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INTERFACE WAVES AT AIR/AIR-FILLED POROELASTIC MEDIA BOUNDARIES

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1. INTRODUCTION

The aim of this work is to study the dispersion and attenuation characteristics for waves in porous media by using a poroelastic theory developed by Biot [1,2] in fifties and sixties, and later modified by Attenborough [4]. Biot's theory for wave propagation in a poroelastic medium predicts the existence of two longitudinal wave types and one transverse wave type. The fast longitudinal wave and the transverse wave correspond to the two types of waves in an elastic solid, and the slow longitudinal wave is sometimes regarded as a diffusion wave. All three of the waves are attenuated.

The problems are concerned with waves at an interface between air and an air filled porous elastic half space, and with waves in a layer between air and an air filled porous elastic half space. In order to obtain the frequency equations which give relations between frequency and the complex wavenumber, we shall solve the equations of motion governing the wave motion in the air and in poroelastic media subject to appropriate boundary conditions. The dispersion curves and the attenuation curves are obtained numerically by solving the frequency equations. It is found that similar to purely elastic systems, for the two media system surface waves are almost non-dispersive, and for the three media system waves are dispersive. In the both cases, the low velocity modes have higher attenuation.

2. THEORY

For the modified Biot theory, the properties of poro-elastic medium are specified by the adiabatic bulk modulus for air K_a, bulk modulus of individual grains K_r, shear modulus G, bulk modulus of the assemblage of particles K_b, average mass density ρ, fluid density ρ_f, porosity Ω, flow resistivity σ, tortuosity q, pore shape factor S_p, the ratio of specific heats for the fluid γ and the Prandtl number N_{pr}.

Governing Equations

A rectangular coordinate system is used. The location of an origin depends on the problems that we shall study. The positive z-axis points to the air. We assume that a plane wave propagates in the x-direction.

In air, the equation of motion is given by

$$\nabla^2\phi_1 = \frac{1}{C_a^2}\frac{\partial^2\phi_1}{\partial t^2} \quad (1)$$

where $C_a = \sqrt{K_a/\rho_f}$ and ϕ_1 is a potential function of displacement field in air.

In a poro-elastic medium, when the time harmonic motion ($exp(-i\omega t)$) is assumed and there is no displacement in the y direction, the equations of motion can be given by [3]

$$\nabla^2(H\phi_3 + \alpha M\phi_2) = \frac{\partial^2}{\partial t^2}(\rho\phi_3 + \rho_f\ \phi_2)$$

$$\nabla^2(\alpha M\phi_3 + M\phi_2) = \frac{\partial^2}{\partial t^2}(\rho_f\phi_3 + \rho_c\ \phi_2) \quad (2)$$

$$\nabla^2\phi_4 = \frac{1}{C_4^2}\frac{\partial^2\phi_4}{\partial t^2}$$

where ϕ_2, ϕ_3, ϕ_4 are potential functions of displacement field and

$$H = (K_r - K_b)^2 / (D - K_b) + K_b + 4G_b / 3$$
$$\alpha M = K_r (K_r - K_b) / (D - K_b) = C$$
$$M = K_r^2 / (D - K_b)$$
$$D = K_r (1 + \Omega (K_r / K_f - 1))$$
$$\rho_c = q^2 \rho_f / [\Omega (1 - 2T(\lambda) / \lambda)]$$
$$\lambda = (2q^2 \omega \rho_f i / \Omega \sigma)^{1/2} / S_p$$
$$T(\lambda) = \sqrt{i} J_1(\lambda) / J_0(\lambda)$$
$$K_f = (\gamma K_a) / [1 + 2(\gamma - 1) T(\sqrt{N_{pr}} \lambda) / (\sqrt{N_{pr}} \lambda)]$$
$$C_4 = (G_b / [\rho (1 - \rho_f^2 / (\rho \rho_c))])^{1/2}$$

When the potential functions are known, the displacement vector can be determined by

$$\bar{u}_a = \nabla \phi_1 \tag{3}$$

in air and

$$\bar{u}_s = \nabla \phi_3 + \nabla \times \phi_4 \hat{y} \tag{4}$$

in solid, where $\hat{y}$ is the unit vector in the y-direction. The relative displacement of fluid and solid, in terms of the potential functions, is given by

$$\overline{W} = \nabla \phi_2 - \nabla \times \frac{\rho_f}{\rho_c} \phi_4 \hat{y} \tag{5}$$

Constitutive Equations

In Air, the expression of pressure in terms of the potential function is given by

$$P = -K_a \nabla^2 \phi_1 \tag{6}$$

Stresses in porous media, in terms of displacements, are obtained from the constitutive equations. The constitutive equations are given by [3]

$$\begin{aligned} t_{ii} &= He - \alpha M \zeta - 2G (e_{jj} + e_{kk}) \\ t_{ij} &= Ge_{ij} \\ P_f &= M\zeta - \alpha M e \end{aligned} \tag{7}$$

where t_{ij} are total stresses, P_f is pressure in the pores, e_{ij} are strain components, $e = \nabla \bar{u}_s$ and $\zeta = \nabla \overline{W}$.

Boundary Conditions

There are two types of boundaries. One is the contact between air and a poroelastic medium. On such an interface, conditions given by Deresiewicz and Skalak [5] are employed. The total stress in the z-direction equals the pressure in air.

$$t_{zz} = P_1 \tag{8}$$

The shear stress t_{zr} vanishes since air can not sustain shear stress.

$$t_{zx} = 0 \tag{9}$$

The pressures in the air and in the pores are equal.

$$P_2 = P_1 \tag{10}$$

The condition of the z-component displacement continuity requires

$$u_{z1} = u_{z2} - W_{z2} \tag{11}$$

The other type of interface is a contact between two different poroelastic media. In this case we use the conditions suggested by Biot [2] so that the total stresses, fluid pressure and solid displacement are continuous, also that the normal component of the relative displacement is continuous at this boundary. The equations are given by

$$\begin{aligned} t_{zz2} &= t_{zz3} \\ t_{zr2} &= t_{zr3} \\ P_2 &= P_3 \\ u_{z2} &= u_{z3} \\ u_{r2} &= u_{r3} \\ W_{z2} &= W_{z3} \end{aligned} \tag{12}$$

3. THE TWO MEDIA SYSTEM

First let us consider the two media system. The origin of a coordinate system is set at the interface. We are seeking a solution for wave field which decays with increasing distance from the interface. Such a solution satisfying the equations of motion subject to the boundary conditions can be given by

$$\phi_1 = A_1 e^{-kq_1 z} e^{(ikx - i\omega t)} \tag{13}$$

where A_1 is an arbitrary constant independent of time and position. $q_1 = \sqrt{1 - (V/C_a)^2}$ and the complex velocity $V = \omega / k$; ω is angular frequency, k is complex wavenumber.

In the half space, the potential functions of wave field can be given by

$$\begin{aligned} \phi_2 &= (A_2 e^{kq_f z} + A_3 Q_s e^{kq_s z}) e^{(ikx - i\omega t)} \\ \phi_3 &= (A_3 e^{kq_s z} + A_2 Q_f e^{kq_f z}) e^{(ikx - i\omega t)} \\ \phi_4 &= iA_4 e^{kq_4 z} e^{(ikx - i\omega t)} \end{aligned} \tag{14}$$

where A_2, A_3, A_4 are constants and it can be found that the equations of motion are satisfied if

$$q_f = \sqrt{1 - (V/V_f)^2}$$

$$q_s = \sqrt{1 - (V/V_s)^2}$$

$$q_4 = \sqrt{1 - (V/C_4)^2}$$

$$\left.\begin{matrix} V_f \\ V_s \end{matrix}\right\} = [2A / (-B \pm \sqrt{(B^2 - 4AC)})]$$

$$A = (\alpha M)^2 - HM$$

$$B = H\rho_c + M\rho_s - 2\alpha M\rho_f$$

$$C = \rho_f^2 - \rho\rho_c$$
$$Q_f = (\alpha M / V_f^2 - \rho_f)/(\rho - H / V_+^2)$$
$$Q_s = (H / V_s^2 - \rho)/(\rho_f - \alpha M / V_-^2)$$

In the present problem, the wavenumber is always complex valued. The phase velocity is given by $V_{wvf} = \omega / Re(k)$ rather than the complex velocity $V = \omega / k$. The condition of exponential decay in the direction of the negative z axis requires that $Re(kq_j) > 0$. Since we assume that waves propagate in the positive x-direction, $Im(k)$ must be greater than zero.

The frequency equation for the two media system

On substituting the general solutions into the boundary conditions by use of the constitutive equations, we can obtain a system of 4 homogeneous equations in unknowns A_1, A_2, A_3 and A_4. The condition for existence of non-trivial solution requires that the coefficient determinant is zero which gives the relationship between the frequency and the complex wavenumber

$$\begin{vmatrix} q_1 & q_f(Q_f - 1) & q_s(1 - Q_s) & \rho_f / \rho_c + 1 \\ K_a(\frac{V}{C_a})^2 / G & ((HQ_+ - \alpha M)(\frac{V}{V_f})^2 - 2GQ_f)/G & ((H - \alpha M Q_-)(\frac{V}{V_s})^2 - 2G)/G & 2q_4 \\ 0 & -2q_f Q_f & -2q_s & (1 + q_4^2) \\ K_a(\frac{V}{C_a})^2 / G & (\frac{V}{V_f})^2 (M - Q_f \alpha M)/G & (\frac{V}{V_s})^2 (MQ_s - \alpha M)/G & 0 \end{vmatrix} = 0 \qquad (15)$$

Eq.(15) represents the frequency equation for interface waves between an air and a poroelastic medium. This frequency equation is in the form of 4 by 4 determinant. Equating the real part and the imaginary part of the frequency equation to be zero respectively gives two simultaneous equations in three unknowns frequency ω, the real part and the imaginary part of the wavenumber, k_r and k_i. For given material constants, we can solve it numerically. We fix ω and solve the simultaneous equations to obtain the corresponding values of k_r and k_i.

Numerical results for the two media problem

In the air, the material constants are given

$$K_a = 1.4 \times 10^5 Pa \qquad \rho_a = 1.2 kg / m^3$$

In the half space, the parameters chosen correspond to an acoustically-hard soil or sand [3]

$$K_r = 3. \times 10^{10} Pa \qquad K_b = 8 \times 10^8 Pa, \quad \gamma = 1.4 \quad q = \sqrt{2.5}$$
$$\rho = 1. \times 10^3 kg / m^3 \quad \rho_f = 1.2 kg / m^3 \quad G = K_b \quad N_{pr} = \sqrt{0.7}$$
$$\sigma = 3 \times 10^5 Ns / m^4 \qquad \Omega = 0.4 \qquad S_p = 0.5$$

We vary frequency from 100 Hz to 500 Hz.

For the parameter we chosen, the bulk speeds are given by

$C_+ = \omega / Re(k_+) = 1366\ m/s$ ---- *Fast longitudinal speed*

$C_s = \omega / Re(k_4) = 894\ m/s$ --- *Transverse speed*

$C_a = 341\ m/s$ --- *Sound speed in air*

$C_- = \omega / Re(k_-) = 38 - 81\ m/s$ --- *Slow longitudinal speed*

It is found that three surface waves are possible on the surface of an air-filled poroelastic half-space with parameters corresponding to a dry soil. One of these corresponds to the surface wave frequently predicted in the solution for the field due to a point source above an impedance boundary. However the dispersion and attenuation are less than predicted over a rigid porous half space. Nevertheless, it is highly attenuated. This mode has a speed close to the sound speed of air and may be called a pseudo-Stoneley wave. The second surface wave type which has a speed slower than the bulk transverse wave speed, is a pseudo-Rayleigh wave having low dispersion but fairly high attenuation. The third type has very little attenuation or dispersion and travels close to the fast longitudinal wave speed.

We also examine changes in the influence of some material constants. Change of the pore shape factor has only little effect on those surface modes. But different porosities change the attenuation for the pseudo-Stoneley mode. The high porosity has a rapidly attenuation and it depends on frequency very much. Change in G effects the pseudo-Rayleigh mode but has little influence on the pseudo-Stoneley mode.

4. THE THREE MEDIA SYSTEM

We assume that the origin is at the interface between the two poroelastic media. The thickness of the layer is H. Similar solutions to the two media system are used for the two half spaces. In the layer, the potential function of wave field can be given by

$$\begin{aligned}
\phi_{2l} &= (A_{2l}\sin(kq_{fl}z) + B_{2l}\cos(kq_{fl}z) + A_{3l}Q_{sl}\sin(kq_{sl}z) \\
&\qquad + B_{3l}Q_{sl}\cos(kq_{sl}z))e^{i\omega t - ikx} \\
\phi_{3l} &= (A_{2l}Q_{fl}\sin(kq_{fl}z) + B_{2l}Q_{fl}\cos(kq_{fl}z) + A_{3l}\sin(kq_{sl}z) \\
&\qquad + B_{3l}\cos(kq_{sl}z))e^{i\omega t - ikx} \qquad (16) \\
\phi_{4l} &= (A_{4l}\sin(kq_{4l}z) + B_{4l}\cos(kq_{4l}z))e^{i\omega t - ikx}
\end{aligned}$$

where A_{2l}, A_{3l}, A_{4l}, B_{2l}, B_{3l} and B_{4l} are constants and

$$\begin{aligned}
q_{fl} &= \sqrt{1-(V/V_{fl})^2} \\
q_{sl} &= \sqrt{1-(V/V_{sl})^2} \\
q_{4l} &= \sqrt{1-(V/C_{4l})^2} \\
\left.\begin{matrix} V_{fl} \\ V_{sl} \end{matrix}\right\} &= [\pm(H_l^2\rho_{cl}^2 - 4H_lC_l\rho_{fl}\rho_{cl} - 2H_l\rho_lM_l + 4H_l\rho_{fl}^2M_l + 4C_l^2\rho_l\rho_{cl} \\
&\qquad - 4C_l^2\rho_l\rho_{fl}M_l + \rho_l^2M_l^2) + H_l\rho_{cl} - 2C_l\rho_{fl} + \rho_lM_l]/(2(H_lM_l - C_l^2)) \\
Q_{fl} &= -(k^2C_l - \omega^2\rho_{fl}^2 + C_lq_{fl}^2)/(k^2H_l - \omega^2\rho_l + H_lq_{fl}^2) \\
Q_{sl} &= -(k^2H_l - \omega^2\rho_l + H_lq_{sl}^2)/(k^2C_l - \omega^2\rho_{fl}^2 + C_lq_{sl}^2)
\end{aligned}$$

where l in a subscript indicates the layer.

The Frequency Equation for the three media system

Similarly from the boundary conditions, Eqs(8-11) at $z=H$ and Eqs(12) at $z=0$, we can obtain a system of 10 homogeneous equations in ten unknowns. The vanishing of the coefficient determinant gives the frequency equation. The elements of the determinant are given by

$$a_{1,1} = -q_1\exp(-Hkq_1), \quad a_{1,2} = -\cos(Hkq_{fl})q_{fl}Q_{fl}, \quad a_{1,3} = \sin(Hkq_{fl})q_{fl}Q_{fl},$$

$$a_{1,4} = \cos(Hkq_{sl})q_{sl}, \quad a_{1,5} = \sin(Hkq_{sl})q_{sl}, \quad a_{1,6} = \sin(Hkq_{4l})\rho_{fl}/\rho_{cl},$$

$a_{1,7} = \cos(Hkq_{4l})\rho_{fl}/\rho_{cl}$. $\quad a_{2,1} = K_a(1-q_1^2)\exp(-Hkq_1)$.

$a_{2,2} = \sin(Hkq_{fl})(q_{fl}^2 Q_{fl}(H_l - aM_l) + q_{fl}^2(M_l - aM_l) + Q_{fl}(H_l - 2G_l - aM_l))$.

$a_{2,3} = \cos(Hkq_{fl})(q_{fl}^2 Q_{fl}(H_l - aM_l) + q_{fl}^2(M_l - aM_l) + Q_{fl}(H_l - 2G_l - aM_l))$.

$a_{2,4} = \sin(Hkq_{sl})(q_{sl}^2 Q_{sl}(M_l - aM_l) + q_{sl}^2(H_l - aM_l) + (H_l - 2G_l - aM_l))$.

$a_{2,5} = \cos(Hkq_{sl})(q_{sl}^2 Q_{sl}(M_l - aM_l) + q_{sl}^2(H_l - aM_l) + (H_l - 2G_l - aM_l))$.

$a_{2,6} = \cos(Hkq_{4l})q_{4l}(2\rho_{fl}/\rho_{cl}G_l - aM_l + M_l)$.

$a_{2,7} = -\sin(Hkq_{4l})q_{4l}(2\rho_{fl}/\rho_{cl}G_l - aM_l + M_l)$.

$a_{3,2} = -2\cos(Hkq_{fl})q_{fl}Q_{fl}$. $\quad a_{3,3} = 2\sin(Hkq_{fl})q_{fl}Q_{fl}$. $\quad a_{3,4} = -2\cos(Hkq_{sl})q_{sl}$.

$a_{3,5} = 2\sin(Hkq_{sl})q_{sl}$. $\quad a_{3,6} = \sin(Hkq_{4l})(1-q_{4l}^2)\rho_{fl}/\rho_{cl}$.

$a_{3,7} = \cos(Hkq_{4l})(1-q_{4l}^2)\rho_{fl}/\rho_{cl}$. $\quad a_{4,1} = K_a(q_1^2 - 1)\exp(-Hkq_1)$.

$a_{4,2} = \sin(Hkq_{fl})(q_{fl}^2 Q_{fl} aM_l - q_{fl}^2 M_l + Q_{fl} aM_2)$.

$a_{4,3} = \cos(Hkq_{fl})(q_{fl}^2 Q_{fl} aM_l - q_{fl}^2 M_l + Q_{fl} aM_2)$.

$a_{4,4} = \sin(Hkq_{sl})(q_{sl}^2 M_l - Q_{sl} q_{sl}^2 M_l + aM_l)$.

$a_{4,5} = \cos(Hkq_{sl})(q_{sl}^2 M_l - Q_{sl} q_{sl}^2 M_l + aM_l)$. $\quad a_{4,6} = -\cos(Hkq_{sl})q_{4l}M_l$.

$a_{4,7} = \sin(Hkq_{sl})q_{4l}M_l$. $\quad a_{5,3} = (q_{fl}^2(Q_{fl}aM_l - M_l) + Q_{fl}aM_l)$.

$a_{5,5} = q_{sl}^2(aM_l - Q_{sl}M_l) + aM_l$. $\quad a_{5,6} = -q_{4l}M_l$. $\quad a_{5,8} = q_s^2(Q_s aM - M) - Q_f aM$.

$a_{5,9} = q_s^2(aM - Q_s M) - aM$. $\quad a_{5,10} = q_4 M$.

$a_{6,3} = (q_{fl}^2 Q_{fl}(H_l - aM_l) + q_{fl}^2(M_l - aM_l) + Q_{fl}(H_l - 2G_l - aM_l))$.

$a_{6,5} = q_{sl}^2 Q_{sl}(M_l - aM_l) + q_{sl}^2(H_l - aM_l) + (H_l - 2G_l - aM_l)$.

$a_{6,6} = q_{4l}(2\rho_{fl}/\rho_{cl}G_l - aM_l + M_l)$.

$a_{6,8} = (q_f^2 Q_f(H - aM) + q_f^2(M - aM) - Q_f(H - 2G - aM))$.

$a_{6,9} = q_s^2 Q_s(M - aM) + q_s^2(H - aM) - (H - 2G - aM)$. $\quad a_{6,10} = -q_4(2\rho_f/\rho_c G - aM + M)$.

$a_{7,2} = 2q_{fl}Q_{fl}G_l$. $\quad a_{7,4} = 2q_{sl}G_l$. $\quad a_{7,7} = (q_{4l}^2 - 1)G_l\rho_{fl}/\rho_{cl}$.

$a_{7,8} = -2q_f Q_f G$. $\quad a_{7,9} = -2q_s G$. $\quad a_{7,10} = (q_4^2 - 1)G\rho_f/\rho_c$.

$a_{8,3} = Q_{fl}$. $\quad a_{8,5} = 1$. $\quad a_{8,6} = -q_{4l}\rho_{fl}/\rho_{cl}$.

$a_{8,8} = -Q_f$. $\quad a_{8,9} = -1$. $\quad a_{8,10} = q_4\rho_f/\rho_c$.

$a_{9,2} = q_{fl}Q_{fl}$. $\quad a_{9,4} = q_{sl}$. $\quad a_{9,7} = -\rho_{fl}/\rho_{cl}$.

$a_{9,8} = -q_f Q_f$. $\quad a_{9,9} = -q_s$. $\quad a_{9,10} = \rho_f/\rho_c$.

$a_{10,2} = q_{fl}$. $\quad a_{10,4} = q_{sl}Q_{sl}$. $\quad a_{10,7} = -1$.

$a_{10,8} = -q_f$. $\quad a_{10,9} = -q_s Q_s$. $\quad a_{10,10} = 1$.

Numerical results for the three media system

In the air, the parameters are used as before.

In the porous layer:

$K_{rl} = 3. \times 10^8\ Pa$ $\quad K_{bl} = 1. \times 10^8\ Pa$ $\quad \gamma = 1.4$ $\quad q_l = \sqrt{2.5}$

$\rho_l = 1.7 \times 10^3\ kg/m^3$ $\quad \rho_{fl} = 1.2\ kg/m^3$ $\quad G_l = K_{b2}$ $\quad H = 1.0\ m$

$\sigma_l = 3.6 \times 10^4\ Ns/m^4$ $\quad \Omega_l = 0.3$ $\quad S_{pl} = 0.36$ $\quad N_{pr} = \sqrt{0.7}$

In the porous half space

$K_r = 8. \times 10^8\ Pa$ $\quad K_b = 5 \times 10^8\ Pa$ $\quad \gamma = 1.4$ $\quad q = \sqrt{2.5}$

$\rho = 2.6 \times 10^3\ kg/m^3$ $\quad \rho_f = 1.2\ kg/m^3$ $\quad G = K_b$ $\quad N_{pr} = \sqrt{0.7}$

$\sigma = 3 \times 10^6 Ns/m^4$ $\quad \Omega = 0.02$ $\quad S_p = 0.36$

Solutions of the frequency equation are plotted in a three dimensional frame, frequency, phase velocity and attenuation. Three dispersive modes are shown in the figure. One of those is found only for a small range of frequency. The three curves have no intersections. It can be seen that not only the phase velocity but also the attenuation change with frequency. At low phase velocities, the attenuation drops very rapidly to the order of 10^{-2}.

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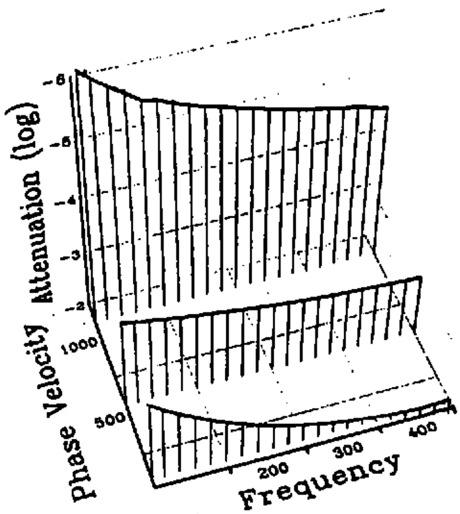

Figure 1. Dispersion and attenuation characteristics of waves in the two media system.

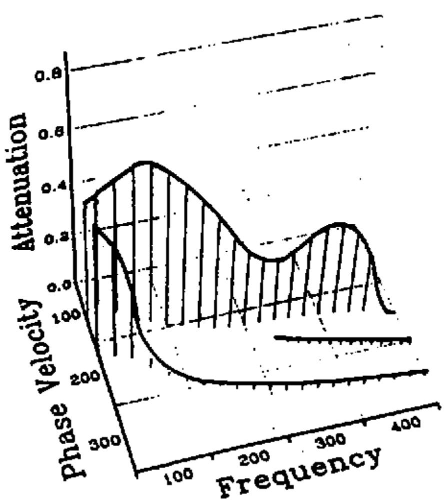

Figure 2. Dispersion and attenuation characteristics of waves in the three media system.