AN EFFICIENT NUMERICAL APPROACH TO EVALUATE THE SOUND INSULATION OF ACOUSTIC METASURFACE

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Aiming to design building façade with both natural ventilation and noise insulation performance, we propose an acoustic metasurface (AMS) by scaling up structured unit cells in a planar array. Each unit cell is in the shape of a short acoustic duct lined with periodic sub-chamber scatters. Acoustic stop-band exists in the periodic structure mainly due to the accumulated scatter resonance effect. An efficient numerical approach to evaluate the sound insulation performance of the proposed AMS is presented. The standard sound reduction index (SRI) is determined from the averaged sound pressure level (SPL) difference between a diffuse source room and a receiving room. The coupling between the AMS with the acoustic fields is formulated using a sub-structuring approach, and the unit cells are treated as a cluster of acoustic elements, modelled by finite element method. Modal based formulations are applied to the source and receiving rooms to characterize the acoustic excitation and the coupling effect, enabling an efficient calculation in the interested frequency range. The predicted SRI suggests it is possible to achieve high sound attenuation using the proposed metasurface, while maintaining the ability to ventilate naturally.

Keywords: sound reduction index, noise insulation, building acoustics, acoustic metamaterial, sub-structuring approach.

1. Introduction

Acoustic comfort plays an important role in preserving a high quality living environment. Although natural ventilation remains an attractive topic from the energy saving perspective, introducing untreated opening to ventilate building often causes significant noise problems. Sometimes people prefer to close the opening and turn on the air-conditioner just to prevent noise. Many studies have attempted to solve the contradiction problem between "noise" and "ventilation". Examples include the use of porous material [1], open double glazing [2], active noise cancellation technique [3] and sonic crystal [4], but so far the practical uses of these structures are limited.

In this study, we propose an acoustic metasurface (AMS) structure with acoustic stop-band property, aiming to realize building façade with both sound insulation and ventilation functions. The unit cell constituting the AMS is an acoustic waveguide lined with periodic resonators. In the following sections, the design and the acoustic property of the unit cell will be described in Sec. 2. The numerical approach to predict the Sound Reduction Index (SRI) of the AMS and the predicted result will be discussed in Sec. 3.

2. Unit cell design

In Fig. 1, the unit cell configuration constituting the AMS is shown, whose outer shape is simply a rectangular acoustic chamber. The chamber has two apertures at the centre of the front and rear surfaces. Inside the chamber, internal partitions are added which divides the whole chamber into several sub-chambers. The cross-section of a unit cell in the thickness direction is sketched in Fig. 1(a). The two-dimensional cross-section is similar to an acoustic duct with periodic resonators in the side-branch. The periodic resonators are used to suppress the incoming sound wave, while the acoustic duct still allows air to pass through.

Several studies have demonstrated the existence of acoustic stop-bands in such periodic wave-guide [5-7]. The forming of the acoustic stop-band is related to both the individual property of the resonator and the periodicity. According to Fig. 1(a), the size of a unit cell is chosen as 0.2×0.2 m, and that of the aperture is 0.06×0.06 m. The width of each sub-chamber is 0.02 m, and the thickness of each inner partition is 0.005 m. The periodicity *d* is therefore 0.025 m. In order to avoid very bulky structure, only four sub-chambers are connected to produce the acoustic stopband effect. The total thickness of a unit cell is around 0.1 m.

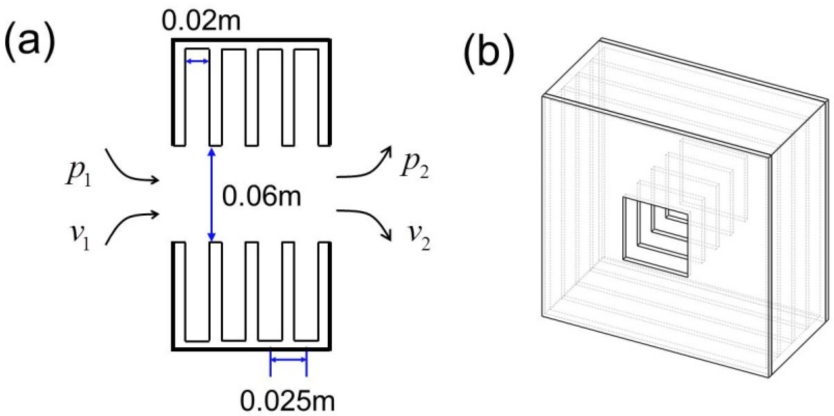

Figure 1: (a) Cross-section of a unit cell: an acoustic waveguide lined with periodic resonators; (b) Three-dimensional view of a unit cell.

The existence of acoustic stop-band in the proposed structure can be estimated by using Bloch wave theory. As a waveguide loaded with periodic scatters, Bradley [8, 9] has analysed the properties of the so-called *Bragg stop-bands* and the *scatterer resonance stop-bands*. The Bloch dispersion relation describes that the Bragg stop-bands is likely to occur when the frequency and periodic resonators having the following condition: $f = nc_0 / 2d, n = 1, 2, 3\cdots$, where d is the periodicity. Given a periodicity of *d*=0.025 m, the first Bragg stop-band appears at frequency higher than 6000 Hz, which is beyond the interested frequency range here.

The scatterer resonance stop-band occurs near the resonant frequency of the side-branch resonator. Considering the cross-section configuration in Fig. 1 (a) as a periodic arrangement of quarter-wave tubes, the resonant frequencies occur when $f = nc_0 / 4l_r$, $n = 1, 3\cdots$, where l_r is the height of the tube. Here, l_r =0.07 m yields the first resonance at around 1200 Hz. Note that the bandwidth of the formed stop-band is also related to the dissipation loss, periodicity, and width of the sub-chambers, etc. In the next section, this unit cell will be scaled up into a surface structure, aiming to realize a building façade with both natural ventilation and noise mitigation effect. As a preliminary step, the SRI performance of the AMS structure will be analysed numerically using a sub-structuring approach.

3. Sub-structuring approach

Figure 2 illustrates the idea of constructing an AMS by scaling up the unit cells as described in Sec. 2. In total 5 by 5 unit cells are considered, resulting in a surface of 1 square meter. The AMS is placed in the partition wall between a source room and a receiving room, in order to predict the SRI. The size of the source room is $6\times4\times5$ m, and that of the receiving room is $4\times3\times3.5$ m. Due to the large room dimension, using commercial finite element method (FEM) software for modelling and prediction is computationally challenging. To tackle this, a sub-structuring approach has been developed in our previous work [10]. Figure 3 describes the methodology, where the system is decoupled into a set of subsystems, and the ensemble system response is solved by assembling these subsystems together. Here, analytical formulations are applied to the large rooms for saving computational cost, whereas FEM tool is used to model the waveguide in the AMS structure. Because hybrid numerical techniques are combined taking into account the characteristics of each subsystem, the total calculation time is greatly reduced.

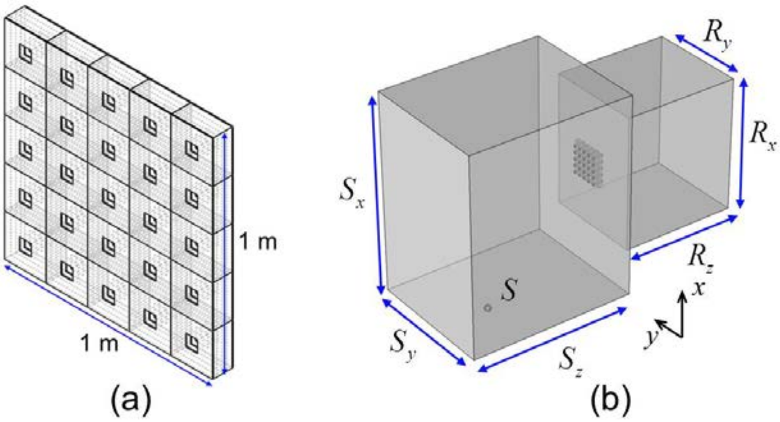

Figure 2: (a) Acoustic metasurface constructed from a planar array (5×5) of unit cells; (b) three-dimensional simulation model for SRI prediction.

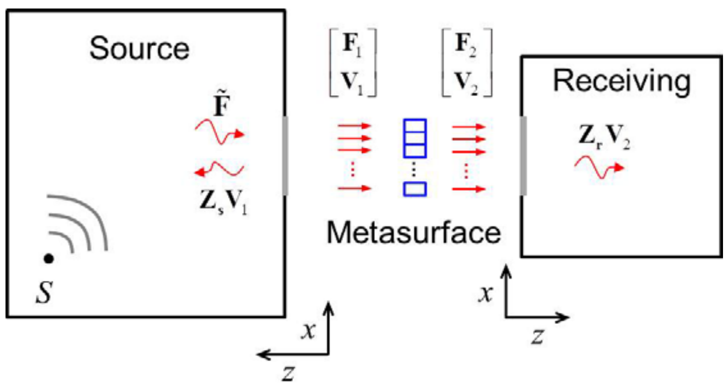

Figure 3: (a) Illustration of the sub-structuring approach.

To generate a diffuse sound source, the rectangular room with rigid wall surfaces is excited by a point sound source *S*. The sound pressure fields p_r inside the rectangular room can be analytically decomposed into the rigid-walled acoustic modes as:

$$P_r(x,y,z)=\sum_m a_r^m\varphi_r^m(x,y,z)=\sum_m a_r^m\cos(k_x x)\cos(k_y y)\cos(k_z z) \qquad (1)$$

where a_r^m and φ_r^m are the amplitude and mode shape function of the eigenmodes; m denotes the modal number; k_x, k_y and k_z are the wavenumbers in the -*x*, -*y* and -*z* directions, respectively. The same modal expansion applies to the receiving room. By incorporating the Helmholtz's wave equation into a Green's formulation, the acoustic response solely due to the point source is [10]:

$$a_r^m N_r^m (k^2 - k_m^2) = \int_{V_r} q \varphi_r^m \delta(x_s, y_s, z_s) dV_r \tag{2}$$

where $k_m = 2\pi f_m / c_0$, f_m is the eigenfrequency of the room. V_r describes the air volume enclosed by the room. q is the strength of the point source, δ is the Dirac delta and coordinate (x_s, y_s, z_s) specifies the location of the source.

The sound pressure generated by the point source is transmitted into the receiving by the AMS. The general formulation by considering the AMS as a vibrating boundary with normal velocity V_n writes [10, 11]:

$$a_r^m N_r^m (k^2 - k_m^2) = \int_{S_r} -\left(\partial p_r / \partial n\right) \varphi_r^m dS_r = \int_{S_r} j \rho_0 \omega V_n \varphi_r^m dS_r \tag{3}$$

where S_r is the surface area of the AMS, *n* denotes the direction normal to the surface.

For the system shown in Fig. 2(b) involving a source field, a receiving field, and a coupling structure (the AMS) in between, the coupling formulations for describing the two interfaces have been documented in Ref. [10]. The remaining work is to link up the AMS with the sound field inside the rooms. To model each unit cell, the sound transmission through the duct can be treated by the transfer matrix method. Assuming the cut-off frequency of the waveguide is higher than the frequency of interest, the four-pore parameters can be used to describe the inlet-outlet relationship:

$$\begin{bmatrix} P_1 \\ V_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} P_2 \\ V_2 \end{bmatrix} \tag{4}$$

where *A*, *B*, *C* and *D* are the four-pole parameter of a unit cell. P_1 and P_2, V_1 and V_2 are the pressure and velocity at the inlet and outlet, respectively [Refer to Fig. 1(a)]. The four-pole parameters for each unit cell can be obtained as:

$$\begin{aligned} A = P_1 / P_2, C = V_1 / P_2, \ \text{when} \ V_2 = 0 \\ B = P_1 / V_2, D = V_1 / V_2, \ \text{when} \ P_2 = 0 \end{aligned} \tag{5}$$

Here, the above relation is solved by using FEM, where all the partitions and walls of the unit cell are deemed as rigid.

Using the proposed approach, the SRI of the AMS in Fig. 2(a) is calculated from 125 Hz to 2000 Hz. Figure 4 presents the SRI result. Numerical simulation shows that strong sound attenuation occurs from 600 Hz to 1400 Hz, with SRI greater than 30 dB. These frequencies agree with the estimated acoustic stop-band for a unit cell, which possibly explains the sound reduction mechanism. This range can be potentially used to prevent traffic noise entering into the building. It is seen that the predicted SRI can have peak value greater than 80 dB, given the assumption that all walls are rigid and sound leakage are neglected. Note that this performance can hardly be achieved in reality. Experimental work to verify the stop-band property of a unit cell and the sound insulation of the proposed AMS will be conducted in the future.

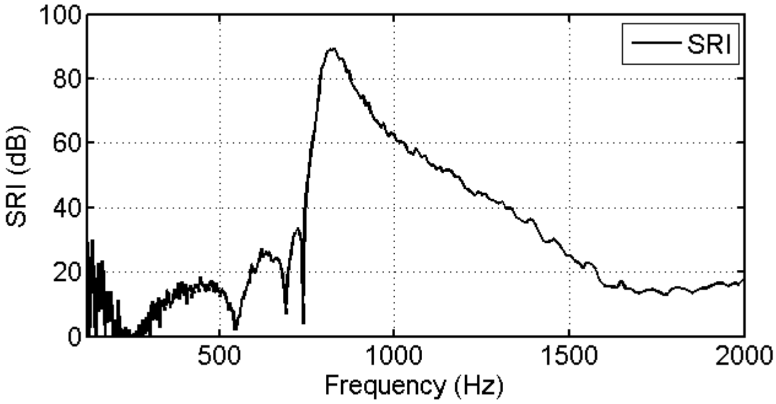

Figure 4: (a) Predicted SRI of the proposed AMS structure.

4. Conclusions

This paper presented a theoretical investigation on the sound insulation of an acoustic metasurface, which was constructed by scaling up acoustic unit cells in a planar array. The unit cell was designed as an acoustic waveguide loaded with periodic resonators. The stop-band property was analysed when choosing the unit cell geometry. The predicted SRI result of the AMS showed significant sound reduction in the design frequency range, which could be related to the stop-band behaviour of the unit cells. Further experimental work will be conducted to verify the existence of the stop-band and the SRI performance.

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