steer light. That is becauee they consisit of layets of atoms with metallic properties interspaced with layers of atoms that have dielectric properties .In effect they are the ultimate metamaterials constructed on the atomic scale. That raises the fascinating prospect that physicists might one day engineer superconducting metamaterials of their own. And with a better understanding of exactly how these layers steer electrons with zero resistance it might even be possible to make materials that superconductor at higher temperatures than are possible today. We argue that the metamaterial approach to dielectric response engineering may considerably increase the critical temperature of a composite superconducting dielectric metamaterial.

## 3. Dielectric response function as a key parameter in high temperature superconductivity

D. A. Kirzhnits et al[2] described the electron-electron interaction in superconductor in terms of the effective dielectric response function and obtained a final equation very similar to the BCS theory. The pairing interaction is expressed directly through the spectral density ρ of the inverse dielectric function. $\varepsilon_{eff}(q,\omega)$.All the quantities entering the final expression for the critical temperature can be expressed through ρ. The dependence of a critical temperature as different characteristic of the initial system (energy of the eigenmode, their damping, the coupling strength etc.) can be inves-tigated in the general form within the framework of this approach. The most interesting part is the calculation of the critical temperature for superconductivity. They are considering the system where beside purely Coulomb interaction between the conducting electrons there is an additional interaction caused by the phonon, exciton and so on. This will pave the way towards a successful theory of high temperature superconductor. With the possibility of using quantum metamaterial to be the unit cell of a resonator, the days of an artificial high temperature superconductor are not far away.

They obtained

$$T_c^{max} = \varepsilon_F e^{-3/g} \quad (1)$$

where $\varepsilon_F$ = Fermi energy, and g = coupling constant.

Eqn(1) for the maximjm critical temperature is obtained in the weak coupling approximation and therefore the numerical estimates on the basis of eqn.(1) are not reliable enough. If we still try to estimate the maximum critical temperature by this formula, by taking g ≈ ½ and $\varepsilon_F$ ≈ 10 ev, we obtain $T_c^{max}$≈ 300K which is a rather optimistic result. In a similar way the estimate of for the case of strong couping gives the value smaller by an order of magnitude. It should be mentioned that the numerical results are rather sensitive to many details of the description of the system due to the exponential dependence of on the parameters.

The key issue in Kirzhnits et al [2]'s reformulation of the BCS theory is the use of the effective dielectric function to describe the electron-electron interaction and the critical temperature. The basic mechanism of superconductivity is unchanged but only the Cooper pairing of electron-electron interaction is enhanced and the strength of the clearing of ways for the movement of the electrons is increased. This paves the way for the manipulation or the steering of electrons artifi-cially leading to the increase in the critical temperature. Kirzhnits et al[2] argued that pnonon-mitigated electron-electron interaction may be expressed in the form of effective Coulomn potential:

$$V(\vec{q},\omega) = \frac{4\Pi e^2}{q^2 \varepsilon_{eff(\overrightarrow{q,\omega})}} \quad (2)$$

where V =4Π$e^2$ /$q^2$ $i$s the usual Fourier-transformed Coulomb potential in vacuum and $\varepsilon_{eff}(\overrightarrow{q,\omega})$is the linear dielectric response function of the superconductor treated as an effective medium. Kirzhnits et al[2] noted that the effective medium consideration assumes homogeneous system so that the influence of the lattice periodicity is taken into account only to the extent that it may be included into $\varepsilon_{eff}(\vec{q}$ ,ω). Igor Smolyaninov et al[1] learnt that the homogeneous system approximation may remain valid even if the basic structure elements of the material are not simple atoms or molecules. It is known that artificial metamaterial may be created from much bigger building blocks, and the electromagnetic properties of these fundamental building blocks (meta-atoms) may be engineered at will. Since the superconducting coherent length(the size of the Cooper pair) ζ is about 100 nm in a typical BCS superconductor, we have an opportunity to engineer the fundamental metamaterial building blocks in such a way that the effective electron-electron interaction will be maximised, while homogeneous treatment of $\varepsilon_{eff}(\vec{q},\omega)$will remain valid. In order to do this, the metamatωerial unit size must fall within a rather large window between ~ 0.3nm (given by the atomic scale) and ζ ~100nm scale of a typical Cooper pair. However, this task is much more challenging than typical application of superconducting metamaterials suggested sofar[3] which only deal with metamaterial engineering on the scale which are much smaller than the microwave or RF wavelength. Our task requires development of superconducting metamaterials which are much more refined. In addition, the coherence length of the metamaterial-superconductor must be predetermined in a self-consistent manner. The coherence length will decrease with increasing of the metamaterial superconductor since the approach of Kirzhnits et al[2] gives rise to the BCS=like relation between the supercon-ducting gap and the coherence length:

$$\Delta(\frac{\zeta}{v_F}) \sim \hbar \quad (3)$$

Wwere $v_F$ = Fermi velocity. Therefore, metamaterial structural parameter such as the inter-layer distance etc which must remain smaller than the coherence length will define the limit of critical temperature increase.

Igor Smolyaninov et al[1] demonstrated that tuning electron-electron interaction is indeed possible in metamaterial scenario. It is obvious from eqn(1) that the most natural way to increase attractive electron-electron interaction is to reduce the absolute value of $\varepsilon_{eff}(\vec{q},\omega)$while keeping $\varepsilon_{eff}(\vec{q},\omega)$ negative within a substantial portion of the relevant four momenta spectrum. Potentially this may be done using the epsilon-near-zero (ENZ) metamaterial approach [4] which is based on intermixing metal and dielectric components in the right proportion. A ENZ metamaterial would maximise attractive electron-electron interaction given by eqn(1). Igor Smolyaninov et al[1] have derived the ENZ condition around

$$\delta_i = (2\varepsilon_m + \varepsilon_i)/2(\varepsilon_m\text{-}\varepsilon_i) \qquad (4)$$

where $\varepsilon_m$ = dielectric constant of superconducting material and $\varepsilon_i$ = dielectric constant of dielectric inclusions.

This means that $\varepsilon_m$and $\varepsilon_i$ must have opposite signs and $\varepsilon_i \approx$ -2$\varepsilon_m$ so that will be small. This simple consideration indicates that attractive electron-electron interaction in a superconducting metamaterial may indeed be increased by using the correct amounts of dielectrics. However, of the dielectric needs to be very large, since of the metal component typically given by the Drude model in the far infrared and THz ranges is large and negative. Ferroelectrics materials have large positive in the same frequency range may be a very good choice of such dielectrics. On the other hand, Maxwell-Garneett based analogy of eqn(2) indicates that even far from ENZ condition a superconductor dielectric metamateial must have larger and higher compared to the original un-diluted superconducting host. Igor Smolyaninov et al[1] have shown that ENZ condition must lead to considerably larger . and diverging photon density of states. Igor Smolyaninov et al[1] demon-strated that hyperbolic metamaterial geometry offers another natural way to increase attractive elec-tron-electron interaction in a layered dielectric-superconducting metamaterial. The effective Coulomb interaction of two electrons may become attractive and very strong in the hyperbolic frequen-cy bands. In order to be valid, the metamaterial effective medium description requires that the structural parameter of the metamaterial(in this particular case, the interlayer distance) must be much smaller than the superconducting coherence length. If the the structural parameter approaches 1nm scale, Josephson tunnelling across the dielectric layers will become very prominent in such an anisotropic layered superconducting hyperbolic metamaterial.

Since metamaterial dimensions required for engineering of electron-electron interaction approaches nanometer scale, another potentially important issue is applicability of nanoscale metal and dielectric layers description using the macroscopic dielectric constants. This issue is well known and extensively studied in nanophotonics and electromagnetic metamaterials. The electro-magnetic response of thin metal layers is indeed known to exhibit weak oscillatory dependence on layers thickness due to quantum mechanical effects such as formation of electron standing waves inside the thin layers. While there is a weak effect on the other hand, the dielectric constant of dielectric materials does not depend on layer dimension until the atomic scale is reached. This fact has been useful in experiments on surface plasmon resonance.

The key issue is to engineer dielectric response of composite superconducting metamaterials on sub-100nm scale with purpose to increase critical temperature of such composite superconductor-dielectric metamaterials.

Igor Smolyaninov et al[1] have plotted the dielectric constant versus the frequency. It showed that around the frequency= 2300 there is a critical value which increases to a maximum then drops to near zero epsilon followed by a huge drop to a large negative value of the dielectric constant.

## 4. Successful fabrication of world's first quantum metamaterial

Pascal Macha et al[5] at the Karlsruhe Institute of Technology in Germany have built and tested the world's first quantum metamaterial. They constructed as an array of 20 superconducting quantum circuits in the form of split ring resonators(SRR) embedded in a microwave resonator. This experiment is a significant challenge. They fabricated their quantum circuits out of aluminium in a niobium resonator which they operated be-low 20 millikelvin. Their success comes from two factors. First was in minimising the difference between each quantum circuit so there was less than a 5 per cent difference in the current passing through each. The second was in clever design. A quantum circuit influences an incoming photon by interacting with it. To do this as a group, the quantum circuits must also interact with each other. The problem in the past is that physicists had arranged the circuits in series so that the combined state must be a superposition of the states of all the circuits. So if a single circuit was out of kilter, the entire experiment failed. Macha et al[5] got around this by embedding the quantum circuits inside a microwave resonator chamber about a wavelength long in which the microwaves become trapped. To interact with a photon each quantum circuit needs only couple with the resonator itself and its nearest neighbours. That is much easier to do with a large ensemble of quantum circuits and the results show that it worked at least in parts. The interaction within the quantum circuits changes the phase of the outgoing photons in subtle but measurable ways. So by studying this changes, Macha et al[5] were able to work out exactly what kind of interaction was occurring. What they saw was that eight of the circuits formed a coherence group that influenced the photons. But over time, quantum circuits that raises the tantalising question of why the large ensemble dissociated into two smaller ones something Macha et al[5] will surely be investigating in future work. It also raises the prospect of a new generation of devices. Quantum circuits, based on proof-of-principle experiment offer a wide range of prospects from detecting single microwave photon to phase switching, quan-tum birefringence and superradian phase transition. This is a significant first step for quantum metamaterial.

## 5. Experimental demonstration of increase in critical temperature

N. Smolyaninov et al [6] performed experiments to test the metamaterial approach to dielectric response engineering my increase the critical temperature of a composite superconductor-dielectric metamarerial using compressed mixture of tin and barium titanate nanoparticle of varying composition. An increase of the critical temperature of the roder of 0.15K compared to bulk tin has been observed for 30% volume fraction of barium titanate nanoparticles. Similar results were also obtained with compressed mixtures of tin and strontium titanate nanoparticles

## 6. Proposal of alternate method to test effective dielectric response approach to increase critical temperature

Our proposal was inspired by the successful fabrication of the world's first quantum metamaterial in Germany in September 2013[5]. This quantum metamaterial is superconductor metamaterial. We extend this work from ordinary superconductor to high temperature superconductor by replacing the quantum circuits made from aluminium by quantum circuits made from BSCCO high tem-perature superconductor. These quantum circuits will again be embedded in a indium microwave resonator. The internal resonance of the geometric structure will enhance the attractive electron-electron interaction and hence the effective dielectric response leading to increase in the critical temperature. This enhancement of the effective dielectric response will take place at the resonance near zero epsilon. It has to be noted that the split-ring resonator (SRRs) used to produce effective negative permeability and effective negative mass density (analogous to effective dielectric constant for the electromagnetic case) all show a similar patterns in the plots versus frequency of these parameters that near the resonance frequency there is an increase in the effective permeability and effective mass density to a maximum values followed by a sudden drop to a huge negative values of these parameters. All these geometric structures are all resonators based confirming that this inter-esting phenomenon is due to the internal resonance of the geometric structure.

## 7. A quantum phononic crystal based on idea of localized resonant structures

Liu Z et al[3]have fabricated sonic crystals, based on the idea of localized resonant structures, that exhibit spectral gaps with a lattice constant two orders of magnitude smaller than the relevant wavelength. Disordered composites made from such localized resonant structures behave as a material with effective negative elastic constants and a total wave reflector within certain tunable sonic frequency ranges. A 2-centimeter slab of this composite material is shown to break the conventional mass-density law of sound transmission by one or more orders of magnitude at 400 hertz

In this paper, the concept of quantum phononic crystal will be introduced which is to shrink this type of phononic crystal to having unit cell the size of ten nanometres. Since artificial quantum metamaterial has been successfully fabricated, the 3D printing method can be investigated for the fabrication of the quantum phononic crystal. The concept of quantum phononic crystal can be applied to artificial high temperature superconductor as the metamaterial is able to control and manipulate the critical temperature of the superconductor.

## 8.Principle of the local resonacne sonic crystal

In phononic crystal,based on multiple scattering theory, there exists a certain energy band gap which will forbide sound transmission. Liu et al [3] extended this concept to local resonance structure which will produce a negative elastic constant. They present a class of sonic crystals that exhibit spectral gaps with lattice constants two orders of magnitude smaller than the relevant sonic wavelength. Their materials are based on the simple realization that composites with locally resonant structural units can exhibit effective negative elastic constants at certain frequency ranges. The geometric structure of the phononic crystal is shown in the following diagram:

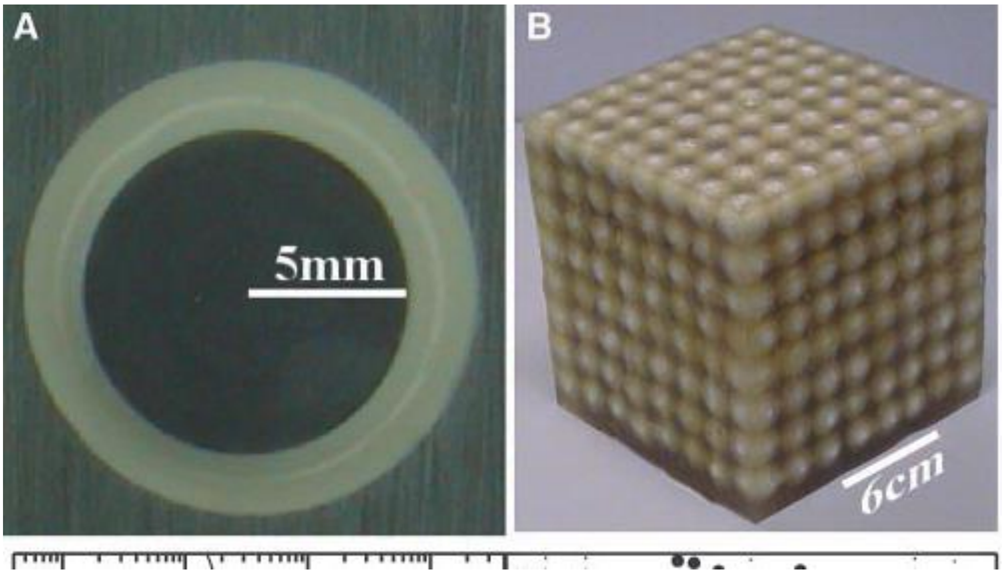


Fig.1. (A) Cross-section of a coated lead sphere that forms the basic unit, (B)for an 8x8x8 sonic crystal.( after Ref (3))

In an artificial high temperature superconductor, one can manipulate and control the dielectric response function to increase the critical temperature. The metamaterial is able to produce a negative dielectric response function near the resonance frequency enabling the increase and manipulation of the crtitical temperature as shown by the epsilon-near-zero (ENZ) metamaterial approach [4] which is based on intermixing metal and dielectric components in the right proportion. A ENZ metamaterial would maximise attractive electron-electron interaction given by eqn(1). In analogous, the local resonance sonic crystal is able to produce a negative dielectric response function for the increase and manipulation of the critical temperature. The negative dielectric response function will occur near the resonance frequency of the plot of the dielectric response function versus the ultrasound frequency. Here one will replace the lead sphere by the BSCCO sphere. BSCCO by itself is a high temperarture superconductor. In so doing, one can increase and manipulate the critical temperature of BSCCO by using the capability of the artificial quantum phononic crystal. One will have to shrink the dimension of the phononic crystal down to the quantum regime of tens of nanometres.

The next stage of the work will be to derive the resonance frequency for the plot of the dielectric response function versus the ultrasound frequency in terms of the parameters of the geometric structure of the unit cell of the phononic crystal such as the diameter of the BSCCO sphere and the spacing between the spheres and the number of the spheres Then it is able to optimise the increase in the critical temperature by the design of the geometric structure of the unit cell of the phononic crystal..This will be the topics of a future paper.

## 9 Metamaterial is Artificial .Phase Transition

It is to be noted that metamaterial is in fact a form of artificial phase transition. The negative permeability, negative permittivity, negative bulk modulus and negative mass density are transport properties of a negative phase material. The double negativity metamaterials are in fact a transformation from the positive phase material to the negative phase material. The magnetism with negative permeability proposed by John Pendry et al[7] is artificial magnetism.It is of interest to note that during the phase transition, at the critical point of phase transition or at the resonance frequency, there is a hyperbolic behaviour of the transport properties of permeability for magnetism, bulk modulus for elasticity and dielectric There is a sudden rise of these transport properties to infinity then a sudden drop to negative infinity and then a gradual increase in values in the negative region. It is experimentally shown that this hyperbolic behaviour of the dielectric response function occurs for the artificial high temperature superconductivity.[1] This is phase transition. This will help to support that the double negativity metamaterial is a phase transition from positive to negative phase material.

This concept is useful for the application of several other transport properties to form various other forms of metamaterial. The artificial magnetism with negative permeability and the artificial elasticity

with negative . bulk modulus and the artificial high temperature superconductivity [1]are only the first examples of this concept . This shows that transport theory which is the theory of the transport properties is the backbone of the metamaterials.

An advantage of using transport theory for exploring new forms of metamaterial is that there is no need of using analogy such as that of obtaining acoustic metamaterial from electromagnetic metamaterial.

Currently I am investigating various other forms of metamaterials such as artificial piezoelectricity and artificial ferromagnetism. Ferromagnetim is a form of quantum magnetism in oppose to the type of magnetism described by John Pendry et al [7] which is classical magnetism.

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