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COHERENCE RESONANCE IN A PREMIXED COMBUSTOR DRIVEN BY A TURBULENCE-INDUCED COLOR NOISE

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Self-excited thermoacoustic oscillations occur in many propulsion systems, such as gas turbines and rocket motors. They are unwanted in these systems due to the detrimental outcomes, which includes the structure vibrations and enhanced thermal load to the combustor wall. Due to the complexity of the operating conditions and combustors' structure, considering the random noisy forcing on the combustion process will be more deeper and accurate to understand the stochastic behaviors of the practical combustion. In the present paper, the effects of turbulence-induced colored noise on the coherence motion of the combustion are theoretically and numerically studied. The resonance-like behavior in signal-to-noise ratio (*SNR*) is a manifestation of coherence resonance, and it is observed as either the noise intensity or the correlation time of the colored noise is varied. In addition, when the combustion system is closer to instability, *SNR* becomes larger and more distinguished.
Keywords: coherence resonance, premixed combustor, turbulence-induced colored noise, signal to noise ratio

1. Introduction

Self-excited thermoacoustic oscillations occur in many propulsion systems, such as gas turbines [1] and rocket motors. The phenomenon is a manifestation of feedback coupling between acoustic disturbances and heat release fluctuation: heat release fluctuation can produce acoustic waves, and part of the reflected waves from the boundary can enter the combustion zone and enhance the unsteady heat release. In order to achieve a good understanding of the mechanism in the thermoacoustic system, knowing the process of transition to instability is significant important.

In practice, studying the stability behaviors of a thermoacoustic system in the deterministic aspect is not enough. Numerous measured experiment data [2, 3] suggests that thermoacoustic oscillations have random features. For example, the amplitude and phase of limit cycle oscillation change from cycle to cycle, or the parameters defining the combustor stability boundaries are varying from test to test. Therefore, considering the stochastic driving in the combustion process will be more appropriate to characterize the thermoacoustic system's response. Actually, interest in combustion noise was initially raised in solid propellant rockets and ramjet engines. In 1986 Poinsot [4] verified the existence of noise in a experimental dump combustor by spatial maps of coherence function. Culick [5] performed an excellent stochastic modelling study, in which the stochastic forcing in the combustor came from the effect of the turbulent flow. By using the stochasticity in the combustor system, Boujo et al [6] excellently confirmed their proposed analytical model and approaches. Noiray and Schuermans [7] made use of the noise-perturbed oscillation data to obtain the linear growth rate in the combustion system near the supercritical bifurcation. Lieuwen et al [8] studied the statistical characteristics of pressure oscillation in a premixed combustor, and obtained a good prediction in probability density function of the amplitude of oscillation. In addition, they [9] also studied the effect of the background noise on the combustion stability, and explained many random features of the combustion driven by external noise perturbation, such as cyclic variability and noise-induced transition. It has also been found that the external noise could not only trigger the linear stable systems [10–12], but also cause stochastic P bifurcation [13]. It is worth of being noticed that most previous analytical investigations are mainly considering the effect of the white noise on the thermoacoustic system's behaviors. Actually, compared with white noise, the colored noise would be more realistic and accurate to characterize the turbulent-induced stochastic forcing [14, 15]. Such colored stochastic forcing is determined by both the noisy intensity and correlation time. It has been found that the correlation time of the colored noise had a significant influence on the statistical properties in some dynamic system [16]. In the present work, the effect of the turbulence-induced colored noise on the coherence motion of the premixed combustor is studied in the vicinity of supercritical Hopf bifurcation point.

It is known [17, 18] that coherence resonance (CR) may incur noise-induced enhancement of deterministic dynamics in some nonlinear systems. When the coherence resonance occurs, the dynamic response of the nonlinear system shows a bell-shape dependence on the noise intensity: an optimum response results from a particular intermediate level of noise intensity. It was first introduced by Hu G et al, and named as 'autonomous stochastic resonance' [19]. CR is a variant of a similar phenomenon known as stochastic resonance (SR), where an external weak deterministic signal can be enhanced by the presence of noise. SR and similar counterintuitive effects of noise have been investigated in both experiments and numerical studies in a variety of nonlinear systems such as biological and chemical systems, lasers, electronic circuits and even geological studies. The noise-induced coherence resonance has been experimentally observed in a premixed combustor [20] near the subcritical Hopf bifurcation point. Lack of systematical analysis of the colored noise-induced coherence resonance in combustion system with supercritical bifurcation motivates the present work.

In this work, the noise-induced coherence resonance is studied in a premixed combustor, when the system is configured in the vicinity of supercritical Hopf bifurcation point and driven by the turbulence-induced colored noise. The signal-to-noise-ratio (SNR) is used to measure the extent of coherent motion of combustion response. The effect of noise intensity and the correlation time of the colored noise on SNR is evaluated theoretically and numerically.

2. Modelling of the premixed combustor driven by turbulence-induced colored noise

Unsteady heat release is an efficient energy source to produce acoustic disturbances. Because of the acoustic impedance change at the boundary, part of these sound waves is reflected back to the combustion zone and further affect the heat release rate [21]. When the unsteady heat release rate and the acoustic disturbances are 'constructively' interacted, thermoacoustic instability occurs. To gain insight on thermoacoustic instability, the basic physics of the momentum and energy equations for the acoustics perturbations are given as [22]

$$\bar{\rho}\frac{\partial \tilde{u}'}{\partial \tilde{t}} + \nabla \tilde{p}' = 0 \qquad (1)$$

$$\frac{\partial \tilde{p}'}{\partial \tilde{t}} + \gamma \bar{p} \nabla \tilde{u}' = (\gamma - 1)\tilde{Q}' \qquad (2)$$

where $\bar{\rho}$, $\bar{p}$ and γ denote the mean density, mean pressure and the specific heat ratio respectively. $\tilde{p}'$ and $\tilde{u}'$ denote the acoustic perturbation and the acoustic velocity. $\tilde{Q}'$ denotes the unsteady heat release rate from the heat source to oncoming air flow. Differentiating Eq. (1) in space and Eq. (2) in time $\tilde{t}$, and then eliminating their cross terms lead to

$$\frac{\partial^2 \tilde{p}'}{\partial \tilde{t}^2} - \bar{c}\,\nabla^2 \tilde{p}' = (\gamma - 1)\frac{\partial \tilde{Q}}{\partial \tilde{t}} \tag{3}$$

In order to make our analysis to be more generalized, Eq. (3) is rescaled in a nondimensional form by introducing the reference scales as follows:

$$\tilde{p}' = \bar{\rho}\bar{c}^2 p',\ \tilde{Q}' = \frac{\rho \bar{c}^2}{(\gamma - 1)R} Q',\ \tilde{t} = \frac{Rt}{\bar{c}}, \tag{4}$$

where $\bar{c}$ and R denote the sound speed and the radius of annular combustor. Considering the annular geometries in gas turbines and acoustic losses from the dissipation at the boundaries and the volumetric losses, Eq. (3) can be modified as

$$\frac{\partial^2 p}{\partial t^2} + \alpha \frac{\partial p}{\partial t} - \frac{\partial^2 p}{\partial \theta^2} = \frac{\partial Q}{\partial t} \tag{5}$$

where α denotes acoustic damping coefficient. Galerkin series is applied to expand the acoustic disturbances here, so that the behaviors of thermoacoustic system can be studied by using dynamic theory. It is assumed that the unsteady pressure oscillation is dominated by a single mode, so $p'(\theta, t) = \varphi(\theta)\eta(t)$. $\varphi(\theta)$ denotes the unstable mode shape in the spatial distribution and $\eta(t)$ denotes the time-dependent variable. Based on the works [22, 23], the unsteady heat release rate to acoustic forcing can be modeled by a third order polynomial, as shown as

$$Q' = \beta p' - l p'^3 \tag{6}$$

where β and l are two constant coefficients. Considering the effect of the external turbulence-induced noise perturbation [14], Eq. (5) can be modified as

$$\ddot{\eta} - 2v\dot{\eta} + \frac{9l}{4}\eta^2\dot{\eta} + k_0^2\eta = \varepsilon(t) \tag{7}$$

where $v = (\beta - \alpha)/2$. $\varepsilon(t)$ denotes the turbulence-induced colored noise. It can be mathematically generated by the linear Ornstein-Uhlenbeck (OU) process [**?**], as shown as

$$\dot{\varepsilon}(t) = -\frac{1}{\tau_c}\varepsilon(t) + \frac{\sqrt{2D}}{\tau_c} n(t) \tag{8}$$

where τ_c denotes the correlation time of the colored noise, and D denotes the noise intensity. $n(t)$ is Gaussian white noise with zero mean and unit variance. Such colored noise has the statistical properties, as shown as

$$< \varepsilon(t) >= 0 \quad \text{and} \quad < \varepsilon(t)\varepsilon(t') >= \frac{D}{\tau_c}\exp\left(-\frac{|t - t'|}{\tau_c}\right). \tag{9}$$

where $<>$ denotes the operator of expectation. Based on [17], the power spectral density (PSD) of η can be obtained, when the combustion system is operated in the vicinity of the supercritical Hopf bifurcation, as shown as

$$PSD(k) \approx \frac{S(k_1)}{8\lambda_1 k_1^2}\frac{\lambda_1 + \lambda_2}{(\lambda_1 + \lambda_2)^2 + (k - k_1)^2} + \frac{r_m^2}{2}\frac{\lambda_2}{\lambda_2^2 + (k - k_1)^2} \tag{10}$$

where $S(k_1)$ denotes the power spectrum density of the colored noise Eq. (8). $k_1 = \sqrt{k_0^2 - v^2}$. $\lambda_1 = 2\sqrt{v^2 - C_r S(k_1)/k_1^2}$. $\lambda_2 = S(k_1)/4r_m^2 k_1^2$. $r_m = \sqrt{-\left(v + \sqrt{v^2 - C_r S(k_1)/k_1^2}\right)/2C_r}$ and

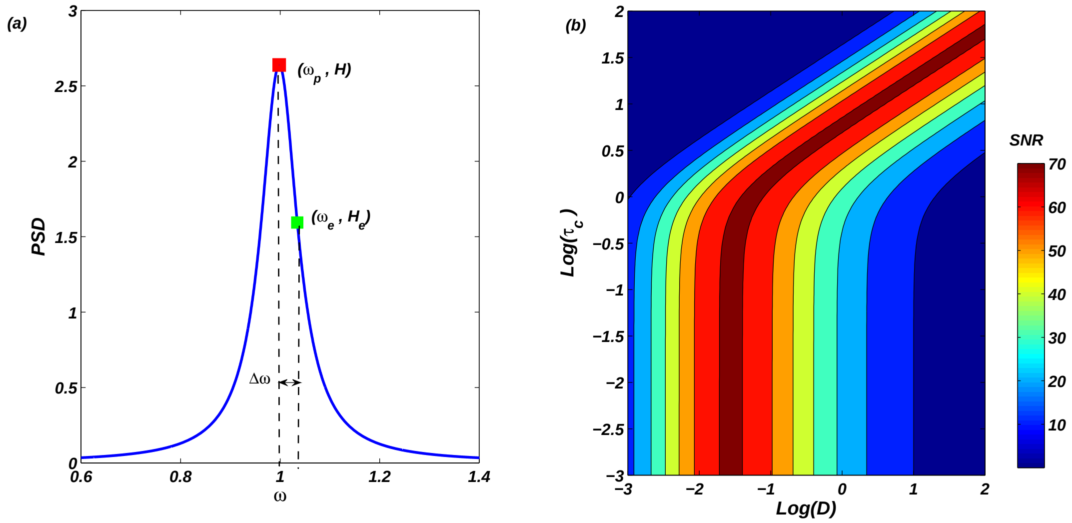

Figure 1: (a) The power spectral density of $\eta(t)$; (b) Contour plots of *SNR* in the plane of the noise intensity D and correlation time τ_c, as $\upsilon = -0.02$, $k = 0.2$.

$C_r = -1/8$. The performance of noise-induced oscillations (NIOs) is characterized by *SNR*, which is defined as

$$SNR = H(\Delta\omega/\omega_p)^{-1} \tag{11}$$

where H denotes the peak hight of the power spectrum. ω_p is the corresponding frequency, as illustrated in Fig. 1a. $\Delta\omega$ is the width of the peak at the hight $H_e = e^{-1/2}H$. Based on Eq. (10), the analytical expression of *SNR* can be obtained.

3. Results and Discussion

In this section, the effects of turbulence-induced colored noise on the coherence motion of the premixed combustion are studied. If the colored noise is not considered, then the combustion system becomes unstable, if the critical parameter υ passes through the supercritical Hopf bifurcation point, i.e., $\upsilon = 0$. As $\upsilon > 0$, the system becomes unstable and periodic oscillation occurs. In the present paper, noise-induced motion is both theoretically and numerically studied in the vicinity of the supercritical Hopf bifurcation point, i. e., $\upsilon < 0$, where the deterministic combustion is linearly stable. In order to verify the theoretical findings, numerical simulations are also conducted. The time step is 0.001 and 400 different trajectories are chosen to estimate the statistical average of system's steady response.

Firstly, the dependence of *SNR* on the noise intensity D and correlation time τ_c is studied theoretically in Fig. 1b and numerically in Fig. 2, as $\upsilon = -0.02$. From Fig. 1b, it is easy to see that the maximum SNR is located in the center of the colored ribbon. As the correlation time τ_c is changed, the red-centered behavior only occurs at larger noise intensity D (larger than $D_c \approx 10^{-1.5}$). Three representative cases are numerically simulated in Fig. 2(a). When $D{<}D_c$, e.g., $D = 0.01$, as τ_c is increased, SNR decreases monotonically. No resonance-like behavior is observed. However, for $D{<}D_c$, e.g., $D = 0.1$ and 0.3, bell-shape behaviors are both observed for SNR with respect to the correlation time τ_c.

Furthermore, Fig. 1b shows that the optimal noise intensity D remains almost the same when the correlation time τ_c is small. However, if the correlation time τ_c is larger than a threshold value (around $\tau_c = 10^{-0.5}$), an increase in correlation time τ_c will almost linearly increase the optimal noise intensity D, which is supported by the numerical results in Fig. 2(b). Secondly, the dependence of *SNR* on the system's parameter υ is evaluated theoretically in Fig. 3 and numerically in Fig. 4. It

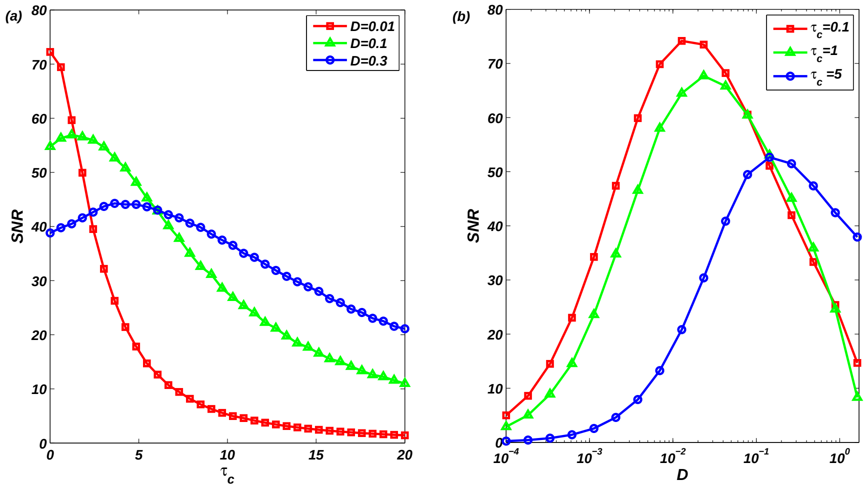

Figure 2: (a) The dependence of *SNR* on the autocorrelation time τ_c, with different noise intensity D; (b) The dependence of *SNR* on the noise intensity D, with different autocorrelation time τ_c, as $v = -0.02$, $k = 0.2$

is worthy of being noticed that $|v|$ is a measure of distance from the supercritical bifurcation point. So it can be used to characterize the distance from stability to instability. Fig. 3(a) and (b) studies the dependence of *SNR* on (v, τ_c) and (v, D). Clearly, the closer the system is configured to the supercritical bifurcation point, the larger the *SNR* is. Specially, from Fig. 3, it can be seen that the bell shape behavior only appears in the vicinity of $v = 0$. When the coherence resonance occurs, larger $|v|$ leads to larger optimal correlation time. This variation agrees well with the numerical results in Fig. 4a. For Fig. 3.b , it can be seen that the bell shape behaviors can be observed in the whole range of v we considered. With the increase of v, the optimal noise intensity approaches to larger value, which is confirmed by the numerical simulation in Fig. 4b. At the meantime, the resonance-like behavior will also become more difficult to identify.

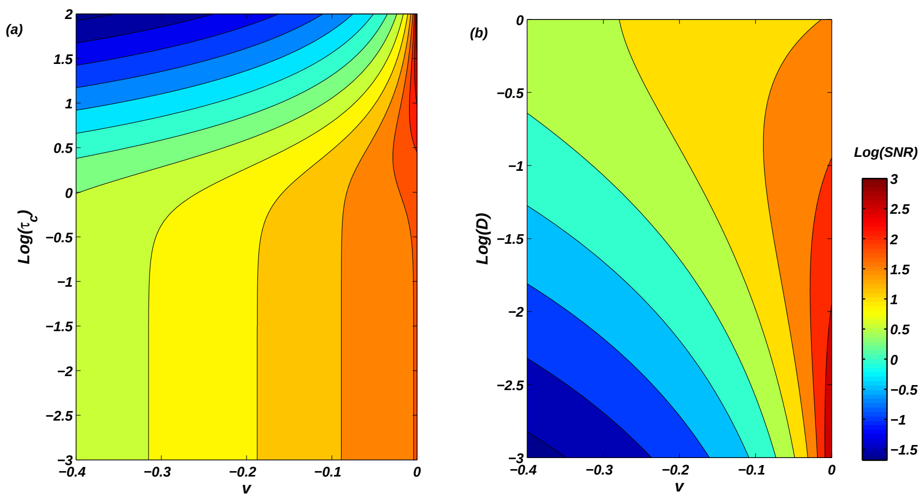

Figure 3: (a) Contour plots of *SNR* in the plane $(v, \tau_c,)$, as $D = 0.1$, $k = 0.2$; (b) Contour plots of *SNR* in the plane (v, D), as τ_c=0.1, $k = 0.2$.

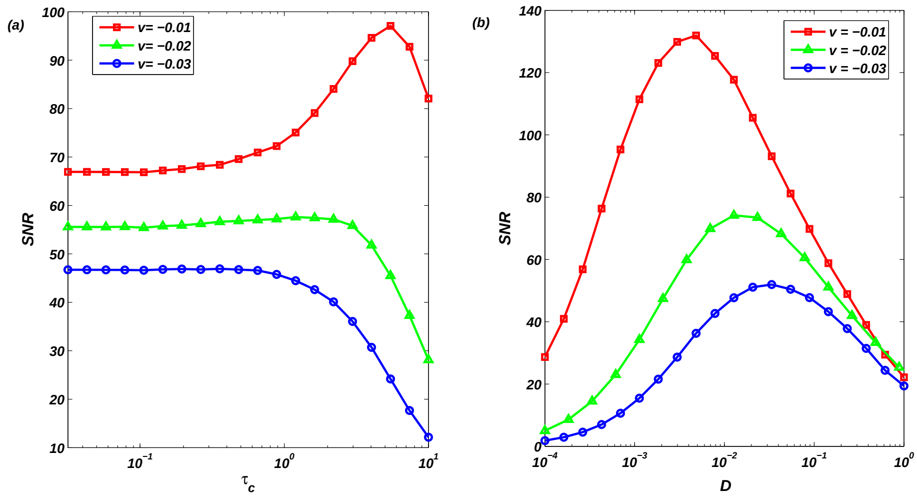

Figure 4: (a) The dependence of *SNR* on the correlation time τ_c, as D=0.1, k=0.2; (b) The dependence of *SNR* on the noise intensity D, as $\tau_c = 0.1$, $k = 0.2$.

4. Conclusions

In this work, the coherence motion of a premixed combustion is theoretically and numerically studied, when the system is configured in the vicinity of supercritical Hopf bifurcation points. The turbulent-induced colored noise is considered, and *SNR* is used to measure the extent of the coherence motion. It has been found that both the noise intensity and the correlation time of the colored noise can lead to resonance-like behaviors in *SNR*, which denote an occurrence of the coherence resonance. But the bell-shape resonance with respect to the correlation time only occurs for large noise intensity. It is also found that the extent of the system's coherence resonance is also significantly affected by the combustion's operating condition. When the combustion system is more closer to instability, *SNR* becomes more larger and the bell shape becomes more distinguished.

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