OPTIMAL DESIGN OF HYDRAULIC DYNAMIC ANTI-RESONANCE ISOLATORS FOR TUNING OF THE ISOLATION FREQUENCY RANGE

Xingjian Dong, Han Xiao, Zhike Peng, Guang Meng

Institute of Vibration Shock & Noise, State Key Laboratory of Mechanical System and Vibration, Shanghai Jiao Tong University, Shanghai, China

email: donxij@sjtu.edu.cn

The hydraulic dynamic anti-resonance vibration isolator (DAVI) which is composed of two metal bellows and a supplementary spring possesses an anti-resonance frequency, at which the pressure variation in the fluid induced by the inertial force is in opposite phase to the spring force of the isolator. A schematic design of the multi-layered DAVI which consists of multiple single layered DAVIs is proposed and the corresponding mathematical model is developed. The analytical prediction of the vibration transmissibility of the multi-layered DAVI suggests that the isolation frequency range can be tuned by placing the poles and zeros properly. This work focuses on development of a design method for high frequency isolator with the lowest possible lower cut off frequency, as well as band stop isolator with the widest possible stop-band bandwidth. For this purpose, the cut off frequencies together with the stop-band bandwidths of the single layered DAVI and the multi-layered DAVI are formulated respectively in a general framework. Optimization process is then couducted to achieve a minimum lower cut off frequency for the high frequency isolator and a maximum stop-band bandwidth for the band stop isolator, respectively. It is demonstrated numerically that the lower cut off frequency and stop band bandwidth can both be flexibly designed with structural parameters so as to meet the requirements of high frequency isolation and band stop isolation applications.

Keywords: cut off frequency, dynamic anti-resonance vibration isolator, band stop isolation, high frequency isolation, cut off frequency

1. Introduction

This paper discusses undamped uniaxial vibration isolators in case of harmonic base excitation. For a passive vibration isolator, stiffness and bandwidths properties are the main two topics. Rivin and Harris [1,2] demonstrated that passive vibration isolators performed poorly in low frequency isolation applications owing to stiffness lack. It is also mentioned that this factor affects the active vibration isolators [3]. Passive vibration isolators usually have quite narrow bandwidth, however, bandwidths of the passive vibration isolation isolators have not been analysed in detail [1,4,5].

The term “high frequency isolator” used in this paper means that the isolator achieves isolation in the frequency range beyond a particular frequency known as cut off frequency. These kinds of isolators are used in gravitational wave detection systems [6,7]. The term “band stop isolator” is used to imply that the isolators achieve isolation in a frequency range denoted by stop-band. That is to say, the maximum transmissibility of this kind of isolator in the stop-band should be less than a particular number. Analogously, band stop isolator isolators are regarded as notch filters. Therefore, the cut off frequency and stop-band bandwidth of the isolators can be designed by optimally placing poles and zeros of the multi multi-degree-of-freedom (MDOF) isolation system.

In the 1960s, Flannelly developed a new kind of isolator which used inertial coupling to obtain anti-resonance frequencies at which the isolation was achieved [8-11]. It is proved that the hydraulically leveraged one offers a better solution for the design of low cut off frequency passive isolators with high static stiffness since the hydraulic leverage has a compact arrangement and higher leverage ratio than a mechanical one[12, 13]. DAVI has the potentials in design of passive vibration isolators of lower cut off frequencies and larger bandwidths if they are designed properly [14].

In this paper, cut off frequency formulations will be done for various low-pass filter type vibration isolators and optimum designs will be obtained that outperformed the existing low-pass filter type isolators. Also, bandwidths of band-stop filter type vibration isolators will be formulated. Some optimum designs will be put forward to obtain larger bandwidths. The discussion will be conducted based on the hydraulic anti-resonance vibration isolators.

2. Dynamic models of hydraulic DAVIs

2.1 SDOF hydraulic DAVI of type I

The hydraulic anti-resonance vibration isolators are similar to the well-known lever-type anti-resonance vibration isolators. The hydraulic anti-resonance vibration isolator shown in Fig. 1 consists of two metal bellows and an additional spring. The bellows form a self-contained unit which is completely filled with a low viscosity fluid. The equation of motion is as follows:

$$\left(1+\mu\alpha^2\right)\ddot{x}+\omega_0^2 x=\left(\mu\alpha\left(\alpha-1\right)\right)\ddot{y}+\omega_0^2 y \tag{1}$$

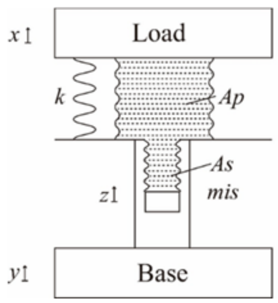

Figure 1: The hydraulic anti-resonance vibration isolator

Here, y is the displacement of the base, x is the displacement of the load, z is the displacement of the isolator mass, k is the mount stiffness, m is the mass of the load, m_{is} is the isolator mass, A_p is the effective cross-sectional area of the primary bellows and A_s is the effective cross-sectional area of the secondary bellows. $\mu=\frac{m_{is}}{m}$ and $\omega_0=\sqrt{\frac{k}{m}}$. The pole ω_p and zero ω_z of the system are

$$\omega_p=\frac{\omega_0}{1+\mu\alpha^2} \qquad \omega_z=\frac{\omega_0}{\sqrt{\mu\alpha(\alpha-1)}} \tag{2}$$

According to definition, α is greater than one. That means the zero is always larger than the pole. The affirmatory relationship may bring many benefits to the design. Then, the base-to-load transmissibility $T(\omega)$ is obtained as

$$T\left(\omega\right)=\frac{1-\omega^2/\omega_z^2}{1-\omega^2/\omega_p^2}=\frac{m-\frac{\omega^2}{\omega_0^2}\left[m_{is}\alpha\left(\alpha-1\right)\right]}{m-\frac{\omega^2}{\omega_0^2}\left(m+m_{is}\alpha^2\right)} \tag{3}$$

2.2 MDOF hydraulic DAVI

A n-dof isolator composed of the hydraulic anti-resonance vibration isolators is shown in Fig. 2.

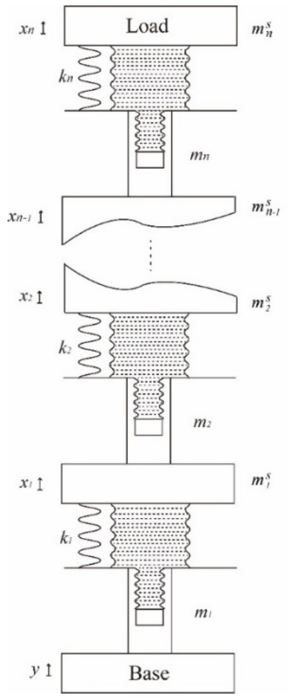

Figure 2: Base excited n-dof hydraulic anti-resonance vibration isolator

In Fig. 2, y is the displacement of the base, x_i is the displacement of the ith stage, m_i^S is the mass of the ith stage, m_i is the mass of the ith isolator, α_i is the area ratio of the ith isolator, and k_i is the spring stiffness of the ith stage. The equations of motion for this system in matrix form are

$$\mathbf{M}\ddot{\mathbf{X}} + \mathbf{K}\mathbf{X} = \mathbf{F} \tag{4}$$

$$\mathbf{M} = \begin{bmatrix} m_1^S + m_1(\alpha_1)^2 + m_2(\alpha_2 - 1)^2 & -m_2\alpha_2(\alpha_2 - 1) & & \\ -m_2\alpha_2(\alpha_2 - 1) & m_2^S + m_2(\alpha_2)^2 + m_3(\alpha_3 - 1)^2 - m_3\alpha_3(\alpha_3 - 1) & & \\ & & \ddots & \\ & & & m_n^S - m_n\alpha_n(\alpha_n - 1) + m_n(\alpha_n)^2 \end{bmatrix}$$

$$\mathbf{K} = \begin{bmatrix} k_1 + k_2 & -k_2 & & & \\ -k_2 & k_2 + k_3 & -k_3 & & \\ & \ddots & & & \\ & & -k_{n-1} & k_{n-1} + k_n & -k_n \\ & & & -k_n & k_n \end{bmatrix}, \mathbf{X} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}, \mathbf{F} = \begin{bmatrix} k_1 y + m_1\alpha_1(\alpha_1 - 1)\ddot{y} \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

we can get n resonance frequencies by solving the eigenvalue problem of the above equations. The n anti-resonance frequencies are in the same form

$$\omega_{zi} = \sqrt{\frac{k_i}{m_i\alpha_i(\alpha_i - 1)}} \tag{5}$$

Hence, the transmissibility can be defined as

$$\frac{\prod_{i=1}^{n}(1 - (\omega_c / \omega_{zi}))^2}{\prod_{i=1}^{n}(1 - (\omega_c / \omega_{pi}))^2} = (-1)^n T_0 \tag{6}$$

3. High frequency vibration isolator

3.1 Cut off frequency

If we regard the isolators as low-pass filters and the isolator acts as high frequency vibration isolator, isolation occurs when the excitation frequency is larger than cut off frequency under the maximum allowable force or displacement transmissibility. Cut off frequency will be formulated by two non-dimensional parameters: T_0 and μ , T_0 is the maximum permissible transmissibility during the isolation frequency range, and μ is the ratio of isolator mass to load mass. According to Fig.1, we can first formulate the cut off frequency of a SDOF hydraulic dynamic anti-resonance vibration isolator.

It can be seen in Eq. (6) that as $\omega\to\infty$, $T(\omega)\to\omega_p^2/\omega_z^2$. Assuming finite ω_z and non-zero ω_p , then $T(\omega)$ converges to a positive number. Therefore, given a maximum allowable transmissibility level, T_0, $T(\omega)$ should converge to T_0 as $\omega\to\infty$. Thus, cut off frequency, ω_c , can be determinded as

$$\omega_c=\sqrt{\frac{\omega_p^2+\omega_z^2}{2}}=\frac{\omega_0}{\sqrt{2}}\sqrt{\frac{1+T_0}{1-\alpha}+\frac{1}{T_0}-T_0} \tag{7}$$

3.2 Optimization of high frequency isolator

There is the opportunity of minimizing the cut off frequency through optimization in an *n*-dof isolator. The aim of optimization is to group the *n* zeros after the highest pole. In that case, there will be $(n-1)$ local peaks in the transmissibility function until the function converges to a constant value. Let us index the peaks in increasing order and let ω_p^i be the *i*th peak frequency. The statement of the optimization problem is as follows:

Minimize ω_c

Subject to $h_i: T(\omega_m^i)=(-1)^{n+i}T_0,\ i=1,2,\dots n-1$

$$h_n: T(\infty)=T_0$$

$$h_{n+1}:\sum_{i=1}^{n-1}m_i^S+\sum_{i=1}^{n}m_i=\mu m$$

$$h_{n+2}:\sum_{i=1}^{n}\frac{1}{k_i}=\frac{1}{k},$$

$$g_1:\omega_{pn}<\omega_{z1}$$

$$k_i\ge k, m_i\ge 0, \alpha_i\ge 1, \text{for } i=1,2,\dots n-1$$

$$m_i^S\ge 0, \text{for } i=1,2,\dots n-1$$

There are $(4n-1)$ variables and $(n+2)$ equality constraints in this problem. In order to obtain some quantitative results, let us try to solve the optimization problem for the case of n=2. There are seven variables in this problem, which are $m_1, m_2, k_1, k_2, \alpha_1, \alpha_2, m_1^S$. Let us focus on the variable m_1^S . This variable represents the mass of the first stage. The product of the poles of this system is given by det(**K**)/det(**M**). Hence, in order to lower the values of the poles, det(**K**)/det(**M**) should be decreased. Since **K** is independent of the variable m_1^S , let us just consider det(**M**). According to Eq. (4), det(**M**) can be calculated as

$$\det(\mathbf{M})=(m_1^S+m_1(\alpha_1)^2+m_2(\alpha_2-1)^2)(m+m_2(\alpha_2)^2-(m_2\alpha_2(\alpha_2-1))^2 \tag{8}$$

m_1^S should be set to zero according to Eq. (8). After elimination of the variable m_1^S , the number of variables in the optimization problem reduces to 6, which are $m_1, m_2, k_1, k_2, \alpha_1, \alpha_2$. There are six variables and four equality constraints. Let us call the variables m_2, k_2, α_1, α_2, as state variables that satisfy the four equality constraints. The remaining two variables, k_1 and m_1, are called the decision variables. The state variables m_2 and k_2 can easily be solved in terms of the decision variables using equality constraints h_3 and h_4, respectively. α_1 and α_2 can easily be determined via Newton's method using the equality constraints h_1 and h_2. To determine the values of the decision variables that minimize ω_c, genetic algorithm based on Matlab software can be used. Results are shown in Tab.1 and Fig. 3 when $k=1, m=1$.

Table 1: Optimization result for μ=0.1, $N=2$, T_0=0.05

m_1	m_2	k_1	k_2	α_1	α_2
0.0294	0.0706	1.6843	5.5369	17.3612	12.7759

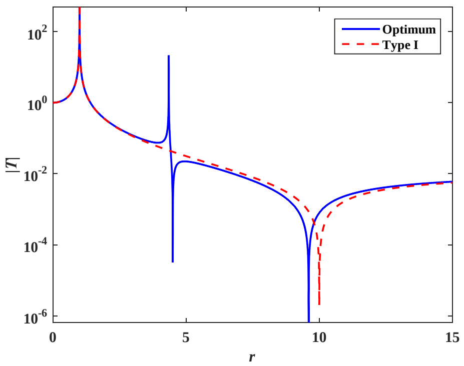

Figure 3: Cut off frequency comparison of the 2dof optimum design

In this case, the normalized cut off frequency of the Type I isolator can be found to be 4.69. Moreover, the normalized cut off frequency of the 2-dof quasi-optimal design is calculated as 4.27. Besides, pole-zero separation is not as critical as before.

4. Band stop vibration isolator

Band stop vibration isolator works in a frequency range denoted by stop-band. In this paper, the frequency range is called bandwidth. We will discuss the bandwidths of hydraulic anti-resonance vibration isolators at low frequencies. In order to make comparisons, a new hydraulic anti-resonance vibration isolator of type II is introduced. The isolator presented in Fig.1 is called type I. The same as above, we formulate the band stop bandwidths by T_0 and α.

4.1 Type II hydraulic DAVI

The hydraulic DAVI of type II is shown in Fig. 4.

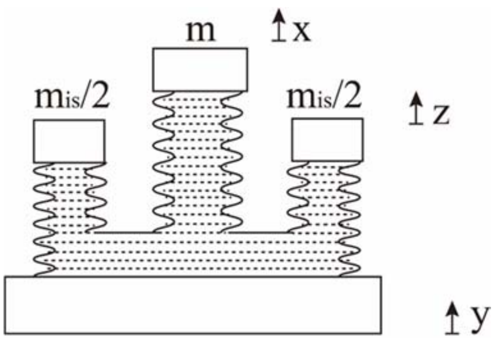

Figure 4: Type II hydraulic anti-resonance vibration isolator

Similar to the previous analysis, the order of the pole and zero is changeable.

$$\frac{\omega_p}{\omega_z} = \sqrt{\frac{m_{is}\alpha(\alpha+1)}{m + m_{is}\alpha^2}} \tag{9}$$

Bandwidth of a type II isolator can be formulated as

$$BW_{\text{II}} = T_0 \left| \frac{1}{N^2} - \frac{1}{\alpha+1} \right| \tag{10}$$

4.2 A design of a 2-dof hydraulic isolator

As mentioned before, both type I and type II isolators have single anti-resonance frequency in their stop-bands. A natural way of increasing the bandwidth of a stop-band is to place additional anti-resonance frequencies. The aim of this part is to synthesize an isolator which has two anti-resonance frequencies placed between two resonance frequencies. It has been proven that the type I isolator always offers a lager zero than pole while the zero-pole position of a type II isolator is uncertain.

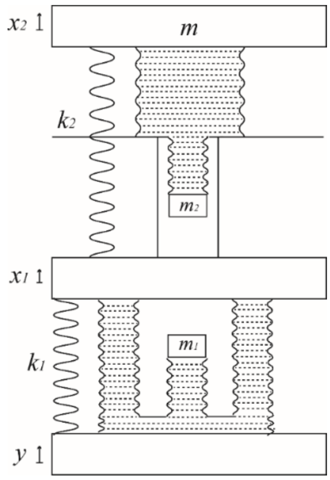

Figure 5: 2-dof hydraulic anti-resonance vibration isolator

Fig. 5 is a schematic design of a 2-dof hydraulic isolator. To obtain the desired pole-zero order in the 2-dof system, one can choose a Type I isolator that has $\omega_{pI} < \omega_{zI}$ and a Type II isolator, which always has $\omega_{zII} < \omega_{pII}$ such that

$$\omega_{pI} < \omega_{zI} < \omega_{zII} < \omega_{pII} \tag{11}$$

In Fig 5, y is the displacement of the base, xi is the displacement of the *i*th stage, m_i is the mass of the *i*th isolator, m is the mass of the load, α_i is the lever ratio of the *i*th isolator stage, and k_i is the spring stiffness of the *i*th stage. the transmissibilit is deduced as

$$T(\omega) = \frac{(k_1 - \omega^2 m_1\alpha_1(\alpha_1+1))(k_2 - \omega^2 m_2\alpha_2(\alpha_2-1))}{(k_1 + k_2 - \omega^2 M_1)(k_2 - \omega^2 M_2) - (k_2 - \omega^2 m_2\alpha_2(\alpha_2-1)^2} \tag{12}$$

Where,

$$M_1 = m_1(\alpha_1+1)^2 + m_2(\alpha_2-1)^2, M_2 = m + m_2\alpha_2^2$$

Let us name the poles and zeros of the 2dof system as $\omega_{z1}, \omega_{z2}, \omega_{p1}, \omega_{p2}$, thus,

$$\omega_{z1} = \omega_{zI}, \quad \omega_{z2} = \omega_{zII} \tag{13}$$

Moreover, let the smaller pole of the 2-dof system be called as ω_{p1} and compare ω_{p1} and ω_{pI}. ω_{pI} is the fundamental frequency of the 2dof system, which has a mode shape such that the both springs are either in tension or in compression. The added compliance from the lower stage decreases the force on the upper stage, which implies that $\omega_{p1} < \omega_{pI}$.

On the other hand, let us compare ω_{p2} and ω_{pII}. ω_{pII} is the higher value pole of the 2-dof system, which has a mode shape such that the springs are working antagonistically, that is, one is in tension while the other is in compression. This increases the force on the lower stage, which implies that $\omega_{pII} < \omega_{p2}$. Therefore, the order of the poles and zeros of the 2-dof system is as follows:

$$\omega_{p1} < \omega_{z1} < \omega_{z2} < \omega_{p2} \tag{14}$$

4.3 Optimization of band stop vibration isolator

Before stating the optimization problem, let us make some definitions. Given T_0 by Eq. (12) and given the maximum allowable transmissibility in the stop-band as T_0 , ω_{s1} and ω_{s2} are the two solutions of $T(\omega) = -T_0$. Since there are two zeros in the stop-band, there is a frequency between the two zeros at which transmissibility attains a local maximum. Let us call this frequency as ω_m. Here is the statement of the optimization problem

Maximize

$$BW = (\omega_{s2} - \omega_{s1}) / \sqrt{\omega_{s1}\omega_{s2}}$$

Subject to

$$h_1 : T(\omega_m) = T_0,$$

$$h_2 : \frac{m_1 + m_2}{m} = \mu,$$

$$h_3 : \frac{k_1 k_2}{k_1 + k_2} = k,$$

$$h_4 : N = \frac{\omega_0}{\sqrt{\omega_{s1}\omega_{s2}}},$$

$$k_1, k_2 \geq k, m_1, m_2 \geq 0, \alpha_1, \alpha_2 \geq 1$$

There are six variables in this problem, which are m_1, m_2, k_1, k_2, α_1, α_2. Regard m_2, k_2, α_1, α_2 as process variables and m_1, k_1 as decision variables. Solve for m_2 and k_2 according to h_2 and h_3, α_1 and α_2 can be established by h_1 and h_4. Given μ, N, T_0, take m and k as 1. Optimazation results are shown in Tab. 2 and Fig. 6.

Table 2: Optimization result for μ=0.1, $N = 2$, T_0=0.05

m_1	m_2	k_1	k_2	α_1	α_2
0.0198	0.0802	1.2826	4.5384	15.3814	15.7640

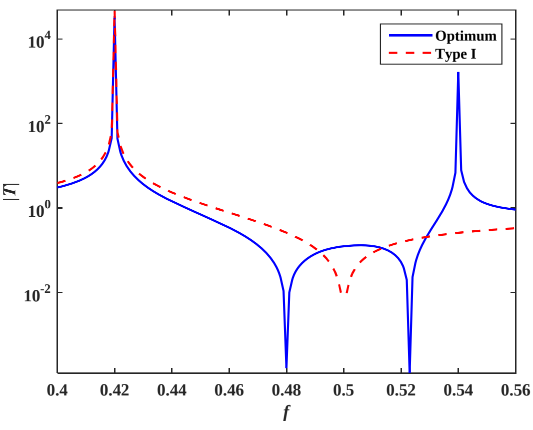

Figure 6: Bandwidth comparison of the 2-dof optimum design (—)and a Type II isolator (--)

It can be seen in Fig. 6 that there are two zeros in a 2-dof isolator system. Given the maximum displacement transmissibly, a larger bandwidth can be obtained by presenting the two zeros. The optimum design has a 2.26 times bandwidth than type II design.

5. Conclusions

In this paper, the vibration transmissibility of two types of DAVI is deduced. It is found that the order of the zero and the pole of the DAVIs can be controled by selecting proper structure parameters. By coupling different type of DAVIs, high frequency vibration isolator and band stop vibration isolator are obtained. It is demonstrated that the minimum lower cut off frequency for the high frequency isolator and the maximum stop-band bandwidth for the band stop isolator can be tuned by optimized design of the DAVIs. The methodolage proposed in this paper will benefit the design of high frequency isolator with the lowest possible lower cut off frequency, as well as band stop isolator with the widest possible stop-band bandwidth.

REFERENCES

1 Rivin, E. I., & Rivin, E. I. Passive vibration isolation (pp. 235-538). New York: Asme press, (2003).

2 Harris, C. M., & Piersol, A. G. Harris' shock and vibration handbook(Vol. 5). New York: McGraw-Hill, (2002).

3 Karnopp, D. Active and semi-active vibration isolation. Transactions of the ASME-L-Journal of Vibration and Acoustics, 117, 177-185, (1995).

4 Macinante, J. A. Seismic mountings for vibration isolation. Wiley, (1984).

5 Schaumann, R. Design of analog filters. Oxford University Press, USA, (2001).

6 J. Winterflood, High Performance Vibration Isolation for Gravitational Wave Detection, PhD Thesis, The University of Western Australia, (2001).

7 J. Giaime, P. Saha, D. Shoemaker, L. Sievers, A passive vibration isolation stack for LIGO: design, modeling, and testing, Review of Scientific Instruments 67 (1), 208 – 214, (1996).

8 Dynamic anti-resonant vibration isolator: U.S. Patent 3,322,379[P]. 1967-5-30.

9 Certain N. Simplified anti-vibration suspension device for a helicopter: U.S. Patent 6,164,915[P]. 2000-12-26.

10 Halwes D R, Simmons W A. Vibration suppression system: U.S. Patent 4,236,607[P]. 1980-12-2.

11 Griffin M D, Hemmen S, Ledbetter M. System and Method of Tuning a Liquid Inertia Vibration Eliminator: U.S. Patent Application 13/337,222[P]. 2011-12-26.

12 Goodwin A J H. Vibration isolators: U.S. Patent 3,202,388[P]. 1965-8-24.

13 Braun D. Development of anti-resonance force isolators for helicopter vibration reduction [J]. Journal of the American Helicopter Society, 27(4): 37-44, (1982).

14 Yilmaz C, Kikuchi N. Analysis and design of passive band-stop filter-type vibration isolators for low-frequency applications [J]. Journal of Sound and Vibration, 291(3-5): 1004-1028, (2006).