THE STUDY ON THE FACTORS INFLUENCING THE PRECISION OF THE BLOCKING FORCE BASED TRANSFER PATH ANALYSIS

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Blocking force is the force applied on the base when the source is fixed on a rigid base, which reflects the independent character of the source. It can prove that the force got by the response and the transfer function without removing the machine is equal to the blocking force. Therefore the blocking force can be measured, and the contribution of each source to the target point could also be calculated by the blocking force and vibration transfer function measured online. This method is the same as the classical transfer path analysis method, but it does not need to disassemble the machine to measure the transfer function. This calculation method was verified by an example of a three-vibration-sources isolation system. At the same time, the error caused by noise in the test signal or a missing source is discussed, which shows that the condition number is the key factor affecting the testing error caused by noise in the test signal. The structure resonance usually causes large condition number, so special attention is needed for such condition. Missing sources will also cause large measurement error.
Keywords: Transfer path analysis, blocking force, online measurement

1. Introduction

In the 1970's the signal processing method was used to solve the vibration transmission problems[1], in the 1980's transfer path analysis[2] was proposed to get the response contribution of each source as

$$a_t = \sum_{i=1}^{n} H_i^f \bullet F_i + \sum_{j=1}^{m} H_j^q \bullet Q_j$$

where F_i is the excitation force, Q_j is the volume sound source, H_i^f , H_j^q is the transfer function from the excitation force i or sound source j to the target respectively. In this method, the vibration source needs to be removed to obtain the transfer function, which is usually not easy to be satisfied. Recently OTPA (operational transfer path analysis) [3, 4] and OPAX (operational path analysis with eXogeneous inputs [5]) were proposed, in which online vibration was used so the machine is not dismantled. The commercial software BBM and LMS adopt the two methods respectively, which have been widely used in engineering. However the theoretical flaw of OTPA, OPAX is that crosstalk is not completely resolved. A free of crosstalk method is needed. The blocking force is the force exerted on the rigid foundation by the vibration source, and the blocking force reflects the independent character of the vibration source. This paper theoretically explains the basic concepts blocking-force-based transfer path analysis, a three source system is analyzed to show the correctness of the method; and the factors affecting measurement accuracy are also studied.

2. The concept of blocking force and verification of blocking force based transfer path analysis

2.1 The concept of blocking force

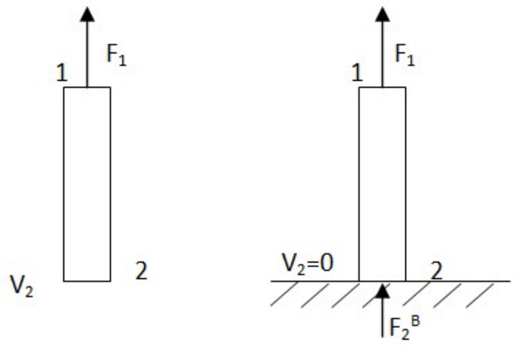

Fig. 1 The force under free end and blocked condition

The structure of the source shown in fig. 1 is subjected to a concentrated force. The response of point 2 of the structure under free state is V_2; if point 2 is fixed on the ground, the force acting on the ground F_2^B can be calculated by the four-pole parameter equation:

$$\begin{Bmatrix} V_1 \\ V_2 \end{Bmatrix} = \begin{bmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{bmatrix} \begin{Bmatrix} F_1 \\ F_2 \end{Bmatrix} \qquad (1)$$

Let $V_2 = 0$,then

$$F_2^B = -F_2 = \frac{H_{21}}{H_{22}} F_1$$

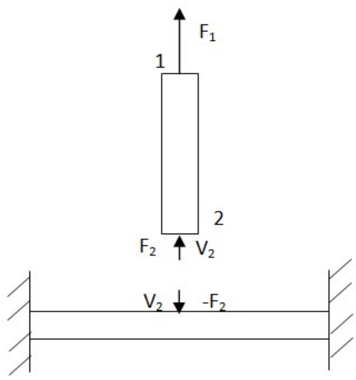

Fig. 2 Force draft to analyze the force acting on the beam

If the structure of the vibration source is connected to a flexible foundation (Fig. 2), the vibration equation is

$$V_2 = -F_2 H^d \qquad (2)$$

where H^d is the input admittance.

According to (1) and (2) , it can be derived that

$$V_2 = F_1 \frac{H^d H_{21}}{H^d + H_{22}} = \frac{H_{21}}{H_{22}} F_1 \frac{H^d H_{22}}{H^d + H_{22}} = F_2^B / \left(\frac{1}{H^d} + \frac{1}{H_{22}} \right)$$
$$= \frac{F_2^B}{Z^{\mathrm{d}} + Z_{22}}$$

It can see that $Z^{\mathrm{d}} + Z_{22}$ is the impedance of the composite structure is at point 2, which can be got by exciting point 2 without disassembling the vibration source. The above analysis shows that the blocking force is only related to the source and the source structure.

2.2 Verification of blocking force based transfer path analysis for multi-source condition

The model of multiple vibration sources is as follows: three machines represented by mass are isolated from the base beam.

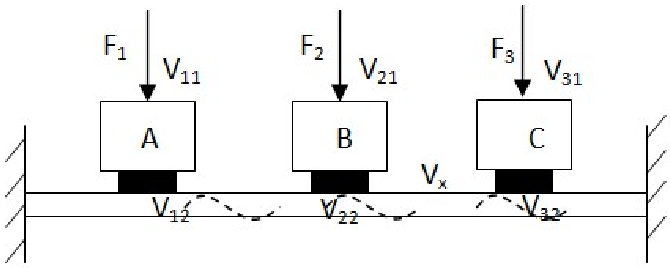

Fig. 3 Three sources fixed on a base beam

The parameters are as follows:

mass: m_1=100kg, m_2=400kg, =200kg; the rubber isolator (a beam model is used): elastic modulus $E_{xj} = 5\times10^6$ Pa, density ρ_{xj} =1100kg/ m^3, area =0.02, height =0.1, damping factor η_{xj} =0.01; the base beam parameters: elastic modulus of E= 2.1×10^{11} Pa, density ρ =7800kg/ m^3, damping factor η =0.001, cross sectional area A=w×h=0.05m×0.05m, length of beam L=1.

2.2.1 The derivation of the calculation model

The response of the up point of the isolator is as follows：

$$V_{i1} = H_m^i (F_i - F_{i1}) \qquad (3)$$

where F_i is the i_{th} excitation force, F_{i1} is the force acted on the up point of the i_{th} isolator, H_m^i is the transfer admittance of i_{th} mass, its value is: $H_m^i = \frac{1}{j\omega m_i}$.

The response of the bottom point of the isolator is as follows：

$$V_{i2} = -\sum_{j=1}^{3} H_{ij}^d F_{j2} \qquad (4)$$

where H_{ij}^d is input and transfer admittance of the base，$H_{ij}^d = \frac{j\omega}{M} \sum_{k=0}^{\infty} \frac{\varphi_k(x_i)\varphi_k(x_j)}{\omega_k^2(1+j\eta) - \omega^2}$， $M = \rho AL$， the calculation formula of ω_k^2 and $\varphi_k(x)$ of the isolator beam are listed as longitudinal vibration beam in the appendix of the paper.

The following equations are established for the isolator:

$$\begin{Bmatrix} V_{i1} \\ V_{i2} \end{Bmatrix} = \begin{bmatrix} H_{11}^i & H_{12}^i \\ H_{21}^i & H_{22}^i \end{bmatrix} \begin{Bmatrix} F_{i1} \\ F_{i2} \end{Bmatrix} \qquad (5)$$

where F_{i2} is the force acted on the bottom point of the i_{th} isolator, $H_{11}^i = H_{22}^i = \frac{j\omega}{M_{xj}} \sum_{k=0}^{\infty} \frac{\varphi_k(0)\varphi_k(0)}{\omega_k^2(1+j\eta_{xj})-\omega^2}$ and $H_{12}^i = H_{21}^i = \frac{j\omega}{M_{xj}} \sum_{k=0}^{\infty} \frac{\varphi_k(0)\varphi_k(L)}{\omega_k^2(1+j\eta_{xj})-\omega^2}$ because x_i =0 and x_j =L , $M_{xj} = \rho_{xj} A_{xj} L_{xj}$, the calculation formula of ω_k^2 and $\varphi_k(x)$ of the isolator beam are listed as flexural vibration of the clamped beam in the appendix of the paper.

By substituting (3) and (4) into (5), the mechanical equations of each isolator can be got as

$$(H_m^i + H_{11}^i)F_{i1} + H_{12}^i F_{i2} = H_m^i F_i$$

$$H_{21}^i F_{i1} + (H_{22}^i + H_{ii}^d)F_{i2} + \sum_{j=1, j\neq i}^{m} H_{ij}^d F_{j2} = 0$$

Write all the isolators equation into matrix equations

$$\begin{bmatrix} H_m^1 + H_{11}^1 & H_{12}^1 & 0 & 0 & \cdots & \cdots & 0 & 0 \\ H_{21}^1 & H_{22}^1 + H_{11}^d & 0 & H_{12}^d & \cdots & \cdots & 0 & H_{1n}^d \\ 0 & 0 & H_m^2 + H_{11}^2 & H_{12}^2 & \cdots & \cdots & 0 & 0 \\ 0 & H_{21}^d & H_{21}^2 & H_{22}^2 + H_{22}^d & \cdots & \cdots & 0 & H_{2n}^d \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & 0 & 0 & \cdots & \cdots & H_m^n + H_{11}^n & H_{12}^n \\ 0 & H_{n1}^d & 0 & H_{n2}^d & \cdots & \cdots & H_{21}^n & H_{22}^n + H_{nn}^d \end{bmatrix} \begin{bmatrix} F_{11} \\ F_{12} \\ F_{21} \\ F_{22} \\ \vdots \\ \vdots \\ F_{n1} \\ F_{n2} \end{bmatrix} = \begin{bmatrix} H_m^1 F_1 \\ 0 \\ H_m^2 F_2 \\ 0 \\ \vdots \\ \vdots \\ H_m^n F_n \\ 0 \end{bmatrix} \quad (6)$$

where n is the number of vibration sources.

The force can be calculated by the above equation, the response of each point on the beam is obtained as follows:

$$V_x = -\sum_{i=1}^{n} H_{xi}^d F_{i2}$$

According to formula (3) and (5) , equation (7) can be derived

$$V_{i2} = \frac{H_{21}^i H_m^i}{H_{11}^i + H_m^i} F_i + \frac{H_{22}^i (H_{11}^i + H_m^i) - H_{12}^i H_{21}^i}{H_{11}^i + H_m^i} F_{i2} \quad (7)$$

2.2.2 The locking force calculated for each source

Let V_{i2} =0 in equation (7)，The blocking force can be calculated as

$$F_{i2}{}^B = -F_{i2} = \frac{H_{21}^i H_m^i}{H_{22}^i (H_{11}^i + H_m^i) - H_{12}^i H_{21}^i} F_i \quad (8)$$

The impedance of the connection point to the isolator of the complex structure can be derived as

$$V_{i2} = \frac{H_{22}^i (H_{11}^i + H_m^i) - H_{12}^i H_{21}^i}{H_{11}^i + H_m^i} F_{i2} \Rightarrow Z_i = \frac{H_{11}^i + H_m^i}{H_{22}^i (H_{11}^i + H_m^i) - H_{12}^i H_{21}^i}$$

2.2.3 Calculation of the blocking force of the complex structure

The blocking force is excited on the connection point of the base beam. At the same time, the driving force of the original position is removed. The inertial force of the mass is acted on the up point of the isolator is $-Z_i V_{i2}$.

The responses of the connection point of the base of the isolators are

$$\begin{bmatrix} V_{12} \\ V_{22} \\ \vdots \\ V_{n2} \\ V_x \end{bmatrix} = [H^d] \bullet \begin{bmatrix} F_1^B - F_{12} \\ F_2^B - F_{22} \\ \vdots \\ F_n^B - F_{n2} \\ F_x \end{bmatrix} \Rightarrow \begin{bmatrix} V_{12} \\ V_{22} \\ \vdots \\ V_{n2} \\ V_x \end{bmatrix} = [H^d] \bullet \begin{bmatrix} F_1^B - Z_1 V_{12} \\ F_2^B - Z_2 V_{22} \\ \vdots \\ F_n^B - Z_n V_{n2} \\ F_x \end{bmatrix}$$

The blocking force can be calculated by substituting the response of (7) into equation (9) as

$$\begin{bmatrix} F_1^B \\ F_2^B \\ \vdots \\ F_n^B \\ F_x \end{bmatrix} = \begin{bmatrix} V_{12} \\ V_{22} \\ \vdots \\ V_{n2} \\ V_x \end{bmatrix} \left[[H^d]^{-1} + \begin{bmatrix} Z_1 & 0 & \cdots & 0 & 0 \\ 0 & Z_2 & & \vdots & \vdots \\ \vdots & & \ddots & 0 & \\ 0 & \cdots & 0 & Z_n & 0 \\ 0 & \cdots & & 0 & 0 \end{bmatrix} \right] \quad (9)$$

Three random input forces are generated, and the force calculated by the formula (9) is compared with the value calculated by the formula (8) , they are shown in fig. 4：

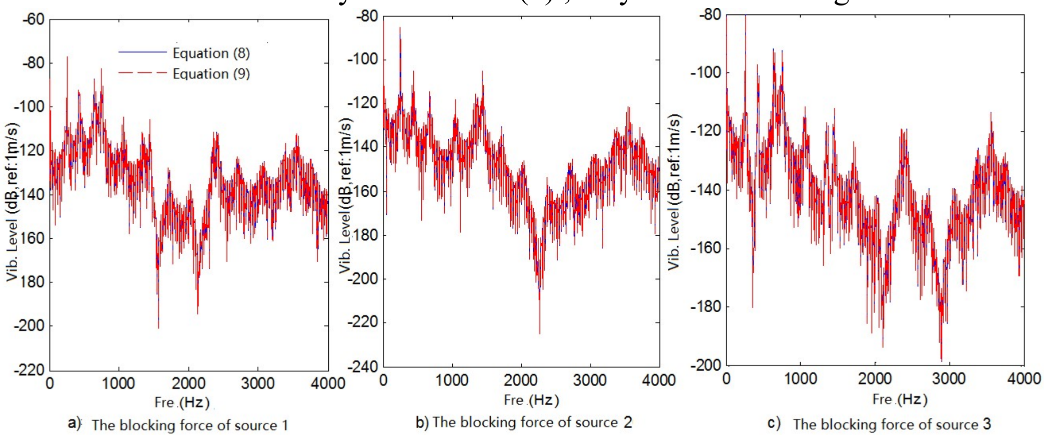

Fig. 4 The calculated blocked force by equation (8) and (9)

It can see that the two curves coincide completely, so the blocking force calculation is correct for complex structure.

2.2.4 The calculation of the contribution of each source by the blocking force

The contribution of each source is calculated by to the blocking force and the admittance of the complex structure：

$$V_x = H_{xi}^B F_{i2}^{\ B}$$

H_{xi}^B （i=1..n）is the admittance of the complex structure，its value is the first n elements of the last row of the matrix $\left[[H^d]^{-1} + \begin{bmatrix} Z_1 & 0 & \cdots & 0 & 0 \\ 0 & Z_2 & & \vdots & \vdots \\ \vdots & & \ddots & 0 & \\ 0 & \cdots & 0 & Z_n & 0 \\ 0 & \cdots & & 0 & 0 \end{bmatrix} \right]^{-1}$.

At the same time, the response of each source is got by input each force into equation (6) respectively， the results of each method are shown in the following figures.

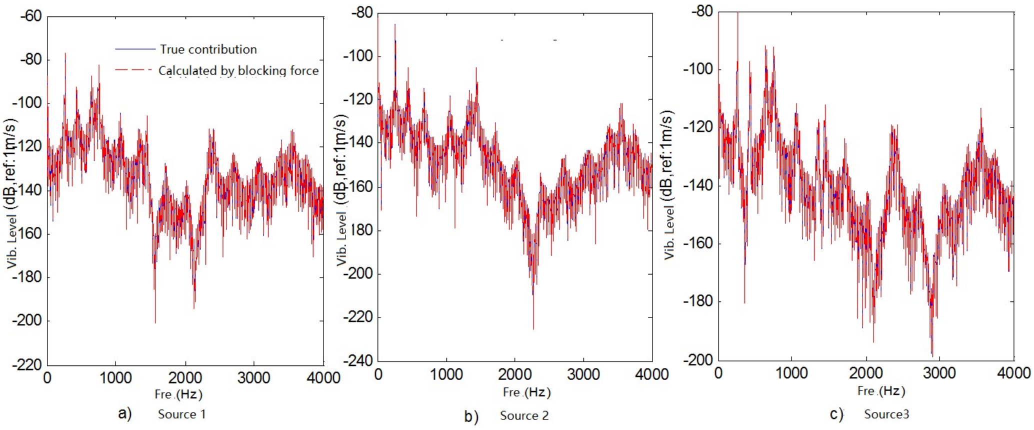

Fig. 5 The calculated response of each blocked force and the true value

The results are in good agreement, which shows the correctness of the calculation method by the blocking-force-based transfer path analysis.

3. Analysis of results with measuring noise

If there is a certain noise in the measured response, and the signal-to-noise ratio is 0.01, the true blocking force and the calculated one are as follows:

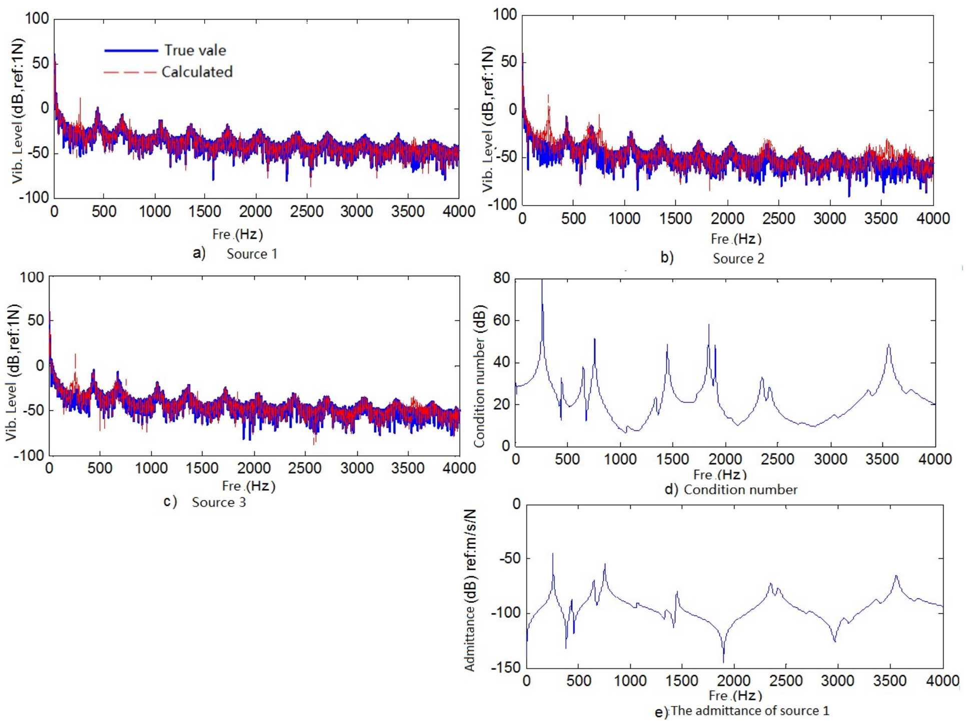

Fig. 6 The calculated blocked force with testing noise

It can be seen that there are much error at the resonance point, which can be explained by the condition number theory in matrix inverse: to solve the linear equation $Ax = b$, if small change in b results in great changes in the solution, the equation are ill conditioned. The error of ill conditioned equation can be expressed as [6]:

$$\frac{\|\delta \mathrm{x}\|}{\|\mathrm{x}\|} \le \mathrm{cond}(A)\frac{\|\delta b\|}{\|b\|}$$

where $\| \ \|$ represent the norm of the matrix, and δb is the test signal error. As the transfer function shown in Fig. 6 e), there is a relatively large condition number at the resonance point in fig. 6 d), which cause larger error in the calculation of blocking force as shown in figure 6 a), 6 b).

4. Analysis of the results with missing source

The influence of some source missing on the results is to be investigated. Assuming only two sources are considered in a three source structure, in other words, one source is missing. The on-line measurement is only made for the transfer function of the two sources, and also only the two blocking force are calculated. The calculated blocking force and true one are shown in following figure.

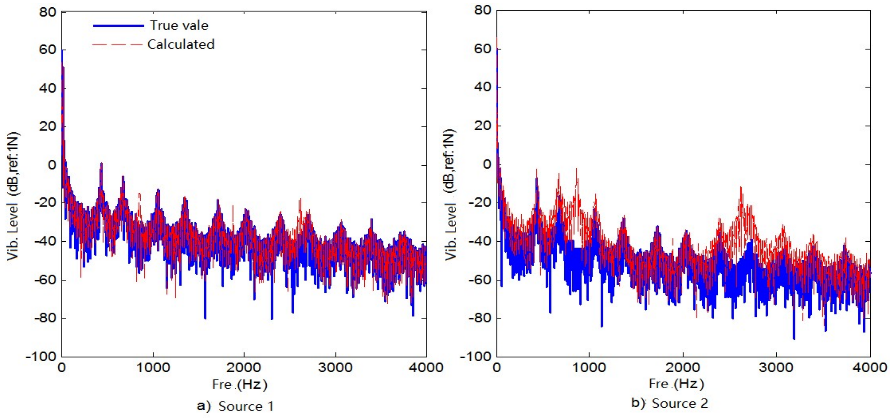

Fig. 7 The calculated blocked force with missing source

It can be seen that the error of calculation results is relatively large at most frequencies.

5. Conclusion

The blocking force reflects the characteristics of independent sources, which can be obtained using the online transfer function without disassembling the machine. Basing on the blocked force and the online transfer function, the source contributions are got and the results are crosstalk free. When applying this method, we should pay attention to the missing recognition of some sources and the resonance condition when the condition number is relatively large and it will cause relatively large errors.

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Appendix modal frequencies and shapes of beams

（1）Natural frequency and modal shape of longitudinal vibration beam

Natural frequency:

$$\omega_i = \frac{i\pi}{l}\sqrt{\frac{E_{xj}}{\rho_{xj}}}$$

Vibration modal shape：

$$\varphi_i(x) = \sqrt{2}\cos(\frac{i\pi}{l_{xj}}x)$$

（2）Natural frequency and modal shape of the flexural vibration of the clamped beam

Natural frequency:

$$\omega_i = \frac{(\lambda_i l)^2}{l^2}\sqrt{\frac{EI}{\rho A}} \quad \text{（i=1,2,..）}$$

$\lambda_1 l = 4.73$, $\lambda_2 l = 7.853$, $\lambda_3 l = 10.996$, $\lambda_i l = (i + \frac{1}{2})\pi$, i>=2

Vibration modal shape：

$$\varphi_i\ (x) = \cosh(\lambda_i x) - \cos(\lambda_i x) - \frac{\cosh(\lambda_i l) - \cos(\lambda_i l)}{\sinh(\lambda_i l) - \sin(\lambda_i l)}(\sinh(\lambda_i x) - \sin(\lambda_i x))$$

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