DESIGN OF FLEXURAL WAVES WAVEGUIDE BASED ON FINITE EMBEDDED COORDINATE TRANSFORMATION THEORY

Xing Chen, Li Cai, Jihong Wen, Haibin Yang

Vibration and Acoustics Research Group, Science and Technology on Integrated Logistics Support Laboratory, College of Mechatronic Engineering and Automation, National University of Defense Technology, Changsha, Hunan, P. R. China, 410073

email: 304590604@qq.com

Coordinate transformation theory offers an unconventional approach to redirecting electromagnetic or acoustic fields at will, and it is expanded to the finite embedded coordinate transformations, which gives a significant amount of flexibilities to coordinate transformation design. In this article, the finite embedded coordinate transformation theory is introduced into the elastic dynamic equation of flexural waves, a design of bend waveguide to control flexural waves to bend at arbitrary angel is proposed. The formula to describe the transformed materials properties in an elastic thin plate is obtained, which contains anisotropic heterogeneous Young modulus and a radially dependent mass density. Through homogenization of layered periodic composite materials, the anisotropic materials are dispersed into discrete layered isotropic materials. Full-wave simulations by using the finite element methods are performed to analyze the behavior of the flexural waves in the finite embedded coordinate transformation device. Results show that the waveguide consisting of 10 layers alternating two types of isotropic elastic materials works over the frequency range [2000, 8000] hertz, it can realize broadband effect on flexural waves control. The flexural waves are redirected to bend at any angle as design by the waveguide, without reflections at the entrance or exit boundary to free space. The finite embedded coordinate transformation theory is useful on flexural waves control. The study can provide technological approaches to flexural waves control in thin plates, and it is expected to provide potential applications in isolating structures from vibrations.

1. Introduction

In 2006, Pendry et al.[1] creatly proposed the coordinate transformation theory, once the theory of coordinate transformation has been put forward, it has aroused wide concern of scholars in related fields. To combine the coordinate transformation theory with electromagnetic metamaterials study, which makes it possible to achieve the characteristics that traditional electromagnetic devices difficult to achieve, and electromagnetic invisible cloak and other novel electromagnetic wave control device can be design with the coordinate transformation theory.

D. Schurig[2] designed the electromagnetic cloak in the microwave band in 2006 and carried out corresponding experiments. Zhang et al.[3] took use of calcite, the anisotropic optical material, to prepared invisible carpet, which makes 2mm thick objects stealth in the optical band.Cui et al.[4] designed a magnetic electromagnetic "black hole" based on electromagnetic materials, and it can achieve the absorption of electromagnetic waves in a particular frequency band. Cummer et al.[5] studied the acoustic equation in the fluid, and the dynamic comparison shows that the kinetic equations of the two are exactly the same in form. Therefore, the sound waves in the fluid can be controlled by the coordinate transformation theory[6].

Electromagnetic waves and elastic waves have the characteristics of interference, diffraction, and researchers in various countries also concentrated their attention on the question to whether the theory of coordinate transformation can be introduced into the elastic wave control. In 2006, Milton et al.[7] analyzed the coordinate transformation of the elastic dynamic equation. It is shown that the general elastic wave equation does not satisfy the form invariance of the coordinate transformation because it includes both the longitudinal wave and the transverse wave terms and the two are coupled with each other, that means the general elastic wave cannot be controlled by the coordinate transformation design theory. Farhat et al.[8] studied the feasibility of introducing the coordinate transformation theory into the thin-plate bending wave control for the first time, and demonstrated the questions to the design of the thin-plate bending wave invisible cloak.

In 2008, Rahm et al.[9] further proposed a finite embedded coordinate transformation, The extension of this theory provides more flexibility for coordinate transformation design.

In this paper, the finite embedded coordinate transformation theory is introduced into the elastic dynamic equation of flexural waves, distribution of materials in an elastic thin plate is calculated, which can make flexural waves deflect at any angle. The simulation model of the waveguide design is built, and it is analyst with finite element methods.

2. Principle description

It is assumed that the dielectric constant and the permeability of the homogeneous medium are ε and μ, respectively. After the coordinate transformation, in the space under the new coordinate system, the dielectric constant $\hat{\varepsilon}$ and the permeability $\hat{\mu}$ satisfy:

$$\begin{aligned} \hat{\varepsilon}(u,v,w) &= J\varepsilon(x,y,z)J^T/\det(J) \\ \hat{\mu}(u,v,w) &= J\mu(x,y,z)J^T/\det(J) \end{aligned} \tag{1}$$

Where (x,y,z) and (u,v,w) represent the coordinates under Cartesian coordinate system and arbitrary curve coordinate system, respectively. The relationship between the two is:

$$\begin{cases} u = q_1(x,y,z) \\ v = q_2(x,y,z) \\ w = q_3(x,y,z) \end{cases} \tag{2}$$

J is the Jacobian matrix, and its representation is:

$$J = \begin{cases} du = \dfrac{\partial u}{\partial x}dx + \dfrac{\partial u}{\partial y}dy + \dfrac{\partial u}{\partial z}dz \\ dv = \dfrac{\partial v}{\partial x}dx + \dfrac{\partial v}{\partial y}dy + \dfrac{\partial v}{\partial z}dz \\ dw = \dfrac{\partial w}{\partial x}dx + \dfrac{\partial w}{\partial y}dy + \dfrac{\partial w}{\partial z}dz \end{cases} \tag{3}$$

After the coordinate transformation, the material parameters such as the dielectric constant and the permeability are converted from the constant form of the uniform material to the general tensor form, and the material parameters show anisotropy in the new coordinate system.

For the bending wave in the thin plate, the kinetic equation is the fourth order partial differential equation [10], writes:

$$D\nabla^2\nabla^2 w + h\rho\frac{\partial^2 w}{\partial t^2} = q \tag{4}$$

Where D is the plate stiffness, ∇^2 is the Laplacian operator, h is the thickness of the plate, ρ is the density of the plate, w is the deflection of the plate, and q is the external force perpendicular to the plate.

To introduce the coordinate transformation shown in Fig. 1 into the equation above.

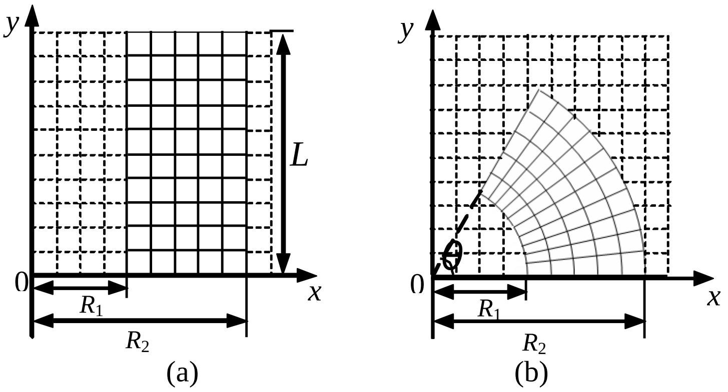

Figure 1: Sketch of the coordinate transformation: (a) Before the coordinate transformation; (b) After the coordinate transformation;

The coordinate transformation can be expressed as the following transformation equation:

$$\begin{cases} r'(x,y) = x \\ \theta'(x,y) = (y/L)\cdot\theta = y/C \end{cases} \tag{5}$$

Where the rectangular area of length L_0 is transformed, $C = L/\theta$, θ is the angle of deflection. (x, y) is the coordinate in the original coordinate system, and (r', θ') is the coordinates in the new coordinate system.

The transformation matrix Λ under the coordinate transformation can be expressed as:

$$\Lambda = \frac{J_{x'x} J_{x'x}^T}{\det(J_{x'x})} \tag{6}$$

Where J^T is the transpose matrix of J, and $J_{x'x} = J_{x'r'} J_{r'x}$.

To combine formula (6) with formula (3) and formula (5) to get the transformation matrix:

$$\Lambda = R(\theta') diag\left(\frac{C}{r'}, \frac{r'}{C}\right) R^{-1}(\theta') \tag{7}$$

According to Milton and Farhat's theory [7, 8], $U'(r', \theta')$ the displacement of the thin plate on the normal direction of the plane satisfies:

$$\nabla \cdot \left\{\xi^{-1} \nabla \left[\beta \nabla \cdot \left(\xi^{-1} \nabla U'\right)\right]\right\} - \beta^{-1} \gamma_0^4 U' = 0 \tag{8}$$

Where $\gamma_0^4 = h_0 \rho_0 \omega^2 / D_0$, ξ is a second diagonal tensor, β is a variable associated with the density of the material, $\xi = \hat{E}^{-1/2}$, $\beta = \rho^{-1/2}$. $\hat{E} = diag(E_r, E_\theta)$ is the tensor form of elastic modulus [11].

Substituting formula (8) into formula (7), the material parameters of the thin plate in the spatial distribution can be expressed as:

$$\begin{cases} E'_{r'} = E_0 \\ E'_{\theta'} = \dfrac{r'^4}{C^4} \cdot E_0 \\ \rho' = \rho_0 \end{cases} \tag{9}$$

According to the equivalent parameter theory of layered periodic composites, elastic modulus anisotropic homogeneous material can be equated by alternating layered media *A* and *B*[12,13].As is shown in Fig. 2:

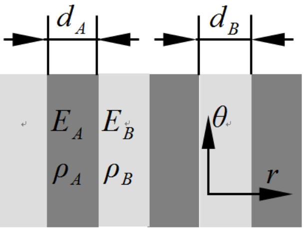

Fig.2. Sketch of the anisotropic lamellar medium

The equivalent relationship can be written as[8]:

$$\frac{1}{E'_{r'}} = \frac{1}{1+\eta}\left(\frac{1}{E_A} + \frac{\eta}{E_B}\right), \quad E'_{\theta'} = \frac{E_A + \eta E_B}{1+\eta}, \quad \rho' = \frac{\rho_A + \eta\rho_B}{1+\eta}, \quad \eta = \frac{d_A}{d_B} \tag{10}$$

Assuming η =1, Further to get:

$$\begin{cases} E_A = \left(\frac{r'^4}{C^4} + \sqrt{\frac{r'^8}{C^8} - \frac{r'^4}{C^4}}\right) \cdot E_0 \\ E_B = \left(\frac{r'^4}{C^4} - \sqrt{\frac{r'^8}{C^8} - \frac{r'^4}{C^4}}\right) \cdot E_0 \\ \rho_A = \rho_0 \\ \rho_B = \rho_0 \end{cases} \tag{11}$$

The design of the bending waveguide can be realized by using the material parameters shown in formula (11).

3. Numerical results and discussion

The simulation model of bending waveguide is built as follows:

$\theta = \pi/6$, L =1m, R_1=2m, R_2=3m . The size of the thin plate is $5\times5\times0.01m$, ρ_0 =2700 kg/m^3 E_0 =72GPa. Poisson's ratio is 0.3. The anisotropic material between R_1 and R_2, is separated into 10 layers, each layer contains one homogeneous anisotropic material, replaced by A and B these two isotropic materials.

Changes of E_A and E_B from R_1 to R_2 along the radial direction is shown in Fig.3.

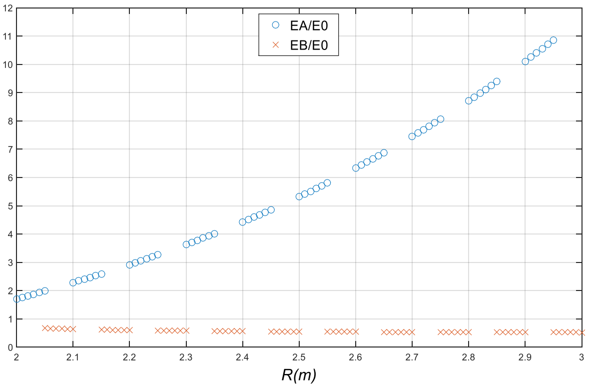

Fig.3. The elastic modulus E_A and E_B change from R_1 to R_2

Using COMSOL for simulation, the excitation source is a sinusoidal excitation between A and B , perpendicular to the plane of the plate, the dynamics simulation results are shown in Fig.4:

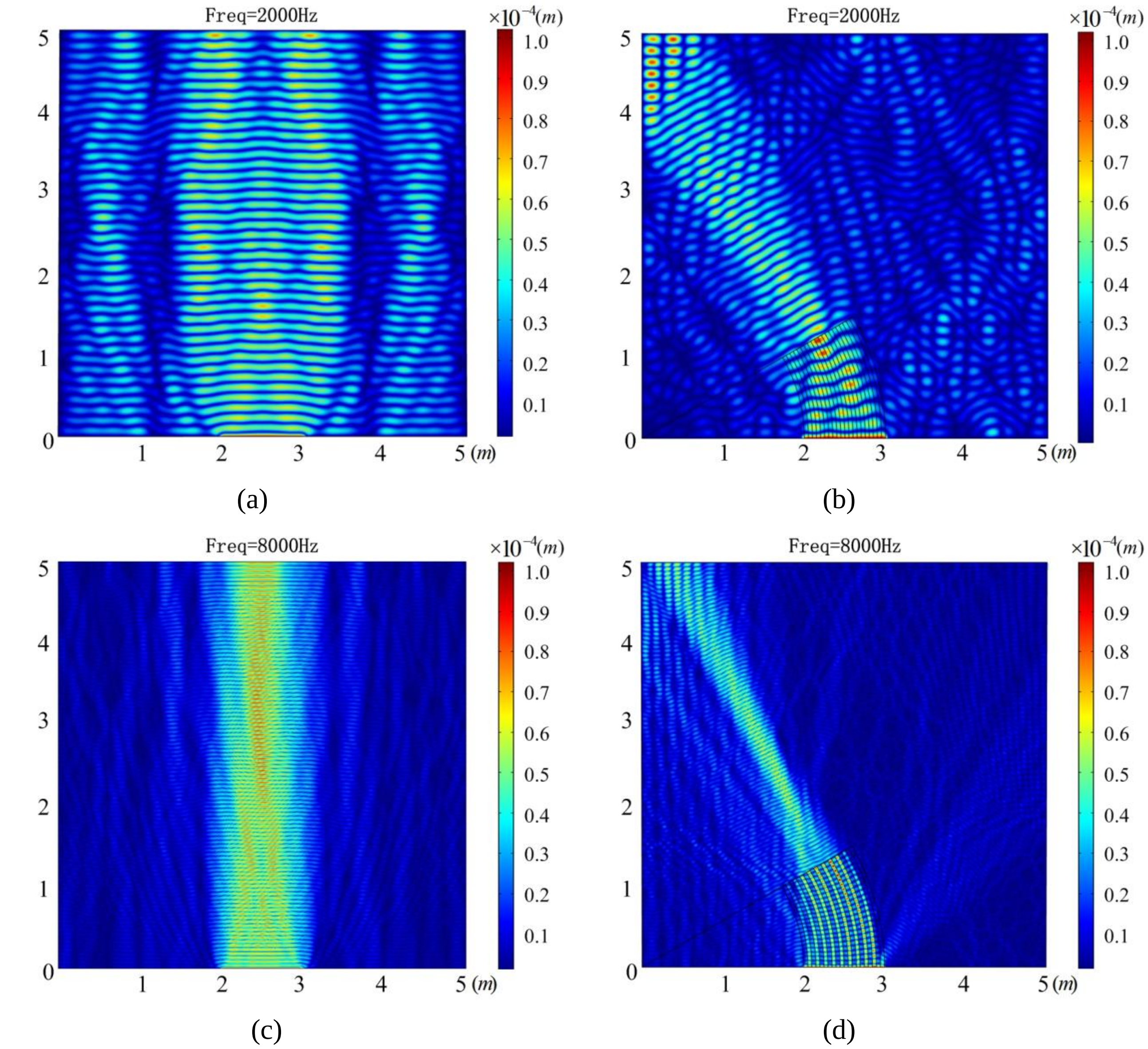

Fig.4. Simulations of the bending waveguide at 2000Hz and 8000Hz(a) The normal displacement of thin homogeneous plate at 2000Hz; (b) The normal displacement of wave-beam with waveguide control at 2000Hz; (c) The normal displacement of thin homogeneous plate at 8000Hz; (d) The normal displacement of wave-beam with waveguide control at 8000Hz

It can be seen from Fig. 4(a) ~ Fig. 4(d), the bending wave is deflected according to the bending path required by the theoretical design. As is shown in Fig. 4(b) and Fig. 4(d) that the vibrational wavefront of the coordinate transformation region always remains flat, the deflection angle is the theoretical design value θ, and the energy of the wave is concentrated in the coordinate transformation area, and there is no scattering from the left and right borders. In addition, after the coordinate transformation area, the bending wave continues to travel along the line, it can be inferred that the bending waveguide can maintain the propagation directivity of the bending wave. And the waveguide can still guide the wave to spread without reflection in a wide frequency range, it has a strong ability on wave field control.

4. conclusion

To summarize, in this paper, the finite embedded coordinate transformation theory is introduced into the elastic dynamic equation of flexural waves, distribution of materials in an elastic thin plate is calculated, which can make flexural waves deflect at any angle. The results of the dynamic simulation analysis show that the finite embedded coordinate transformation design can effectively control the bending wave in thin plate with ultra-wideband characteristics, the research of this paper is expected to provide a new technical approach to the vibration and noise reduction of thin plate structure.

REFERENCES

1 Pendry, J. B., Schurig, D. & Smith, D. R. Controlling electromagnetic fields, *Science*, **312**, 1780–1782, (2006).

2 Schurig D, Pendry J B, Smith D R,et al. Metamaterial Electromagnetic Cloak at Microwave Frequencies[J]. *Science*, **314**-977 ,(2006).

3 Zhang B,Luo Y,Liu X G,et al. Macroscopic Invisibility Cloak for Visible Light[J]. *Physical Review Letters* **106-**033901 (2011).

4 Shen X P, Cui T J, Ye J X *Acta Phys. Sin*. **61** 058101(2012).

5 Cummer S A, Schurig D. One path to acoustic cloaking[J]. *New Journal of Physics* **9**-45(2007).

6 Cummer S A, Popa B I, Schurig D,et al. Scattering Theory Derivation of a 3D Acoustic Cloaking Shell[J]. *PhysRevLett* **100-**024301 (2009).

7 Milton G W, Briane M, Willis J R,et al.On cloaking for elasticity and physical equations with a transformation invariant form[J]. *New Journal of Physics* **8-**248(2006).

8 Farhat M,Guenneau S,Enoch S,et al. Cloaking bending waves propagating in thin elastic plates[J]. *Physical Review B* **79-**033102 (2009).

9 Rahm M, Roberts D A, Pendry J B, Smith D R *Opt. Express* **16** 11556(2008).

10 Timoshenko S, Woinowsky-Krieger S Theory of Plates and Shells （New York: McGraw-Hill Book Company）pp79—83(1959).

11 Farhat M, Guenneau S, Enoch S *Phys. Rev. Lett*. **103** 024301(2009).

12 Torrent D, Sánchez-Dehesa J *New J. Phys*. **10** 063015(2008).

13 Chen H Y, Chan C T *Appl. Phys. Lett.* **91** 183518(2007).