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EXPERIMENTAL INVESTIGATIONS OF A NONLINEAR MULTIMODAL VIBRATION ENERGY HARVESTER BASED ON MAGNETIC LEVITATION

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In this paper, we propose a nonlinear multimodal vibration energy harvesting method using an array of coupled levitated magnets. This approach is validated with an experimental prototype of two magnets. The advantage of this device is the combination of the benefits of nonlinearities and modal interactions in order to enlarge the bandwidth and increase the harvested power. At first, the governing equations are derived while considering the magnetic nonlinearity and the electromagnetic damping. This model includes also the identified magnetic stiffness thanks to a specific experimental protocol. The device is designed with closed eigenfrequencies in order to enlarge the harvester bandwidth and dynamic experiments have been performed under a harmonic base excitation. A resistive electrical circuit permits the measurement of the harvested power and the optimal load resistance enabling the maximal harvested power has been identified and compared with good agreement to the theoretical value. The proposed harvester enables scavenging the vibration energy in the operating frequency range of $8 - 13.5\,Hz$, and a normalized harvested power of $2\,mW\,cm^{-3}\,g^{-2}$.

Keywords: nonlinear dynamics, vibration energy harvesting, magnetic levitation, multimodal interactions, oscillator arrays

1. Introduction

The natural energy sources exist around a system in different kinds, among them vibration sources are the most ubiquitous and can be found everywhere in our daily life and, hence, have attracted much research attention. Therefore, several designs of Vibration Energy Harvesters (VEHs) have been developed recently. Most vibration energy harvesters (VEHs) work as linear vibration resonators in which the system performance largely relies on the resonant frequency. For such a device, it mostly works at its first resonant frequency, while its high-order modes are usually neglected because they are far away and provide much lower response level as compared to the first mode. In practice, the source of vibrations in the environment is usually frequency variant or random with energy distributed over a wide frequency range. Thus, a conventional linear VEH using only a single mode is limited in terms of performances [1, 2]. To overcome this limitation, a great number of approaches have been proposed with the capability of broadband energy harvesting. The multimodal energy harvesting technique is one approach widely pursued for enlarging the frequency range [3, 4]. Although multimodal techniques enable wide bandwidth energy harvesting, they require more sophisticated interface circuits [5]. Therefore, several researches have been oriented towards the study of nonlinear systems. For instance, Mann and Sims [6] showed analytically and experimentally how magnetic levitation could be used to extend the VEH bandwidth through a hardening response. Mahmoudi et al. [7] proposed an alternative to overcome the dry friction by guiding the moving magnet vertically in an elastic way with an hybrid piezoelectric and electromagnetic transductions. More recently, Abed et al. [8, 9] proposed an optimized model of two and three coupled levitated magnets for nonlinear multimodal harvesting.

In this paper, an experimental prototype of a two dofs vibration energy harvester using levitated magnets is realized and a proof of concept of nonlinear multimodal VEH is achieved.

2. System modeling

The proposed VEH as shown in figure 1, is composed of an array of magnets where the first and the last magnets are fixed and the other central magnets are levitated. The levitated magnets oscillate inside coils made with copper wire which are wounded around the outer casing in order to induce electric current.

2.1 Equation of motion

The tube is under harmonic excitation $Y = Y_0 \cos(\Omega t)$, the vibration amplitude of the j^{th} magnet is x_j, so since the first and the last magnets are fixed, we have $x_0 = x_{n+1} = Y$. The moving magnets are subjected to gravitational forces $\vec{P_{0j}} = m_j \vec{g}$, where m_j is the mass of the j^{th} magnet and two magnetic forces are applied by both upper and bottom magnets. After applying the change of variable $v_j = x_j - Y$, the resulting magnetic force applied on each moving magnet M_j can be expressed in Taylor series up to the third order as follows:

$$\overrightarrow{F_j^m} = (\frac{2\lambda}{d^3}(2v_j - v_{j+1} - v_{j-1}) + \frac{3\lambda}{d^4}((v_j - v_{j-1})^2 - (v_j - v_{j+1})^2) + \frac{4\lambda}{d^5}((v_j - v_{j-1})^3 + (v_j - v_{j+1})^3))\overrightarrow{x} \quad (1)$$

Where Q_{M_j} is the magnetic intensity of the j^{th} magnet, μ_0 is the magnetic permeability, d is the gap between two adjacent magnets, x_j is the vibration amplitude of the j_{th} magnet and $\lambda = \frac{\mu_0}{4\Pi} Q_{M_j}^2$ is a magnetic parameter.

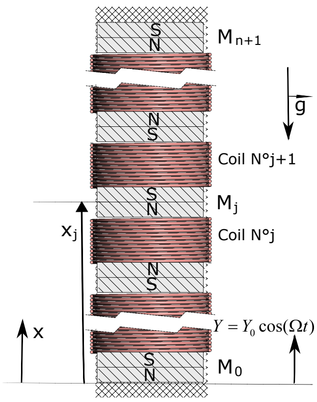


Figure 1: A schematic diagram of the MDOF VEH based on magnetic levitation.

The resulting electrical damping force can be expressed as follows:

$$f_j^e = c_{ej} \dot{v}_j = \frac{\alpha_j^2}{R_{load,j} + R_{int,j}} \dot{v}_j \quad (2)$$

Where α_j is called the electromechanical coupling coefficient for the j^{th} circuit, $\alpha_j = B_j l_j$, where B_j is the average magnetic field strength of the j^{th} magnet, l_j is the length of the j^{th} coil. $R_{int,j}$

and $R_{load,j}$ are respectively the internal resistance of the j^{th} coil and the load resistance of the j^{th} circuit. On the other hand, the mechanical disspative force is $f_j^m = c_m \dot{v}_j$, where c_m is the mechanical damping of the vibrating system. This damping results from the friction between the tube and the moving magnets.

The equation of motion of each moving magnet M_j is:

$$M_j \ddot{v}_j + c_m \dot{v}_j + c_{ej} \dot{v}_j + \frac{2\lambda}{d^3}(2v_j - v_{j+1} - v_{j-1}) + \frac{3\lambda}{d^4}\left[(v_j - v_{j-1})^2 - (v_j - v_{j+1})^2\right] + \frac{4\lambda}{d^5}\left[(v_j - v_{j-1})^3 + (v_j - v_{j+1})^3\right] = P_j + A_j \cos(\Omega t) \quad (3)$$

Where M_j is the mass of each magnet, $P_j = M_j g$ and $A_j = M_j \Omega^2 Y_0$ the base acceleration.

2.2 Power transferred to the electrical circuit

The proposed harvester contains n coils that insure the electromagnetic energy harvesting. The oscillations of the moving magnets inside these coils provide an induced current (Lenz's Law). The induced current can be expressed as a vibration velocity function $i_j = \dfrac{\alpha_j}{R_{load,j} + R_{int,j}} \dot{v}_j$. The instantaneous electrical power $P_j(t)$ can be written as follows:

$$P_j(t) = R_{load,j}.i_j^2 = \frac{1}{2}\left(\frac{\alpha_j \Omega}{R_{load,j} + R_{int,j}}\right)^2 R_{load,j} V_j^2 (1 - \cos(2\Omega t + \phi)) \quad (4)$$

At the steady state regime, the average power transferred to the electrical load over a vibration cycle can be expressed as follows:

$$P_{av,j} = \frac{\Omega}{2\pi}\int_0^{\frac{2\pi}{\Omega}} P_j(t) dt = \left(\frac{\alpha_j \Omega}{R_{load,j} + R_{int,j}}\right)^2 R_{load,j} V_j^2 \quad (5)$$

The total average power P_{av} harvested from the whole device is equal to the sum of all the powers transferred to each circuit $P_{av} = \sum_{j=1}^{n} P_{av,j}$.

3. Two-degree-of-freedom harvester

3.1 Equations of motion

The experimental study deals with two levitated magnets. Thus, the 2-dofs coupled equations derived from Equation (3), where $v_0 = v_3 = 0$ are:

$$M_1 \ddot{v}_1 + (c_m + c_{e1})\dot{v}_1 + \frac{4\lambda}{d^3} v_1 - \frac{2\lambda}{d^3} v_2 + \frac{3\lambda}{d^4}\left[v_1^2 - (v_1 - v_2)^2\right] + \frac{4\lambda}{d^5}\left[v_1^3 + (v_1 - v_2)^3\right] = P_1 + A_1 \cos(\Omega t) \quad (6)$$

$$M_2 \ddot{v}_2 + (c_m + c_{e2})\dot{v}_2 - \frac{2\lambda}{d^3} v_1 + \frac{4\lambda}{d^3} v_2 + \frac{3\lambda}{d^4}\left[v_2^2 - (v_1 - v_2)^2\right] + \frac{4\lambda}{d^5}\left[v_2^3 + (v_2 - v_1)^3\right] = P_2 + A_2 \cos(\Omega t) \quad (7)$$

The stable equilibrium positions are calculated using the linearized equations derived from equations (6) and (7). The equations of motion are then expanded around the equilibruim positions leading to the following linear 2-dofs system:

$$M_1\ddot{v}_1 + (c_m + c_{e1})\dot{v}_1 + (k_1 + k_c)v_1 - k_c v_2 = P_1 + A_1\cos(\Omega t) \quad (8)$$

$$M_2\ddot{v}_2 + (c_m + c_{e2})\dot{v}_2 - k_c v_1 + (k_2 + k_c)v_2 = P_2 + A_2\cos(\Omega t) \quad (9)$$

Where $M_1 = M_2 = 6\,gram$, $k_1 = 18\,Nm^{-1}$, $k_2 = 3.5\,Nm^{-1}$ and $k_c = 13\,Nm^{-1}$. The identified magnetic parameter is $\lambda = 4.75 * 10^{-5}\,Nm^2$. The measured eigenfrequencies are $f_1 = 6\,Hz$ and $f_2 = 13\,Hz$, while the model permits to calculate the corresponding eigenfrequencies with errors of $2\,\%$ and $2.6\,\%$ respectively.

In order to enlarge the bandwidth of the 2-dofs harvester, the closeness of the two eigenfrequencies is sought thanks to the linearized model. The tuning of the coupling stiffness leads to the measured eigenfrequencies $f_1 = 7\,Hz$ and $f_2 = 8.4\,Hz$.

3.2 Optimization of the load resistance

With the chosen design, each magnet is able to oscillate inside its own coil independently. Thus, each electrical circuit produces its own current and has its own load resistance and the average powers harvested from the two circuits are as follows:

$$P_{av,1} = \left(\frac{\alpha_1\Omega}{R_{load,1} + R_{int,1}}\right)^2 R_{load,1}V_1^2 \quad (10)$$

$$P_{av,2} = \left(\frac{\alpha_2\Omega}{R_{load,2} + R_{int,2}}\right)^2 R_{load,2}V_2^2 \quad (11)$$

and the total average harvested power is $P_{av} = P_{av,1} + P_{av,2}$.

Since the two internal resistances are set by design, the expression of the average power depends only on the load resistance for given excitation frequency and amplitude. Thus, the harvested power could be optimized with respect to the load resistance. The theoretical calculations show that the optimal power is reached $\left(\frac{dP_{av,j}}{dR_{load,j}} = 0\right)$ for $R_{load} = R_{int}$. In order to validate this result experimentally, two coils fabricated with copper wire $R_{int,1} = R_{int,2} = R_{int} = 11K\Omega$ were wound around the outer casing and the device is excited at $6\,Hz$ and $0.5\,g$, the total power response was monitored with respect to the load resistance as shown in figure 2. The graph shows that the optimal load resistance is approximately $11\,K\Omega$, which is in good agreement with the theoretical calculations. In the following section, this optimal condition will be considered.

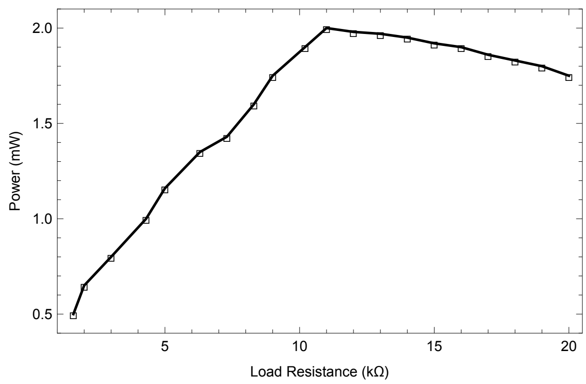


Figure 2: Average power as function of the load resistance at $6\,Hz$ and $0.5\,g$ for $R_{int} = 11\,K\Omega$.

3.3 Power responses

The experimental setup used to investigate the power responses of the system is shown in Figure 3. The device is mounted on a shaker in order to apply the harmonic base excitation and the measurements of the acceleration of the shaker are performed thanks to an accelerometer placed on the support that holds the tube. Finally, the output data acquisition is achieved using a SigLab device. The frequency responses are obtained by sweeping the frequency up and down in order to fully capture the bifurcation topology.

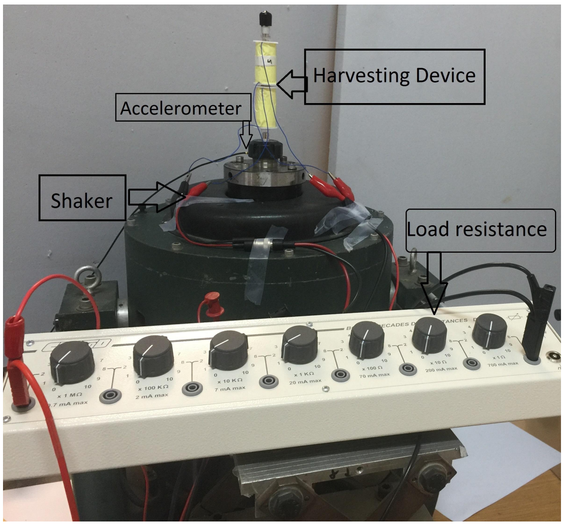


Figure 3: Experimental setup for dynamic characterization of the designed VEH in closed loop circuit.

Figure 4a shows the linear responses of the average power for a low excitation level of $0.3\,g$ in the frequency range between $7\,Hz$ and $8.4\,Hz$ containing the two eigenfrequencies. The two peaks have nearly the same power around $0.014\,mW$.

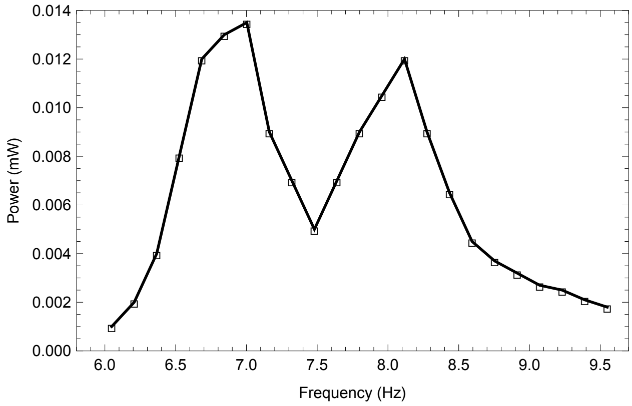
Figure 4b displays the nonlinear responses of the average power beyond the critical amplitude [10, 11, 12] for an excitation level of $0.4\,g$. The harvested powers are $0.78\,mW$ at the first linear resonance and $0.5\,mW$ at the bifurcation point.

On the other hand, in order to extend the nonlinearity to both modes, the excitation level is increased up to $0.8\,g$. Figure 4c shows that the bandwidth has been enlarged and the prototype is able to provide a total maximum power of $27\,mW$ over a frequency range of $8 - 13.05\,Hz$.

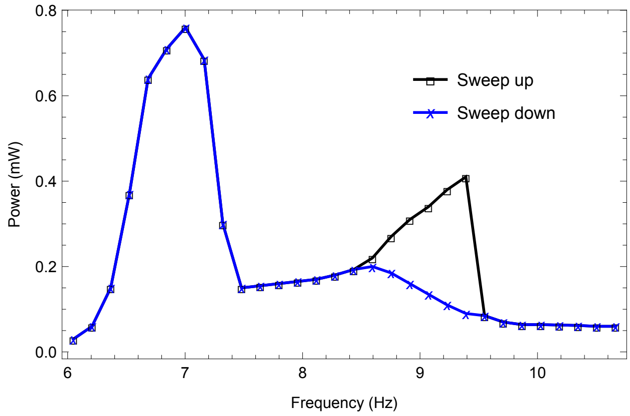
The advantages of the nonlinear coupling are pointed out by comparing the joint performances of two uncoupled VEHs and those of a two-DOFs VEH. The uncoupled VEHs, excited at $1\,g$ provide an overall power of $22\,mW$ over a frequency range of $13.5 - 15.4\,Hz$. However, the two-DOFs VEH excited at $0.8\,g$ has better performances in terms of harvested power (about $27\,mW$) and frequency bandwidth $(8 - 13.05\,Hz)$.

4. Conclusion

In this paper, we propose a novel multi-modal energy harvesting device based on coupled levitated magnets. The equations of motion are derived taking into account the magnetic nonlinearity and electromagnetic damping. The experimental investigations proved that combining the nonlinearties with the modal interactions [13] can enhance both the harvested power and the frequency bandwidth. The proposed harvester enables scavenging the vibration energy in the operating frequency range of $8 - 13.5\,Hz$, and a normalized harvested power of $2\,mW\,cm^{-3}\,g^{-2}$. Notably, the main drawback of this device is the high level of the mechanical damping due to the dry friction between the moving magnets and the lateral surface of the tube. This disadvantage can be avoided using mechanical suspensions with springs [14, 7, 15].



(a)



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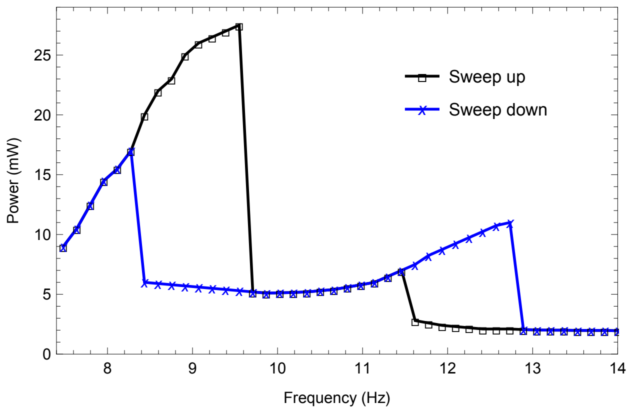

(c)

Figure 4: Frequency responses in term of average power for several excitation amplitudes (a) $0.3\,g$, (b) $0.4\,g$ and (c) $0.8\,g$.

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