
INVESTIGATION OF ELLIPTICAL SONIC METAMATERIALS

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Sonic metamaterials have important applications in both military and commercial fields such as noise control, sonar technology, submarine stealth and acoustic communication. As for sonic metamaterials with elliptical scatterers, different arrangements of scatterers will influence the band gap properties of sonic metamaterials. By placing multiple elliptical scatterers in a unit cell, multi-oscillators sonic metamaterials are constituted. By using finite element method, the band gaps and effective parameters are investigated. The results show that the same kind of elliptical scatterers with different spatial arrangements correspond to different band structure, which is obviously different from the traditional circular scatterers sonic metamaterials. Moreover, it found that the coupling effect among the multi-oscillators strongly affected the band-gap formation. In this paper, it provides a new idea for the design and preparation of sonic metamaterials.

Keywords: sonic metmaterials; scatterers arrangements; finite element methods

1 Introduction

Acoustic metamaterials are specific artificial composite materials that possess certain extraordinary acoustic characteristics (e.g., negative effective mass density, negative effective modulus) [1]. Acoustic metamaterials can break through the law of mass density [2-4], readily facilitate wider band gaps in low frequency, and simultaneously achieve the control of long wavelength sonic wave using smaller structural size. Further studies discovered the negative parameter characteristics in acoustic metamaterials, which means it can be achieved for the design of particular sonic devices. Therefore, the investigation for acoustic metamaterials had profound significance in many respects, for instance, the control of sonic and elastic waves, the design and achievement for new material and specific device. However, the study found that, for single negative acoustic metamaterials, its frequency band is narrow [5, 6], and the regulation factors are less. Meanwhile, the double negative acoustic metamaterials [7, 8] must have complex structure, resulting in relatively difficult for achievement. The investigation for the problem about how to

widen the band gap is less [9]. The narrow bandgap width becomes the key issue for application of acoustic metamaterials.

The local resonant bandgap can be broadened by adjusting the elastic constants of the component and the local resonant element geometry, but most of these practices are based on changing the intrinsic properties of the local resonant unit, the effect on band gap extension is not significant [10-12]. Researchers have conducted studies on acoustic metamaterials, such as Helmholtz metamaterials, acoustic super surface, cylindrical metamaterials and defective metamaterials[13-16], and found that the coupling effect in unit interior can facilitate the achievement of broad bandgap and superior acoustic properties. Otherwise, the extraordinary acoustic properties of the acoustic metamaterials can be controlled by the design of cell unit. Sheng et al. [17] demonstrated that the acoustic properties of the local resonant acoustic metamaterials do not depend on the strict periodic structure and symmetry in the study of the acoustic properties of local resonant structures, but this conclusion is based on the study of traditional acoustic metamaterials with cylindrical scatterers (2D) or spherical scatterers (3D). When the shape of the scatterer is not cylindrical or spherical, but the ellipse, the symmetry of a single scatterer is broken, causing different resonance characteristics in different directions and the existence of coupling effect between different scatters. Hence, the arrangement of scatter may affect the width of bandgap, which in turn affects the extraordinary acoustics characteristics of the acoustic metamaterials.

In this paper, we introduce elliptical scatterers into acoustic metamaterials to study the coupling effect of elliptical acoustics and metamaterials under different arrangement of elliptical scatterers, and in the multi-oscillator structure. The change of bandgap when the vibrator changes is given.

We introduced elliptical scatterer into acoustic metamaterials to study the coupling effect under different arrangement of elliptical scatterer. Moreover, we ascertained how the coupling effect of oscillators influence band structure. The interaction between the oscillators at a plurality of frequencies was obtained. And the variation law of the band gap when oscillators changing was investigated.

2 Research theory and model construction

In the existing research, the multi-oscillator acoustic metamaterial system is complex and has many degrees of control freedom. Its formation mechanism of special acoustic properties is more complicated and difficult to be studied systematically. Through the simplification of model and the comparison for research, we designed a new elliptical acoustic metamaterial model by introducing a series of elliptical scatterers in the structural unit, which is composed of different permutations and combinations. The schematic diagram of a single elliptical scatterer is shown in Figure 1.

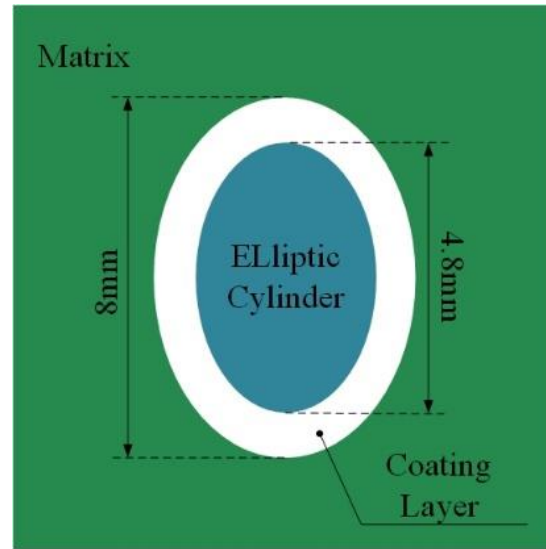


Figure 1. Schematic diagram of the single cell of the elliptical scatterer

The structure consists of substrate, coated soft layer, central column body, the elliptic cylinder center for steel materials, steel column short axis is 4.8mm and the long axis is 6.4mm. Among them, the coating layer is soft silicone rubber, the scatterer short axis is 6mm, long axis is 8mm, which has the matrix for epoxy resin.

Table 1 Mass density and elastic constants of materials

Materials	Epoxy	Si-rubber	Steel
Mass density, ρ (kg/m ³)	1180	1300	7780
Young's modulus, E (MPa)	0.435×10^{10}	1.175×10^5	21.06×10^{10}
Shear modulus, μ (MPa)	0.159×10^{10}	4×10^4	8.1×10^{10}

In this paper, the lumped mass method is used to study the energy band structure and vibrational modes of elliptical acoustic metamaterials. Compared with other methods (such as plane wave expansion method, transfer matrix method and the finite difference time domain method), the lumped mass method has the advantages of good convergence and high calculation accuracy.

The problem of continuous system is transformed into a discrete problem by limiting the mass in continuous medium to finite node or section. Using the basic idea of the lumped mass method, meanwhile, learning from the method of calculating the stiffness matrix of the discrete element with the finite element method, introducing boundary conditions and the Bloch theorem, we investigated the two-dimensional acoustic metamaterials. For two-dimensional acoustic metamaterials, the displacement can be divided into in-xy-plane and out-xy-plane. In order to simplify the calculation, the Bloch model is used to simplify the calculation model to a cell, then the dispersion relation of the whole structure can be obtained by solving the scattering relation of the Brillouin zone boundary. When calculating transmission loss, only finite element is considered in one direction, and the periodic boundary conditions are considered in other directions. Assume that the plane wave incident to the surface of the material, the transmission coefficient is used to characterize the transmission loss of the elliptical scatterer. At the same time, the perfectly matched layer is used to characterize the infinite medium in the incident direction to reduce the reflection echo at the boundary. According to the equivalent medium theory, in the long wave assumption, the acoustic

metamaterials can be regarded as homogeneous materials with special properties. And the dynamic behavior of the system is described by the equivalent medium parameters to determine the propagation of acoustic waves. In this paper, the partial calculation refers to the Wang [18,19], using lumped mass method to study the energy band structure of two dimensional square lattice acoustic metamaterials. And the physical model is constructed by using the finite element software Comsol and the vibration modes are simulated.

In order to study the influence of the arrangement of elliptical acoustic metamaterials on the band gap characteristics, we introduce a different number of scatterers in a cell in and arranged in different ways, according to the number of scatterers are called single-oscillator, dual oscillator, triple-oscillator, quadruple-oscillator and six vibrator elliptical acoustic metamaterials, cellular structure diagram is shown in figure 2. In addition, in order to eliminate the influence of filling rate, they have the same filling rate of 37.7%.

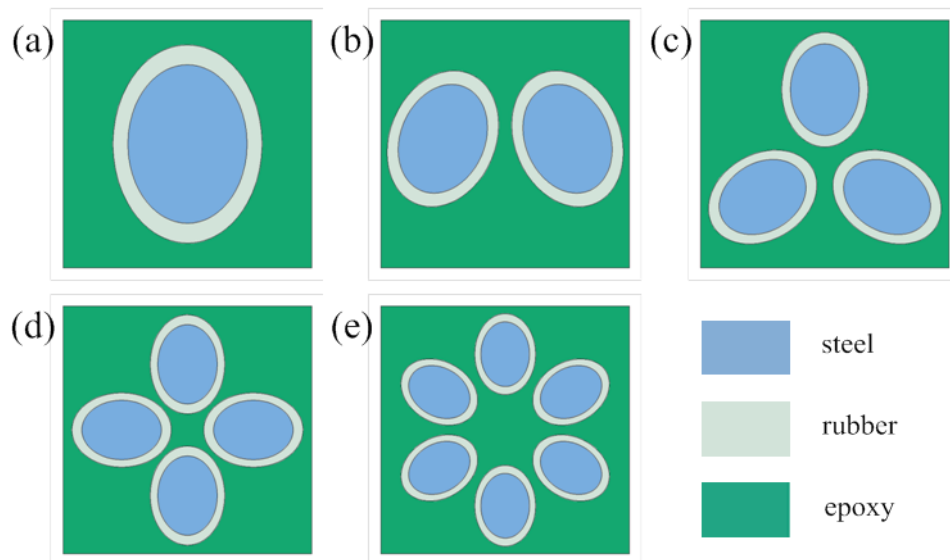


Figure 2. Cellular structure elliptical acoustics metamaterial: (a) single-oscillator; (b) dual-oscillator; (c) triple-oscillator; (d) quadruple-oscillator and six-oscillator elliptical acoustic metamaterials.

3. Results and discussion

As mentioned above, we built a single-oscillator, dual-oscillator, triple-oscillator, quadruple-oscillator, and six-oscillator system. In order to study the acoustic properties of elliptical acoustic metamaterials, we calculated their band structure respectively.

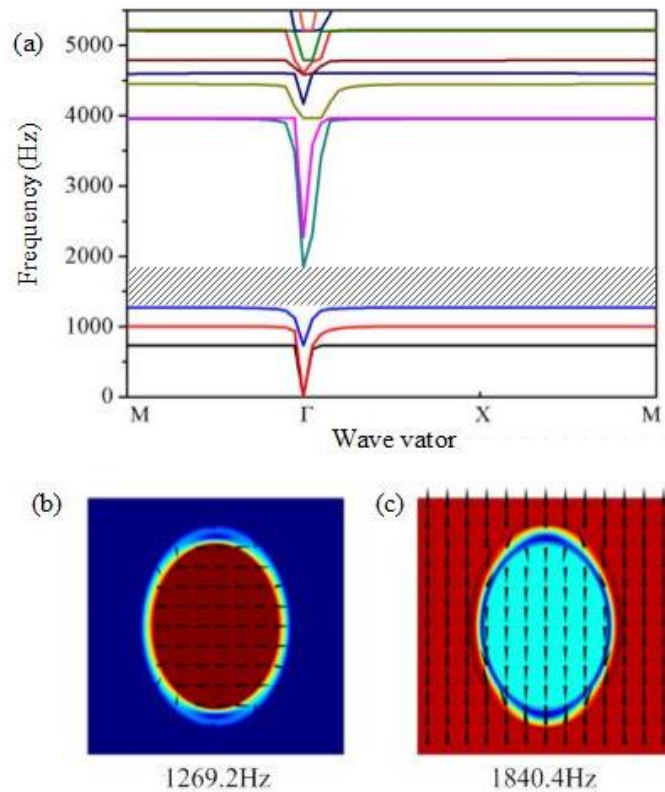


Figure 3. (a) Band structure of single-oscillator elliptical acoustic metamaterial; Vibration modes at lower edge (b) and upper edge (c) of the first band gap.

The band structure of a single-oscillator elliptical acoustic metamaterial is shown in Fig. 3 (a). It can be found there is just one bandgap when the cell contains only one scatterer, with the frequency range from 1269.2Hz to 1840.4Hz. Fig. 3 (b) and (c) show the corresponding vibration modes at the band gap edges, respectively. At the lower edge, steel columns moved along minor axis and the matrix remained almost completely still, where the coating layer connected the steel column and the matrix like a spring. At the upper edge, the matrix and steel columns moved along opposite directions and the coating layer still acted as a spring. It can be seen from Fig. 3 (a) that the Fano interference phenomenon [20] (the response spectrum has an antisymmetric formant) appears in the band structure of single-oscillator elliptical acoustic metamaterial, which leads to the smaller attenuation of the wave in local resonant bandgap, and a limit of the bandgap width.

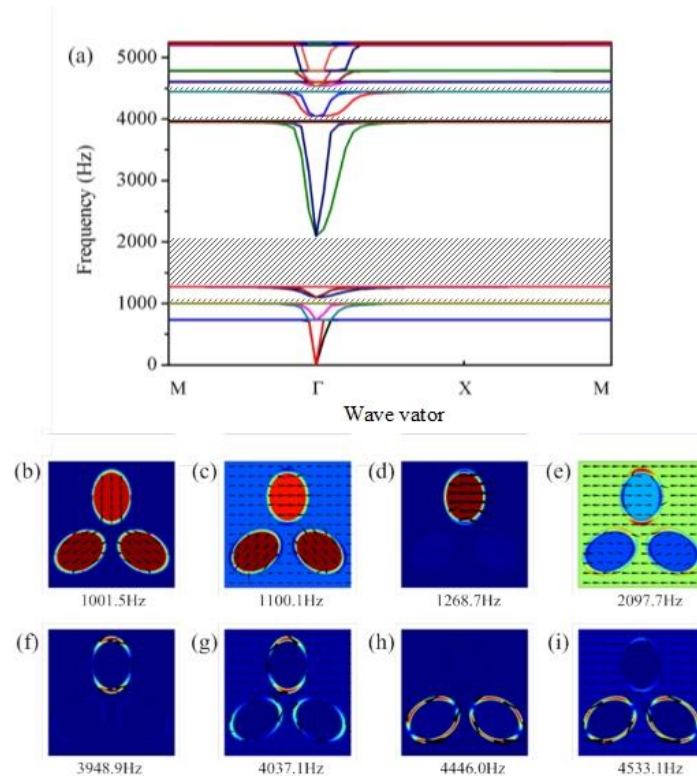


Figure 4. (a) Band structure of triple-oscillator acoustic metamaterial; vibration modes at the lower edge (b) and upper edge (c) of the first band gap; vibration modes at the lower edge (d) and upper edge (e) of the second band gap; vibration modes at the lower edge (f) and upper edge (g) of the third band gap; vibration modes at the lower edge (h) and upper edge (i) of the fourth band gap of the triple-oscillator acoustic metamaterial system.

Figure 4(a) shows the band structure of the triple-oscillator acoustic elliptical metamaterial, which forms a bandgap in four frequency ranges from 1001.5Hz-1100.1Hz, 1268.7Hz-2097.7Hz, 3948.9Hz-4037.1Hz and 4446.0Hz-4533.1Hz. And there's one more bandgap than dual-oscillator elliptical acoustic metamaterial with different bandgap frequency range. To analyze the physical essence of their difference, we have investigated the vibration mode in this paper. As shown in Figure 4 (b)-(i), Figure 4 (b) gives the vibration mode at the lower edge of the first band gap, where three scatterers vibrate along the long axis but the matrix does not move. Figure 4 (d) is the vibration mode at the lower edge of the second band gap. The three scatterers vibrate along the minor axis, but their amplitudes are different and the matrix doesn't move. Figure 4 (c) and (e) are the vibration modes at the upper edge of the first bandgap and the second bandgap, respectively, where the vibration modes are different from each other. Figure 4(f) shows the vibration mode at the lower edge of the third bandgap. The soft layers of the three scatterers vibrate along the minor axis, and the vibration amplitudes are different. Meanwhile, the matrix and steel column are fixed. Fig. 4 (g) shows the vibration mode at the upper edge of the third bandgap where the soft layer vibrates along the minor axis while the matrix vibrates and the steel column doesn't move. Figure 4 (h) is the vibration mode at the lower edge of the fourth bandgap. The soft layers of the three scatterers vibrate along the long axis and the amplitudes of the vibrations are different. The matrix and the steel column doesn't move; Figure 4 (i) is the vibration mode at the lower edge of the fourth bandgap, the soft layer vibrates along the long axis while the matrix also vibrates, and the steel

column is basically fixed.

Compared with the dual-oscillator elliptical acoustic metamaterials, it can be found that: (1) In the vicinity of 1001 Hz, for the dual-oscillator and the triple-oscillator elliptical acoustic metamaterials, The first band gap is induced by vibration of the steel column along long axis; (2) For the dual-oscillator and the triple-oscillator elliptical acoustic metamaterials, the second bandgap is induced due to the vibration of steel column along the minor axis near 1268 Hz, but the latter's bandgap is wider about 160.5Hz than the former; (3) There's the third bandgap in triple-oscillator elliptical acoustics metamaterials because of vibration of the soft layer along the minor axis at 3948.9Hz, while the dual-oscillator elliptical acoustics don't generate the third bandgap, although soft layer vibrates along the minor axis; (4) At 4445 Hz, , bandgap exists in the dual-oscillator and the triple-oscillator acoustic metamaterials with the vibration of the soft layer along the long axis, which is wider in the triple-oscillator system than that in triple-oscillator system.

4 Conclusion

By introducing multiple elliptical scatterers into a cell, we created multi- oscillator elliptical acoustic metamaterial systems: one with a single oscillator, one with two oscillators, one with three oscillators and one with four oscillators. We studied the bandgap characteristics of the different arrangement modes and the variation law of the band structure with the increasing number of oscillators. The study illustrated the physical mechanism of bandgap generation based on vibration modes. The following conclusion is obtained: By adding elliptical oscillators in a certain range , the number of bandgap will increase and the total frequency range of the bandgap will widen. From the analysis of vibration mode, there is a weak coupling effect between the oscillators, which is one of the reasons why the bandgap is more abundant. By comparing the band structure, it is found that the difference between the single-oscillator, dual-oscillator, and triple-oscillator is more obvious than that between triple-oscillator, the quadruple-oscillator and the six-oscillator.

This investigation can provide a theoretical basis for the design of element structures in acoustic metamaterials. The multi-oscillator system is an ideal structure for studying the extraordinary acoustic properties of the multi-oscillator structure. By artificially adjusting the arrangement of the elliptical scatterer, the specific acoustic properties can be achieved. What's more, this study provides basic reference to understand the physical mechanism of the acoustic metamaterials.

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