Proceedings of the Institute of Acoustics

DETERMINING FE PATCH SIZE FOR INPUT RESPONSE MODELLING OF LARGE STRUCTURES

X.H. Wang, G. Stimpson & N. Lalor

Institute of Sound and Vibration Research, The University, Southampton, SO9 5NH, U.K.

1. INTRODUCTION

For the modelling of structural noise radiation one of two approaches is generally adopted: Finite Elements or Energy Balance methods (Statistical Energy Analysis, Energy Flow Analysis). Both techniques are complementary and their use in the conventional sense is really related to different aspects of noise radiation. The Finite Element technique incorporates all the fine details of a structure and thus can be used to predict the effects of small local changes. The fineness of the technique, however, makes its use very complex and cumbersome to apply to all but relatively simple shaped structures and low frequencies (low modal orders). Energy based techniques [1,5], on the other hand, are best suited to high frequency, high modal order noise radiation where an averaged response of the structure in a particular frequency band can be assumed. However, energy based techniques will only predict the effects of averaged or global changes to the structure. This does not really matter in relation to the radiation characteristics of the structure (in the high modal order frequency range) as these characteristics are only affected by global changes. The addition of a single stiffening rib across a plate, for example, will have little effect on its high frequency noise radiation characteristics whereas an overall change of material or thickness may have. The level of noise radiation from a particular structure, however, also depends upon the level of excitation, i.e. the input vibrational energy. The input of vibrational energy depends upon two factors: the applied forces and the response of the structure at the point of application of the force, i.e. the input mobility. For large structures, especially those such as a car body or an aircraft fuselage, the input mobility will be very much related to local structural details around the input point. At this point local structural changes could well have a significant effect upon the overall input of vibrational energy and hence overall noise radiation. The techniques of Finite Elements and energy balance can thus be linked to provide a combined technique where input response is predicted from a relatively simple patch of finite elements around the excitation input point or points and the distribution of vibrational energy around the structure calculated using energy methods.

To apply this technique to a particular structure a method is required to determine the size of the FE patch around the input point. In early work using this technique [4] a car body was progressively physically cut down whilst monitoring mobility at a particular input point. This served to verify that only a small section of body structure was actually contributing to the 'input' response. This method was obviously, however, rather drastic and recent work has been involved with developing a measurement technique to determine the necessary size of a FE patch.

2. CONSIDERATIONS OF PATCH SIZE VIA SUBSYSTEM COUPLING METHOD

Starting with a very simple physical model, consider a simply-supported long beam as shown in Figure 1. The beam is considered as being made up of two parts or subsystems joined together at

point B with a force f_{i1} applied at point A. Each subsystem of the beam can be considered as a two port device with four "terminal" variables shown in Figure 2.
f_{i1} is the force applied at point A; V_{i1} is the vibration velocity at point A; V_{o1}, V_{i2}, f_{o1}, f_{i2} are the interior force and vibration velocities at the imaged joint point B (coupling point). The definition of V_{o2} and f_{o2} is not fixed but they can be thought of as the force and vibration velocity at any point of part 2 except the support point.

The relations between these "terminal" variables can be written in matrix form [2] (frequency dependence is assumed).

$$\begin{Bmatrix} f_{i1} \\ V_{i1} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{Bmatrix} f_{o1} \\ V_{o1} \end{Bmatrix} \qquad \begin{Bmatrix} f_{i2} \\ V_{i2} \end{Bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{Bmatrix} f_{o2} \\ V_{o2} \end{Bmatrix} \tag{1}$$

Here the elements of the two square matrices can be obtained by the following formulas:

$$a_{12} = \frac{f_{i1}}{V_{o1}} \Big|_{f_{o1}=0} \qquad a_{22} = \frac{V_{i1}}{V_{o1}} \Big|_{f_{o1}=0}$$

$$a_{11} = \frac{f_{i1}}{f_{o1}} \Big|_{v_{o1}=0} \qquad a_{21} = \frac{V_{i1}}{f_{o1}} \Big|_{v_{o1}=0}$$

$$b_{12} = \frac{f_{i2}}{V_{o2}} \Big|_{f_{o2}=0} \qquad b_{22} = \frac{V_{i2}}{V_{o2}} \Big|_{f_{o2}=0}$$

$$b_{11} = \frac{f_{i2}}{f_{o2}} \Big|_{v_{o2}=0} \qquad b_{21} = \frac{V_{i2}}{f_{o2}} \Big|_{V_{o2}=0}$$

Since subsystem 1 and 2 are coupled together, $f_{o1} = f_{i2}$, $V_{o1} = V_{i2}$, therefore:

$$\begin{Bmatrix} f_{i1} \\ V_{i1} \end{Bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{Bmatrix} f_{o2} \\ V_{o2} \end{Bmatrix} \tag{2}$$

As no third device is attached and no external force is applied at the output of subsystem 2, so $f_{o2} = 0$. Thus, the driving point mobility of the whole system at point A is:

$$\beta = \frac{V_{i1}}{f_{i1}} = \frac{a_{21}\,b_{12} + a_{22}\,b_{22}}{a_{11}\,b_{12} + a_{12}\,b_{22}} \tag{3}$$

If the two subsystems are discoupled by physically cutting the beam at the point B, i.e. let $f_{o1} = 0$, then we can get another driving point mobility,

DETERMINING FE PATCH SIZE

$$\beta' = \frac{V_{i1}}{f_{i1}}\Big|_{fo1=0} = \frac{a_{22}}{a_{12}} \tag{4}$$

Here, the β is the real driving point mobility of the beam, and the β' is the actual driving point mobility obtained if only part one of the beam is the FE patch. Substituting equation (4) into (3) we obtain:

$$\beta = \frac{a_{21}\ b_{12}/a_{12}\ b_{22} + \beta'}{a_{11}\ b_{12}/a_{12}\ b_{22} + 1} \tag{5}$$

Defining the "unloaded" input impedance of subsystem 1 and the "generalised" input impedance of subsystem 2 respectively as

$$Z_{i1} = \frac{f_{i1}}{V_{i1}}\Big|_{vo1=0} = \frac{a_{11}}{a_{21}} \qquad Z_{i2} = \frac{f_{i2}}{V_{i2}}\Big|_{fo2=0} = \frac{b_{12}}{b_{22}}$$

and defining the cross impedance of subsystem 1 as

$$Z_{i0} = \frac{f_{i1}}{V_{o1}}\Big|_{fo1=0} = a_{12}$$

we get $$\beta = \frac{\beta'}{1 + Z_{i2}/Z_{i0}} + \frac{1}{Z_{i1} + Z_{i0}/Z_{i2}\ a_{21}} \tag{6}$$

Equation (6) provides a relationship between β and β', the problem exists, however, that most of Z_s in this equation are not readily measurable. For the purpose of finding the FE patch size using an experimental method, it is necessary to relate these two mobility terms with some readily measurable quantity.

For the system under consideration, we have the following relationships from equation (1):

$$f_{i1} = a_{11}\,f_{o1} + a_{12}\,V_{o1}$$
$$V_{i2} = b_{21}\,f_{o2} + b_{22}\,V_{o2} \tag{7}$$
$$f_{o2} = 0$$

In case of two subsystems coupled together, $V_{o1} = V_{i2}$. The cross impedance between points A and B (i.e. the cross impedance between the driving point and the boundary of the "patch") therefore will be:

$$Z = \frac{f_{i1}}{V_{o1}} = \frac{f_{i1}}{V_{i2}} = \frac{a_{11}\,f_{o1} + a_{12}\,b_{22}\,V_{o2}}{b_{22}\,V_{o2}} = \frac{a_{11}\,f_{o1}}{b_{22}\,V_{o2}} + a_{12}$$

DETERMINING FE PATCH SIZE

Here, $\frac{f_{o1}}{V_{o2}} = \frac{f_{i2}}{V_{o2}} \Big|_{f_{o2}=0} = b_{12}$

$$\frac{b_{12}}{b_{22}} = Z_{i2}$$

$$a_{12} = Z_{io}$$

So $Z = Z_{io} + a_{11} Z_{i2}$

$$Z_{io} = Z - a_{11} Z_{i2} \qquad (8)$$

Substituting equation (8) into equation (6) we obtain

$$\beta = \frac{\beta'}{1 + \frac{1}{Z/Z_{i2} - a_{11}}} + \frac{Z_{i2} a_{21}}{Z} \qquad (9)$$

This is a general result which can be used for any structure and clearly if $Z \to \infty$, then $\beta' \to \beta$. For large structures such as car body and aircraft fuselage, the terms Z_{i2}, a_{21} and a_{11} will be small and thus, if Z is large enough, then $\beta \approx \beta'$. This means that the extent of the necessary FE patch can be determine experimentally from measurements of the cross impedance between the input driving point and various points on the structure around the input point.

These ideas can be further extended to simplify measurement and interpretation by applying a coherence function approach.

3. MEASURING THE PATCH SIZE USING THE COHERENCE FUNCTION

Again referring to the physical model shown in Figure 1 with, this time, one shaker at point A and another at the point B and considering the two exciting forces as two inputs, X_1, X_2, and the vibration signal at point A as the output y. Both shakers are driven by independent uncoherent sources.

The second shaker, away from the input point, is to simulate a "back reaction" from the structure. The force level of the second shaker is thus set to be equal to that of the first as this is the maximum level that any back reaction could have. Thus, we have the transfer function model as shown in Figure 3.

If $x_1(t)$ is independent of $x_2(t)$, i.e. $S_{12} = 0$, then[3]:

$$\gamma_{1y}^2 = \frac{|H_1|^2 S_{11}}{S_{yy}} \qquad \gamma_{2y}^2 = \frac{|H_2|^2 S_{22}}{S_{yy}} \qquad (10)$$

DETERMINING FE PATCH SIZE

$$S_{yy} = |H_1|^2 S_{11} + |H|^2 S_{22} + S_{nn} \tag{11}$$

In the case of $S_{nn} = 0$, $S_{22} = S_{11}$, we have

$$\frac{S_{11}}{S_{yy}} = \frac{1}{|H_1|^2 + |H_2|^2} \tag{12}$$

Substitute (12) into (10), then

$$\gamma_{1y}^2 = \frac{|H_1|^2}{|H_1|^2 + |H_2|^2} = \frac{1}{1 + \frac{|H_2|^2}{|H_1|^2}} \tag{13}$$

Here, $H_2 = \frac{1}{Z}$ and Z has the same context as in equation (9). Defining $Z_1 = \frac{1}{H_1}$ as the driving point impedance of point A, then:

$$\gamma_{1y}^2 = \frac{1}{1 + \frac{|Z_1|^2}{|Z|^2}} \tag{14}$$

If $\gamma_{1y}^2 \rightarrow 1$, then Z must be very large and, according to equation (9), $\beta \approx \beta'$. Thus, the point B can be chosen as the boundary point of the FE patch.

Thus, by monitoring the coherence function between the input force and response at the input point, i.e. γ_{1y}^2, as the second shaker is moved progressively away the size of its zone of effect can be obtained and hence the size of the necessary FE patch. With the second shaker close to the input point its effect will be large corrupting the direct relationship between input force and acceleration and giving a low value for the coherence function γ_{1y}^2. As the second shaker is moved out of the zone of effect the coherence function γ_{1y}^2 should approach unity.

4. APPLICATION EXAMPLE - PATCH SIZE AROUND AN ENGINE MOUNT BRACKET ON A CAR BODY

The measurement technique has been employed to determine the size of patch required to model the input characteristics around an engine mount bracket on a car body. Figure 4 shows the measurement set-up. Shaker 1 is attached via an impedance head to the mount position and shaker 2 moved around in the vicinity to determine the zone of influence on the coherence between input force and acceleration and hence the necessary patch size.

Figure 5(a) shows the measured coherence between input force and acceleration without the second shaker attached and figure 5(b) how this is reduced when the second shaker is placed nearby. As the second shaker is moved progressively away from the input point the coherence between input force and acceleration is restored to the values without the second shaker (i.e. close to unity). Figure 5(c) shows this situation with the second shaker attached outside the zone of influence. By moving the second shaker in all directions around the input point the patch size is determined.

The patch size obtained from the measurements is indicated in figure 4. The patch size obtained from these tests compared well with results previously obtained when the size on a similar vehicle body was obtained by physically cutting down the structure.

5. CONCLUSIONS

The technique appears to provide a practical method for determining the necessary size of FE patch to model input characteristics of large and complex structures. However, under certain conditions and at certain frequencies, interpretation of the results can be difficult and work is continuing to gain further experience with a variety of structures and further refine the technique.

6. ACKNOWLEDGEMENTS

The authors would like to express their thanks to the Ford Motor Co. and to Nissan Motor Co. for their help in supporting the work described in the report.

7. REFERENCES

[1] N. LALOR, The analysis and prediction of the internal noise of passenger cars. Paper presented to 2nd KIA Academic Seminar, Seoul, Korea (1989).
[2] O. ERNEST, 'System modelling and response: theoretical and experimental approaches', John Wiley and Sons.
[3] J.S. BENDAT, Engineering applications of correlation and spectral analysis. Wiley, New York (1980).
[4] J. LEA, N. LALOR and A.V. PHILLIPS, A combined Finite Element and energy balance model for car body vibration prediction, Proc. of Inter-Noise '88.
[5] G. STIMPSON, The use of energy techniques for the prediction of machinery noise application to the Diesel engine, Proc. of Inter-Noise '88.

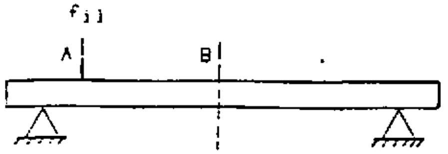


Fig. 1 2 port beam model

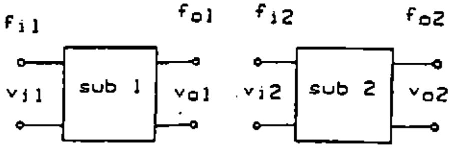


Fig. 2 2 port representation of beam

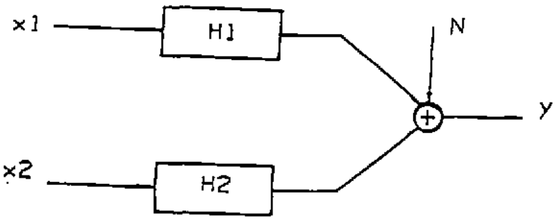


Fig. 3 2 input coherence model

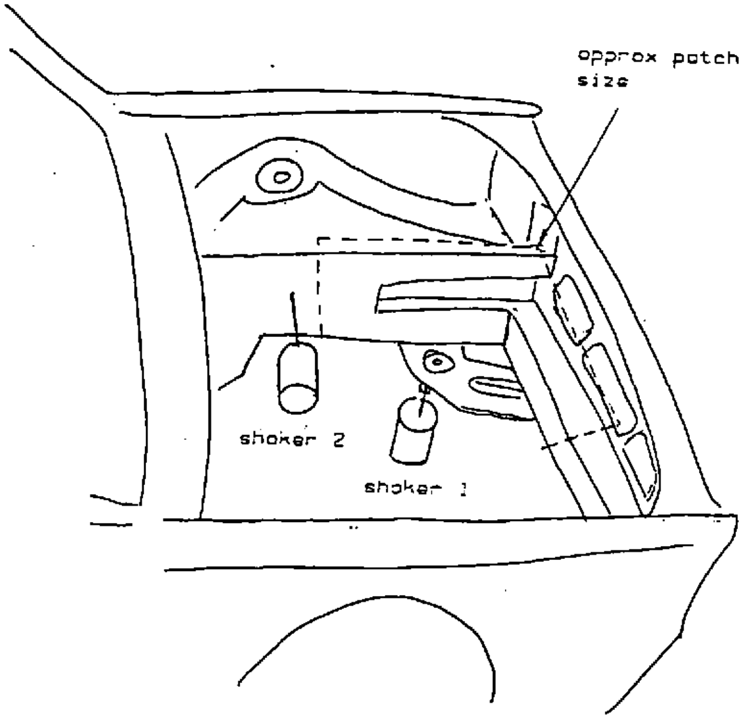
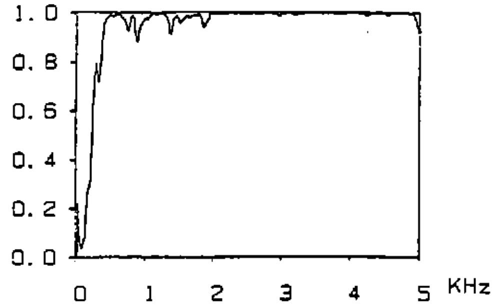
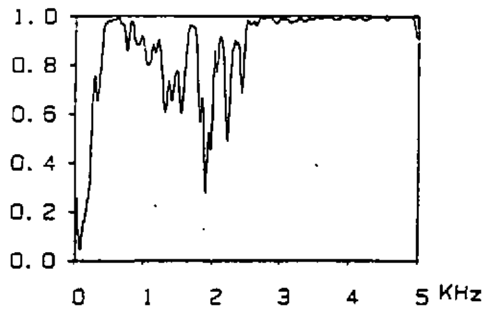


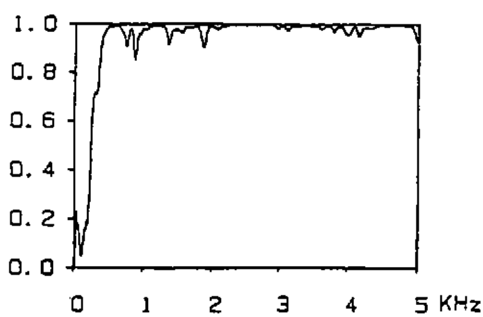
Fig. 4 Set up for tests on car body, engine bay



a) without second shaker attached



b) with second shaker close to mount position



c) with second shaker outside patch

Fig. 5 measured coherence between input force and acceleration at mount position (1)