# DYNAMIC LOAD IDENTIFICATION ON NONLINEAR DAMPING STRUCTURE USING POSITIVE DEDUCTION APPROACH

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Linear and time-invariant hypotheses are the basic requirements to identify dynamic load in frequency-domain by the traditional dynamic identification method. However, these hypotheses for avoiding the ill-posed problem of inverse transfer function matrix might bring about large errors of identification. To this problem, a novel dynamic load identification method is proposed using positive deduction approach for nonlinear damping structure in this paper. This method starts with predetermined dynamic excited forces to solve the dynamic response of sampling site and the frequency response functions (FRFs), and the obtained FRFs in previous step and dynamic response by experiments will be re-used for further modifying excited forces and FRFs, until the dynamic response errors reach the fulfilling requirements of identification precision. For intuitively descripting this method, a fixed plate attached particle dampers are employed to compose a nonlinear damping structure and to demonstrate the process of load identification. In addition, the identification precision of proposed method is compared with the traditional method. It is worthwhile to indicate that the identification method developed in this paper can accurately identify the dynamic load in a nonlinear damping structure, as a result of avoiding the inverse calculation and revision for the transfer function matrix by the traditional method.
Keyword: dynamic load identification, nonlinear damping structure, particle damper, dynamic response, transfer function matrix

## 1 Introduction

In the structural design process, the knowledge of actually exciting load is a necessary requirement for optimizing structure and characteristic parameters. In consideration of the non-measurable feature of dynamic loads, accurate estimation of loads inspires greater confidence in theoretical investigation, and reduces the reliance on expensive experiments and finite element method for studying the dynamic properties. Therefore, scholars greatly expand their effort to identify the dynamic load by the measured responses, such as dynamic displacements, accelerations and strains, etc.

The dynamic load identification (DLI) methods include various frequency domain and time domain methods. The corresponding algorithms were also developed used professional techniques. Method formulation in time domain rely on the convoluting relationship between the system dynamic response and loads base on the kinetic equation [1-5]. In addition, DLI method in frequency domain rest with the spectrums relationship between the dynamic response spectrums and the frequency response function (FRF) of primary system [6-10]. Comparing with time-domain method, the result of DLI using frequency-domain method have relatively higher accuracy and clear physical meaning. Therefore, frequency-domain method have widely application prospects in engineering.

In 70s last century, Bartlett and Flannelly [11] predicted the lateral and vertical hub loads of experimental helicopter model using the frequency domain method. Giansante [12] predicted the external loads of the spindle and tail rotor of the helicopter used the inverse matrix of frequency response function. Subsequently the inverse identification procedures were widely used for dynamic loads identification. However, inverse identification procedures displayed the defects of ill-posed in some frequency region. For reducing the ill-posed, some methods were proposed to improve the accuracy. O'Callahan [13] used singular value decomposition technique to resolve the ill-posed problem, and proving the effectiveness of the regularization method by the experimental result. Meanwhile, Jia [14] also proposed weighted total least squares method to avoid the ill-pose. Although these methods can resolve the ill-pose problem, the accuracy of dynamic loads identification is tolerated for technical application.

In view of the above-mentioned problem, a novel DLI method is proposed for resolving the problem. The method employed the positive deduction approach (PDA) and co-simulation (CS) technique, and successfully avoided inverse calculation and ill-posed problem. In this paper, the process of proposed DLI method will be fully described.

## 2 Basic theory

A continuous system can be discretized as a MDOF system, its equilibrium equation of motion can be written as:

$$\boldsymbol{M}\ddot{\boldsymbol{x}}(t) + \boldsymbol{C}\dot{x}(t) + \boldsymbol{K}x(t) = \boldsymbol{F}(t), \tag{1}$$

where, $\boldsymbol{M}$ is mass matrix, $\boldsymbol{C}$ is structural damping matrix, $\boldsymbol{K}$ is structural spring matrix, $\boldsymbol{F}$ is external excited force and $x$ is dynamic displacement.

According to frequency response function method, dynamic system is described by the following input–output equation:

$$\boldsymbol{Y}(\omega) = \mathbf{H}(\omega)\boldsymbol{F}(\omega), \tag{2}$$

where $\boldsymbol{Y}$ is dynamic response.

For identifying excited force, both side of equation (2) multiply by the inverse matrix of FRF $\mathbf{H}^{-1}(\omega)$. It can be written as:

$$\boldsymbol{F}(\omega) = \mathbf{H}^{-1}(\omega)\boldsymbol{Y}(\omega). \tag{3}$$

If dynamic system with nonlinear element $C_{\dot{x}}$ (for example particle damper), equilibrium equation of motion can be transformed into the form as:

$$\boldsymbol{M}\ddot{x}(t) + (\boldsymbol{C} + \boldsymbol{C}_{\dot{x}})\dot{x}(t) + \boldsymbol{K}x(t) = \boldsymbol{F}(t), \tag{4}$$

where, $\boldsymbol{C}_{\dot{x}} = [0,\ \ 0,\ \ 0 \ldots c_{\dot{x}} \ldots 0, 0, 0]$ is nonlinear element.

The frequency response function includes the nonlinear element, matrix inversion will become more different and ill-pose problem also is easy to occur. In this paper, PDA is used for overcoming these difficulties. Two particle dampers taken as nonlinear element are installed at the composite plate for describing the proposed method. Recently, C.J. Wu etc. [15, 16] have performed studies to mathematically evaluate the damping mechanisms based on the multiphase flow theory of gas-particle, the feasibility and precision of the prediction methods have been validated on cantilever beam and plate. Therefore, the precise mathematical model is beneficial to estimate the efficiency of proposed method in composite plate. The damping effect is defined as:

$$c_{\dot{x}} = 4\bar{c}(\alpha^{1/2} f^{1/2} |\dot{x}|^{1/2} + \alpha|\dot{x}| - \alpha^{3/2} f^{-1/2} |\dot{x}|^{3/2} + \alpha_1^{1/2} f^{-1/2} |\dot{x}| + \alpha_1 |\dot{x}|^2 - \alpha_1^{3/2} f^{-1/2} |\dot{x}|^3, \tag{5}$$

with $$\bar{c} = \left(\frac{9}{16}\right)\pi^3 d^2 h \rho_m, \tag{6}$$

$$\alpha = \frac{\frac{1}{5}\sqrt{\frac{6}{\pi}}(1-e_p)\alpha_p^2 g_p \rho_p d_p}{\pi d^2 \rho_m}, \tag{7}$$

$$\alpha_1 = \frac{[\alpha_p \rho_p + \rho_p(1+e_p)\alpha_p^2 g_p]\sin\emptyset}{12\pi d^2 \rho_m \sqrt{I_{2D}}}, \tag{8}$$

where $\alpha_p$ is the packing ratio, $\rho_p$ and $d_p$ is the density and the mean diameter of particles respectively, and the other parameters please see Ref. [16].

## 3 Detailed process of PDA

In this section, a detailed process of PDA will be introduced. It is well known that COMSOL Multiphysics is a programmable FEM software and derived from toolbox of MATALB. Users might compile some plug-in programs for special computations using the second-development interface (COMSOL with MATLAB). Herein, the interface will be employed for performing the proposed DLI technique. The detailed process can be shown in Figure 1.

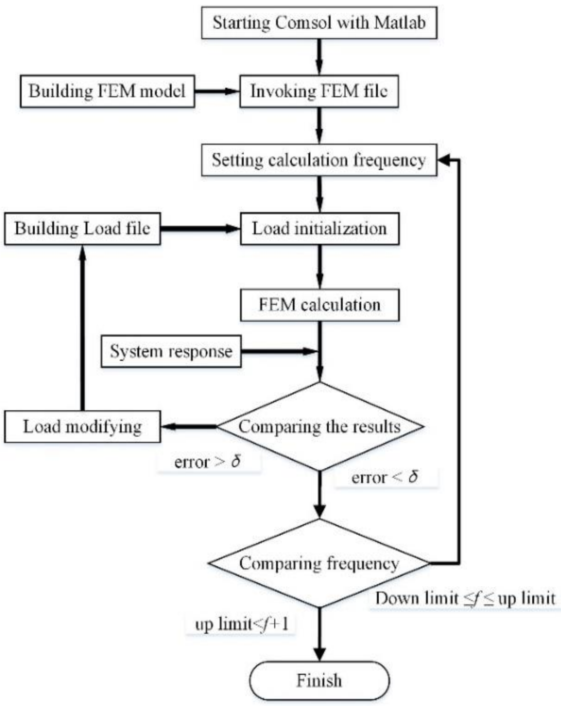



Figure 1: The flowcharts of PDA

The steps are listed as following:

**1st step**: Establishing FEM model in COMSOL. The proposed method invokes FEM software acting as computing module. Then, a FEM file (which includes geometry, material property, boundary conditions, meshing and so on) should be established in COMSOL. Then， the FEM file can be run independently.

**2nd step**: Starting COMSOL with MATLAB. COMSOL has a GUI interface, users might compile some plug-in programs for special computation in MATLAB. After Starting COMSOL with MATLAB, a ‘listener’ will be started on specific port (usually 2036) for ensuring the process of information exchange between COMSOL and MATLAB.

**3rd step**: Invoking COMSOL file in MATLAB. During the process of CS, the responsibility of MATLAB are parameter modification and process control. The command "mphload" will be used for invoking FEM file into MATLAB and reading for identifying dynamic load.

**4th step**: Computing dynamic response. Although COMSOL undertake the calculating task, the calculating process is performed in background and starting the running order must be implemented in MATLAB using the command "model.sol(' resolver name').runAll".

**5th step**: Reading response and comparing with measuring one. The relative error is a common standard to weigh the practicability. It will be used for judged the identification accuracy. The relative error is defined as follow:

$$\delta = \left|\frac{R_s - R_e}{R_e}\right|, \tag{9}$$

where $\delta$ is the absolute value of the relative error, $R_s$ is simulation results of acceleration dynamic response, $R_e$ is experiment results.

**6th step**: Modifying exciting load by obtained simulation FRF. The FRF of nonlinear damping structures is constantly changed by excited force. If the obtained FRF and dynamic response are applied to modify the presupposed excited force, the error of excited force is reduced gradually during an iteration. The equation is as follow:

$$F_{i+1}(\omega) = \frac{R_e}{H_i} = \frac{R_e}{R_{si}} \times F_i(\omega), \tag{10}$$

where $F_{i+1}$ is exciting force of $i$+1th iterative step, $F_i$ is exciting force of $i$ th iterative step, $H_i$ is the frequency response function of $i$ th iterative step by simulation, and $R_{si}$ is acceleration dynamic response of $i$ th iterative step by simulation.

When the relative error of dynamic load is lower than accuracy requirement, the DLI in single frequency is finished. Then, the program will modify the frequency parameter and identify next frequency up to complete the identification process in entire frequency range.

# 4 Experimental verification

## 4.1 Experimental setup

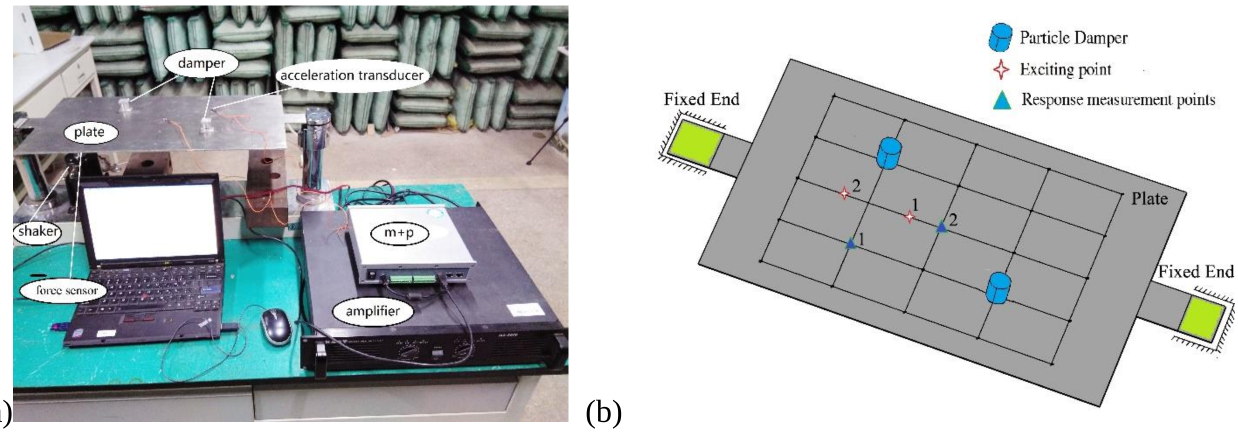


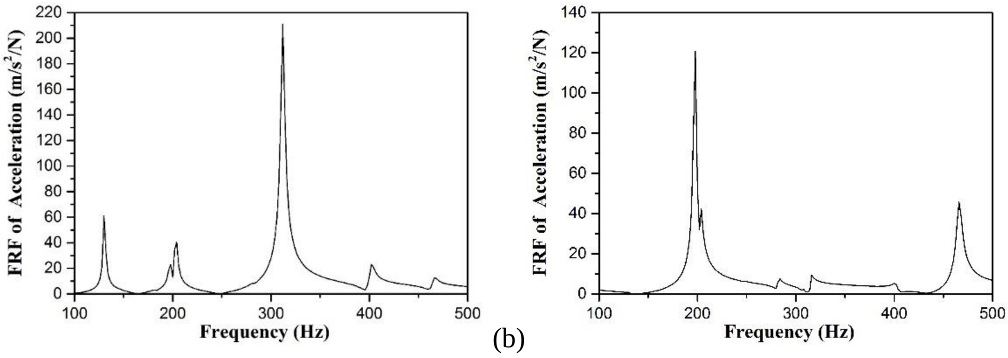

Figure 2: Experiment setup and research object. (a) Experiment setup. (b) Research object.

To validate the feasibility and accuracy of proposed method, an experiment platform is designed for measuring dynamic response and excited force which consist of a Dynamic Signal Analyzer (M+P

SO Analyzer), an amplifier, a smart electromagnetic shaker (MTK2004E01), an acceleration transducer (Dytran3133B1), a force sensor (Dytran1051V4) and a particle damping composite plate. The overall experiment set-up is shown in Figure 2(a). The dimension of composite plate is 550 mm×300 mm×3 mm. The physical parameters are as follows: Young's modulus 69.9 GPa, the density 2690 kg/m$^3$, and the Poisson’s ratio 0.3. And two particle dampers are installed at composite plate denoted by blue cylinders in Figure 2(b). The inner diameter of particle damper is 14 mm, the depth is 20 mm and the mass is 12 g. According to the analysis result of Ref [16], high particle density will achieve better damping effect. As a result, tungsten powder are selected as the particles filled in the dampers whose density is 17000 kg/m$^3$ and the equivalent diameter of the particle is 0.3 mm. In this paper, two group of excited points and measurement points are used for verifying the accuracy of proposed method. The locations is marked at composite plate as shown in Figure 2(b).

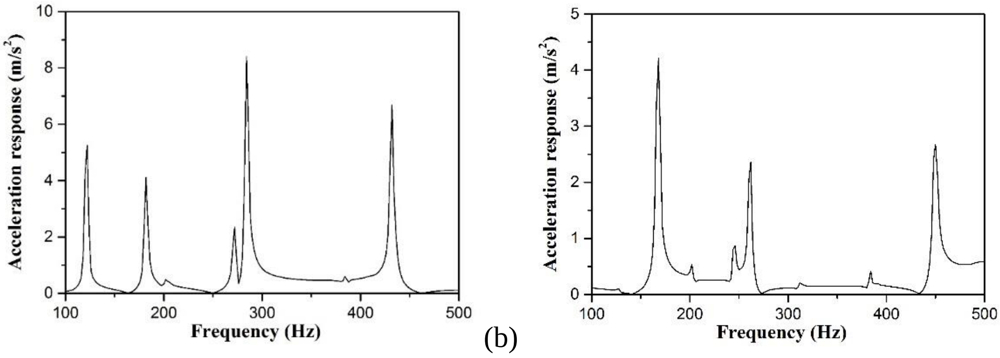
### 4.2 Results comparison

For reducing the measuring error, mathematical averaging method is adopted to process the measurement data, and the measurement are repeated 6 times under the same experimental condition. The frequency response function is shown in Figure 3, and corresponding acceleration dynamic response is given in Figure 4.




(a) (b)

Figure 3: Frequency response function. (a) group 1, (b) group2.




(a) (b)

Figure 4: Acceleration dynamic response. (a) group 1, (b) group 2.

The acceleration dynamic response by simulation will be saved in a txt file and be compared with measuring one used CS program. When the relative error satisfies the accuracy requirement, the excited force in simulation is approbated equaling to experimental excited force. Finally, the identified excited loads by CS technique are shown in Figure 5.

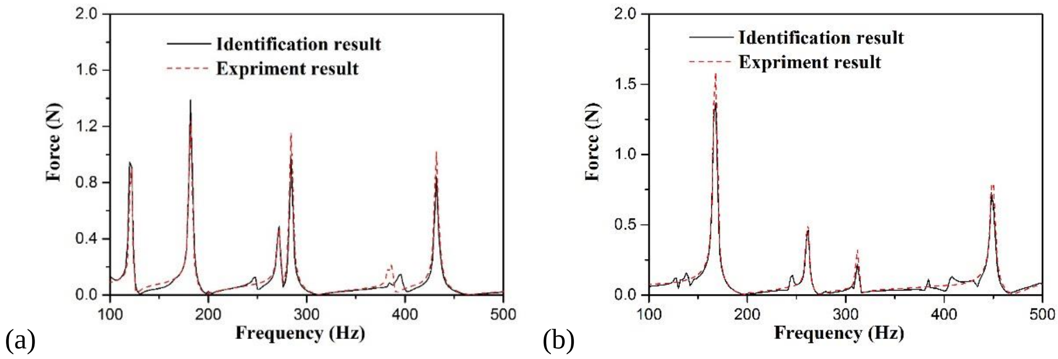



Figure 5: Comparison of the identified load (a) group 1, (b) group 2.

By the contrast of the identification results and test data， it can be noted that identification result is anastomotic with experimental one. However, the real existed error at vibrating formant is inevitable. In engineering, error percentage is common standard to judge error. Then, it will be used for comparing the error of proposed method with additional. The relative percentage is following:

$$\zeta = \frac{\left(\sum_1^n \left|\frac{Fe_i - Fs_i}{Fe_i}\right| \times 100\%\right)}{n}, \tag{11}$$

where $Fs_i$ is identification result of $i$ th frequency, $Fe_i$ is experiment result of $i$ th frequency.

Table 1: Comparing the identified result of exciting point 1

| Method | Identified results of exciting point 1 | | | Relative error percentage |
|---|---|---|---|---|
| | Frequency 1 (182Hz) | Frequency 2 (266Hz) | Frequency 3 (452Hz) | |
| Proposed method | 1.3882 | 0.4901 | 0.8410 | |
| Actual load | 1.2459 | 0.4737 | 1.0195 | |
| Error percentage | 11.4% | 3.4% | 25.6% | 13.5% |
| Tradition method [12] | 20.5% | 23.5% | 35% | 26.3% |

Table 2: Comparing the identified result of exciting point 2

| Method | Identified results of exciting point 2 | | | Relative error percentage |
|---|---|---|---|---|
| | Frequency 1 (182Hz) | Frequency 2 (266Hz) | Frequency 3 (452Hz) | |
| Proposed method | 1.3708 | 0.4642 | 0.7293 | |
| Actual loads | 1.5819 | 0.4959 | 0.7872 | |
| Error percentage | 13.3% | 6.4% | 7.4% | 9.03 |
| Tradition method [12] | 10.1% | 16.3% | 26.1% | 17.5% |

The relative error percentages as listing in Table 1 and Table 2 indicate the identification accuracy is better than tradition method. The relative error percentage reaches double times of the results by tradition method. At key natural frequency (182 Hz, 266 Hz and 452 Hz)，the relative error percentage is also evidently lower the tradition method. These results have verified the proposed method is an accurate and practicable DLI method.

## 5 Conclusions

In this paper, a DLI method is proposed to identify the dynamic load acting at a nonlinear particle-damping plate. The method presupposes initial excited force to solve the dynamic response and FRFs, and the obtained FRF in previous step and dynamic response by experiments will be used for further modifying presumptive excited force, up to dynamic response error fulfilling requirements. Then, eventual presumptive excited force is identification result. For validating the accuracy of proposed method, the method is employed to identify the dynamic excited force acting at a particle damping composite plate. The results indicate the accuracy of proposed method is more excellent than traditional DLI method. Finally, the following conclusions are obtained:

(1) Resolving ill-pose problem. In view of the cause of ill-pose due to inverse calculation of transfer function matrix, the proposed method which combines the PDA and CS technique avoid the inverse derivation successfully, and the ill-pose problem is also resolved.

(2) Excellent identification accuracy. Comparing with traditional method, PDA plays excellent performance in identification accuracy. The identification precision reaches double times of the results by the traditional method. The least error is down to 3.4% at 266 Hz.

It is worthwhile to note that the proposed PDA is only verified by a simple nonlinear structure (i.e. the presented particle damping plate). As a matter of fact, the work presented here could be a valuable tool for obtaining design guidelines in the optimization of dynamic performance of practical structures in engineering. Delightedly, this method have also been applied successfully in the prediction on the load identification of a complicated rack for diesel engine set, and the detailed studied results will be presented in the future.

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