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ANALYSIS OF THE MULTIPLE CYLINDRICAL SHELL-PLATE COUPLED SYSTEM VIBRATION CHARACTERISTIC WITH ELASTIC BOUNDARY

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Based on the energy method, the dynamic model of the multiple cylindrical shell-plate coupled system with arbitrary boundary conditions is constructed. By applying four types of coupling springs with arbitrary stiffness at the junction of the coupled structure, the mechanical coupling effects are completely considered. Each of the plate and shell displacement functions is expressed as the superposition of a two-dimensional Fourier series and several supplementary functions. The unknown series expansion coefficients are treated as the generalized coordinates and determined using the familiar Rayleigh-Ritz procedure. The steady responses of forced vibration of the coupled system are obtained. Results of present method show good agreement with the results calculated by finite element method (FEM). In addition, the effects of the dynamic vibration absorber on the vibration characteristic of multiple cylindrical shell-plate coupled system are studied.

Keywords: Shell-plate coupled system, Fourier series, Rayleigh-Ritz procedure, Vibration

1. Introduction

The vibration characteristic analysis of underwater structure is an important process during underwater vehicle design. There are many literatures [1, 2] study the single part of underwater structures, such as cylindrical shells, stiffened shells, plates and the other structures. On the basis of these dynamic models, the simulation method which can calculate the cylindrical shell-plate combined structure was developed in the past several decades.

Tavakoli and Singh [3] presented the state space method to analyze the free vibration of a hermetic can which is composed of a circular cylinder with two circular end plates, and the analytical results are compared with model measurements. In 1992, Cheng and Nicolas [4] study the inherent characteristics of a circular cylindrical shell closed at one end by a circular plate with various boundary supports. However, they neglect the in-plane motion of the circular plate. Huang and Soedel [5] used the receptance method to solve the cylindrical shell-circular plate structure with shear diaphragm-shear diaphragm boundary conditions. Tso [6] studied the wave propagation through cylinder/plate junctions and provided a theoretical basis for the study of the bulkhead. Meixia Chen et al. [7] study the free vibration of ring stiffened cylindrical shell with intermediate large frame ribs. It's worth noting that the equations of motion of annular circular plates are used to describe the motions of ribs.

From above analysis, it's known that the emphasis in the cited references is the free vibration of a circular cylindrical shell coupled with end plates, and few literature are available on the free and forced vibration of the multiple cylindrical shell-plate coupled structure. In this paper, a unified dynamical model for multiple cylindrical shell-plate coupled system with arbitrary boundary conditions is present. Firstly, the whole model is divided into multiple substructures according to the junctions of shell–shell and shell–plate. In order to eliminate the potential discontinuities and accelerate the convergence of the displacement functions, three displacements for the cylindrical shell and the annular plate are invariably expressed as a modified Fourier series. The translational and rotational springs with independent stiffness are introduced to simulate the complex boundary and coupling conditions. Finally, an analytic method is presented to analyze free and forced vibration of coupled cylindrical shell-plate structure. To validate the present method, some results for classical boundary conditions are compared with those calculated by FEM. Furthermore, the effects of the dynamic vibration absorber on the frequency response of multiple cylindrical shell-plate coupled system are discussed in detail.

2. Analysis procedure

2.1 Description of the model

Consider a two cylindrical shells and one circular plate coupling system as shown in Fig. 1. The cylindrical shell is described with the (r,θ,x) cylindrical coordinate system, in which x_s, r_s and θ_s denote the axial, circumferential and radial directions, respectively. The displacements of the cylindrical shell with respect to this coordinate system can be defined by u_{si}, v_{si} and w_{si} ($i=1,2$) in the x_s, θ_s and r_s directions, respectively. L_{si} ($i=1,2$), h_s, R_s are the length, thickness and middle surface radius of the shells, and E_s, μ_s, ρ_s are Young's modulus of elasticity, Poisson's ratio and density. For the plate, annular plate model is utilized in the coupled system as a basic structure component, which can be used to model the annular and circular plate. The geometrical and material parameters of the annular plates are described by a_p, b_p ($\equiv R_s$), h_p, μ_p, ρ_p and E_p. The geometric dimensions of the annular plate are defined in a local orthogonal co-ordinate system (s_p,θ_p,x_p). The width of plate in the radial direction is $R_p(=b_p-a_p)$.

For the circular plate, the equations of strain energy appear to become singular at the pole of the coordinate system. However, it has been observed that the existence of the singularity makes a negligible difference to the physical response of the plate. Therefore, the method adopted here is to assume the inner radius of the annular plate tends to 0, which simply avoids the co-ordinate dependent singularity.

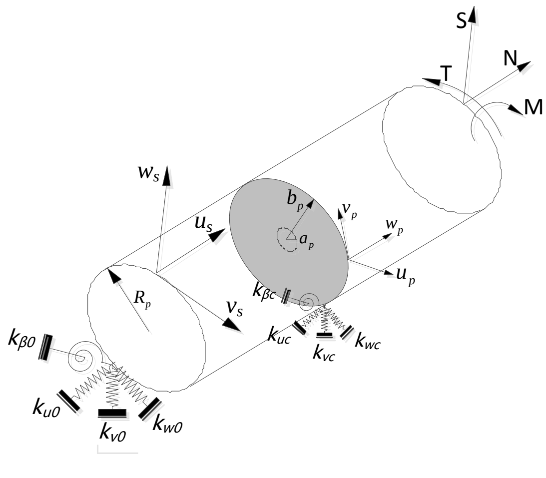

Figure 1: Coordinate system of the multiple cylindrical shell-plate coupled system.

For the sake of simulating the arbitrary elastic boundary and the coupling conditions, artificial spring technique is adopted here. According to the boundary conditions, four distributed springs along the boundary are taken to match bending moments M, transverse shear S, tangential shear force T and axial force N separately. The different boundary and coupling conditions can be easily realized by changing the stiffness of the corresponding springs. At the left end of the cylindrical shell, the k_{u0}, k_{v0}, k_{w0} denote the linear springs in x, θ and r direction and $k_{\beta 0}$ denotes the rotational spring stiffness around θ direction. Similarly, a set of springs $k_{u1}, k_{v1}, k_{w1}, k_{\beta 1}$ can also be applied to the right side.

2.2 Theoretical formulations

The displacement admissible functions of the cylindrical shell and annular plate can be expediently expressed by an improved Fourier series composed of the standard Fourier cosine series and auxiliary functions. For conciseness, the detailed expressions are not shown here. They could be found in reference [8].

Once the form of solutions has been established for the plate and cylindrical shell, the remaining task is to find a suitable set of expansion coefficients that will ensure the series satisfies the governing equations, boundary conditions and joint continuity in some way. A solution can be obtained either in strong form by letting the series satisfy the relevant equations exactly or in weak form by solving the series coefficients approximately. The Ritz method is a direct method to find an approximate solution for boundary value problems. Since the solutions are constructed sufficiently smooth over the solution domain, the unknown series coefficients are calculated by using the Rayleigh–Ritz technique, which is actually equivalent to solve the governing equations, the boundary conditions and coupling conditions directly.

The entire energy for the coupled cylindrical shell-plate system includes five parts: the stain energy V_s and the kinetic energy T_s for the cylindrical shell, the stain energy V_p and the kinetic energy T_p for the annular plate, the spring elastic potential energy V_b denote the energy caused by boundary conditions at the ends of the shell, the potential energy V_c stored at the junction between adjacent substructures and the energy function V_f caused by the external loads.

According to the thin plate theory (Leissa, 1993), the strain energy and kinetic energy for the annular plate can be written as in reference [8]. Consider displacement continuity conditions at the junction and boundary conditions of the coupled system, the potential energy stored in the boundary and coupling springs can be written as

(1) $$V_c = \frac{1}{2}\int_0^{2\pi} [k_{cu}(u_s - w_p)^2 + k_{cv}(v_s - v_p)^2 + k_{cw}(w_s - u_p)^2 + k_{c\beta}(\frac{\partial w_s}{\partial x} - \frac{\partial w_p}{\partial r})^2]\Big|_{x=x_c, s=b} R_s d\theta$$

(2) $$\begin{aligned} V_b &= \frac{1}{2}\int_0^{2\pi}\left[k_{u0}u_s{}^2 + k_{v0}v_s{}^2 + k_{w0}w_s{}^2 + k_{\beta 0}\left(\frac{\partial w_s}{\partial x}\right)^2\right]_{x=0} R_s d\theta \\ &+ \frac{1}{2}\int_0^{2\pi}\left[k_{u1}u_s{}^2 + k_{v1}v_s{}^2 + k_{w1}w_s{}^2 + k_{\beta 1}\left(\frac{\partial w_s}{\partial x}\right)^2\right]_{x=L} R_s d\theta \end{aligned}$$

The frequency response function (FRF) of the coupled structure can be calculated considering the potential energy V_f caused by an external point loads. Under the application of the point force located at (x_0, θ_0), the potential energy V_f can be written as

(3) $$V_f = \int_0^{2\pi}\int_0^{L} (f_{ui}u_i + f_{vi}v_i + f_{wi}w_i)\delta\left(x - x_0, \theta - \theta_0\right) R(x_i) dx_i d\theta_i$$

Where f_{ui}, f_{vi} and f_{wi} are the external force in the x_s, θ_s and r_s directions, respectively. $i=s, p$ denote the cases of the external force acting on the cylindrical shell or circular plate respectively and δ denotes the Dirac function.

When all of the energy expressions are prepared, the Rayleigh-Ritz technique will be used to obtain a weak form of solutions. Thus the Lagrangian energy function can be written as

(4) $$L=\sum_{i=1}^{N_s}(V_s+T_s)+\sum_{i=1}^{N_p}(V_p+T_p)+V_c+V_b+V_f$$

Where N_s and N_p is the number of the cylindrical shells and annular plates in the coupling structure. For simplicity and generality in the analysis, only one annular plate and two cylindrical shells are considered in the following formulation. The current solution procedure can be utilized to derive the characteristic equation of the coupled cylindrical shell-plate structure with more substructures readily. Substituting the total energy into Eq. (4), the eigenvalue problem is formulated by minimizing the Lagrangian function with respect to the arbitrary coefficients. This corresponds to the equations:

(5) $$\frac{\partial L}{\partial q}=0$$

Where q denotes the coefficient vector of the series expansions. The equations (5) yield a set of liner, homogeneous algebraic equations in the unknown coefficients. Then the final system equation can be obtained and summarized in a matrix form as

(6) $$\left(\boldsymbol{K}-\omega^2\boldsymbol{M}\right)\boldsymbol{q}=\boldsymbol{F}$$

Where $\boldsymbol{K}$ and $\boldsymbol{M}$ are the stiffness and mass matrices of the coupling structure respectively. $\boldsymbol{F}$ represents the external force vector. They are written as

(7) $$\boldsymbol{K}=\begin{bmatrix} K_{s1} & K_{s1p1} & 0 \\ K^T_{s1p1} & K_{p1} & K^T_{s2p1} \\ 0 & K_{s2p1} & K_{s2} \end{bmatrix},\boldsymbol{M}=\begin{bmatrix} M_{s1} & 0 & 0 \\ 0 & M_{p1} & 0 \\ 0 & 0 & M_{s2} \end{bmatrix}$$

For conciseness, the detailed expressions for the stiffness and mass matrices will not be shown here. Finally, the natural frequencies and eigenvectors of the coupled structure can be obtained by solving the eigenvalue problem of Eq. (6). Then, the vibration frequency responses can be obtained by modal superposition method.

3. Numerical examples and discussions

3.1 Validation

In a companion paper [8], the authors show that the free vibration characteristics of coupled cylindrical shell-plate systems with arbitrary elastic boundary. The availability of present method is demonstrated by comparisons of results with the FEM and hammer experiment. This section will concentrate on the forced vibration analysis of the cylindrical shell-plate coupled system subject to point force. Unless otherwise specified, the cylindrical shell and the annular plate have the following geometry properties: length of the shell $L_{s1}=L_{s2}=0.6\text{m}$, thickness $h_{s1}=h_{s2}=0.002\text{m}$, mean radius $R_{s1}=R_{s2}=0.24m$, outer diameter of circular plate $b_p=0.24m$, inter diameter $a_p=10^{-4}b_p$. Material properties are as follows: $\rho_{s1}=\rho_{s2}=\rho_p=7800kg/m^3$, $E_{s1}=E_{s2}=E_p=2.1\times10^{11}N/m^2$, $\mu_1=\mu_2=\mu_p=0.3$. The circular plate is coupled to the middle of shells along the axial direction. For reasons of space, we just consider the coupled cylindrical shell-plate structure with free-free end boundary and a rigid coupling condition.

Three points including point A$(0.24, 0, 0.2)$ and point B$(0.24, 0, 0.9)$ at the cylindrical shell and point C$(0.12, 0, 0.6)$ at the middle circular plate are introduced in the local coordinate systems respectively. A unit harmonic force $f_s=e^{j\omega t}$ is applied on the cylindrical shell at point A. In following calculation, the truncated numbers of shell and plate will be uniformly set as $M_s=M_p=12$, $N_s=N_p=10$ to obtain accurate response results. The range of analysis frequencies is from 1 to 200 Hz and the frequency step is set as 1 Hz. The accuracy of present model is validated by making comparisons with the finite element program ANSYS. The finite element model of the cylindrical shell-plate coupled structure, consisting of 4-node SHELL181 elements, is meshed into 15360 meshes to obtain reasonably converged results. The full calculation procedure (direct solver) is employed in ANSYS.

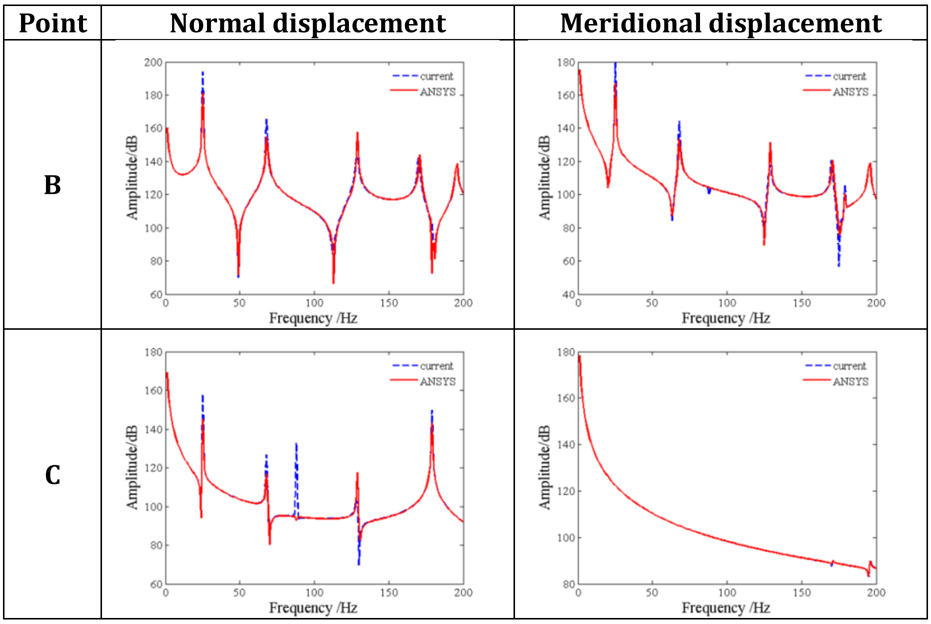

Figure 2: Responses for the cylindrical shell-plate coupled structure subjected to the normal point force. (dB re=$10^{-12}N/m$)

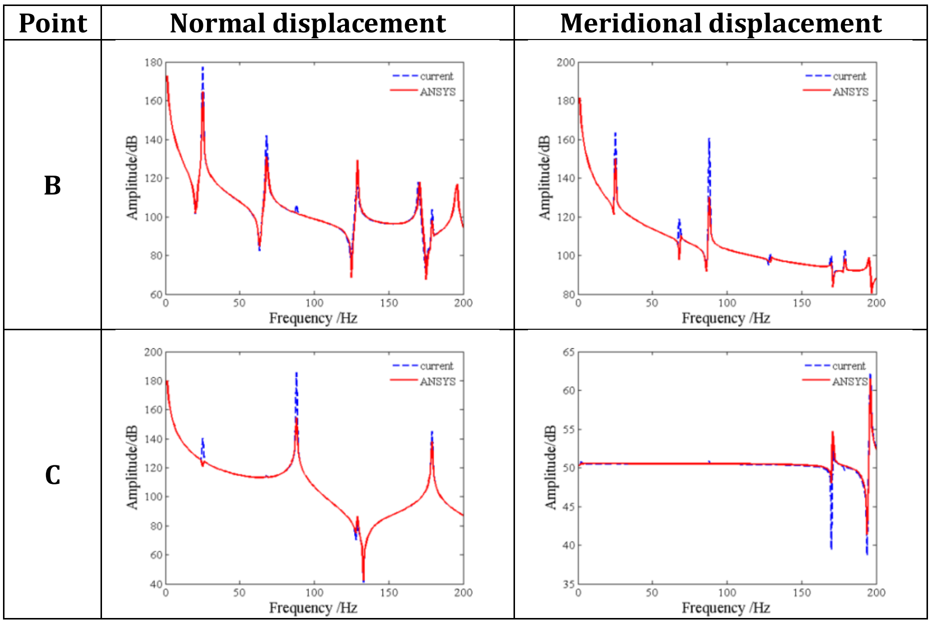

Figure 3: Responses for the cylindrical shell-plate coupled structure subjected to the meridional point force. (dB re=$10^{-12}N/m$)

Figures 2 and 3, respectively, show the comparison of responses at Point B and C for the coupled structure subjected to the normal and meridional point force at Point A between ANSYS and present method. It can be found that the responses of the cylindrical shell-plate coupled structure subjected to the point force show many resonant peaks. This is expected since these external loads can both excite symmetric and antisymmetric vibration modes of the coupled structure. Since no damping has been introduced on the present and finite element models, resonant peaks are supposed to reach an infinite level but are limited thank to a chosen frequency calculation step. Therefore, the amplitude of the resonant peaks are not significant because they are strongly affected by the finite discretization of the frequency range. Except for the locations of resonant peaks, the displacement responses of the two methods are in excellent agreement, which validates the accuracy of the present method to predict the forced vibration of the coupled structure. It should be noted that the computational time as well as the number of degree of freedom (DOF) accounts for the advantages of present method compared to the traditional finite element method. In the calculation, only 2772 DOFs are needed for present method and ANSYS requires more than 90000 DOFs. Thus, the present computational procedure is simple and effective, which make it of great interest to engineers.

As shown in the strain energy expressions of the circular plate, in-plane and out-plane vibration separate completely. When the independent annular plate is subjected to normal or meridional point loading, one of the in-plane and out-plane vibration just will be aroused theoretically. However, due to the coupling effects among the junction of the two components and three displacements of the cylindrical shell, the point load at any direction can excite both in-plane and out-plane vibration of cylindrical shell and circular plate simultaneously. From the comparison of the two figures, it can be observed that the number of resonant peaks to cylindrical shell is more than that of the circular plate and the response at point B is larger than that at point C, which owing to the fact that the vibration distribution is mainly concentrate on the cylindrical shell rather than the middle plate in the considered frequency range.

3.2 Dynamic vibration absorber

In vibration analysis, dynamic vibration absorber (DVA) is a tuned spring-mass system which is commonly designed and tuned to suppress the vibration of a harmonically excited system. In this section, the effects of the single-degree-of-freedom DVA on the vibration characteristics of the cylindrical shell-plate coupled structure will be investigated based on the present method.

For many years there has been considerable interest in the design of dynamic vibration absorbers which change the location of the resonant peaks and reduce the undesirable vibration. The basic assumption of this method is that the vibration response of the objective structure is reduced to zero at its resonance frequency when the resonance of an attached absorber is tuned to match that of the main structure. That is, the energy of the main structure is completely absorbed by the tuned dynamic absorber. Since the lower resonant frequencies are of interest in many engineering problems, the method can be applied to any elastic body including cylindrical shell for suppressing the amplitude of vibration at a certain frequency.

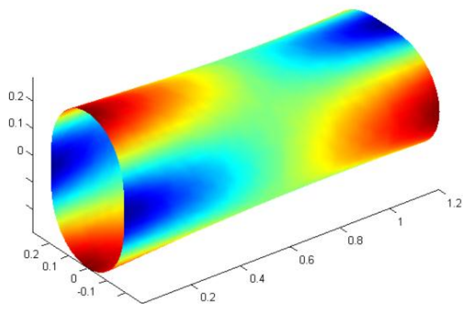

Figure 4: The first mode shape of coupled structure.

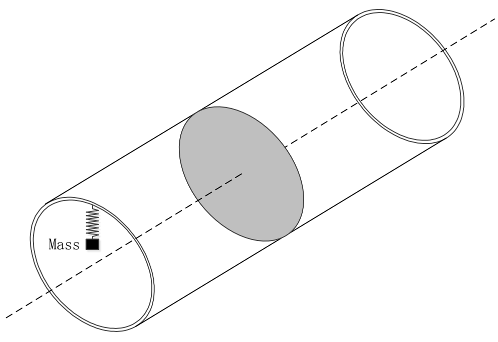

Figure 5: The coupled structure with DVA.

As shown in Fig. 4, the mode shape corresponding to the fundamental frequency (25.01Hz) of the cylindrical shell-plate coupled system is exhibited. It can be seen that the maximum deformation of the coupled structure generate at both ends of the cylindrical shell. Thus, the following problem can be addressed by the use of the spring-mass system to avoid the excitation of structural resonance at fundamental frequency. We consider a cylindrical shell-plate coupled structure with the attached single-degree-of-freedom spring-mass system at point $\mathrm{D}(0.24, 0, 1.2)$. The mass of the DVA is $m_i (i = 1, 2, 3)$ and the stiffness coefficient of the spring between the mass and the main structure is $k_i(i = 1, 2, 3)$. For comparison's purposes, table 1 shows the different absorber parameters used in the analysis. It is worth noting that the mass m_1 is the modal mass corresponding to the fundamental frequency, which is determined by performing the displacement-normalized of vibration mode.

Table 1: properties of the mass-spring system used

DVA No.	mass	stiffness	mass ratio μ
I	m_1=0.15315	k_1= 3781.85	4.933×10^{-3}
II	m_2=0.31048	k_2= 7666.92	1/100
III	m_3= 0.62096	k_3= 15333.83	2/100

The mass ratio μ is defined as the ratio of the absorber mass to that of the coupled system. m_i and k_i meet the following relationships:

(8) $$k_a = (2\pi f)^2 m_a$$

Where the subscript a denotes the absorber and f is the frequency of the vibration mode which is suppressed.

The displacement of the mass block has a form of

(9) $$\boldsymbol{X} = Xe^{j\omega t}$$

The kinetic energy and the potential energy stored in the spring-mass system can be described as

(10) $$V_a = \frac{1}{2} k_a \left(w_{s|x=L_a} - \boldsymbol{X} \right)^2$$

(11) $$T_a = \frac{1}{2} m_a \dot{\boldsymbol{X}}^2$$

Substituting Eqs. (10-11) into the Lagrangian energy function, we can obtain the vibration characteristics of the cylindrical shell-plate coupled system with dynamic vibration absorbers.

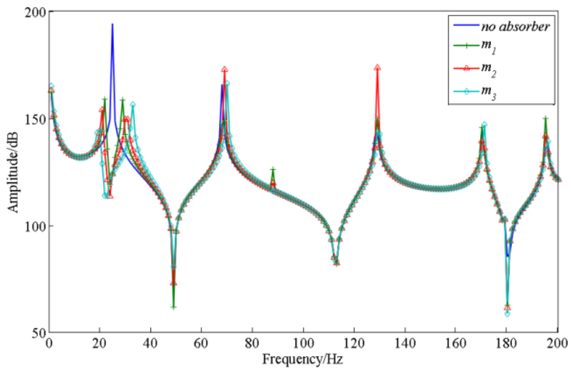

Figure 6: Responses for the cylindrical shell-plate coupled structure attached absorber.

Figure 6 displays the dynamic responses in normal directions at point B. It is interesting to observe that there are two resonance frequencies on the both sides of the fundamental frequency of the objective system, and neither of which equals the original fundamental frequency of the the cylindrical shell-plate coupled system. That is, the objective system is tuned to move the fundamental frequency away from a troubling excitation frequency, which avoids the occurrence of the resonance. Besides the changed fundamental frequency, the other natural frequencies remain unchanged as expected. In addition, as the mass ratio μ increases, the width of the split between the two peaks increases. However, the amplitude of the displacement response has no explicit trend against the mass ratio. From the above analysis, the spring-mass system can reduce the undesirable vibration and achieve the simulation of the vibration absorber.

4. Conclusion

This paper studies the forced vibration characteristics of cylindrical shell-plate coupled system with general boundary conditions. All kinds of boundary conditions can be easily obtained through changing the stiffness of the corresponding springs. Hence, a uniform solution can be obtained for predicting the forced vibration characteristics of the coupled cylindrical shell-plate system regardless of the boundary or coupling conditions. The theoretical results are verified by comparing the present solutions with those obtained from the finite element method. In addition, the present method is also employed to solve the cylindrical shell-plate coupled structure with an attached DVA, which proves a favourable performance to suppress the vibration amplitude at a specified frequency.

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