DIFFUSION CONTROL FOR MULTI-CHANNEL ANC SYSTEMS USING FILTERED-X ALGORITHMS

Yijing Chu

The Hong Kong Polytechnic University, Hung Hom, Kowloon, HK
email: yijing.chu@polyu.edu.hk

Cheuk-Ming Mak

The Hong Kong Polytechnic University, Hung Hom, Kowloon, HK
email: cheuk-ming.mak@polyu.edu.hk

The centralized control for multi-channel active noise control (ANC) systems usually cost considerable processing power due to the transfer functions between a large number of loudspeakers and error microphones; while the decentralized control has the increased risk of global instability. This paper studies the diffusion control for multi-channel ANC systems over a distributed adaptive network based on filtered-x algorithms. Distribution of the controller network saves computational burden and yields spatial diversity, which has been explored alongside the temporal properties in order to enhance the robustness of the multi-channel control system and improve the convergence of the controllers. A group of diffusing Fx-like adaptive algorithms employing either the single- or multiple-measurement has been proposed for ANC. Different strategies, including the centralized, decentralized, incrementally distributed and diffusion control, have been compared. Simulation results show the improved performance of the proposed algorithms in terms of the convergence performance and stability.

Keywords: active noise control, diffusion control, distributed adaptive network, filtered-x adaptive algorithm

1. Introduction

The active control of noises usually involves an introduction of a number of loudspeakers, or known as secondary sources, and error microphones in order to achieve the global control over a region of space [1]. One of the most widely used active noise control (ANC) algorithms for the controller is the filtered-x LMS (FxLMS) algorithm [2] as shown in Fig. 1, where the undesirable sound $\{d(n)\}$ generated from the source signal $\{x(n)\}$ is to be cancelled by the acoustic signal $\{y(n)\}$ generated from a loudspeaker through an appropriate excitation using an adaptive filter-based controller $\{w_k(n)\}$. An error microphone is used to pick up the residual signal $e(n)$ at time instant n to be minimized. The acoustic path from source to microphone is called the primary path, which is modelled as a finite impulse response (FIR) $\{p(n)\}$. Similarly, the acoustic path from loudspeaker to microphone is called the secondary path and modelled as another FIR $\{s(n)\}$.

For multi-channel ANC systems, the corresponding ANC algorithm is called the multiple error FxLMS (MEFxLMS), which has been proved to perform very well for centralized processing [1]. An important issue in the MEFxLMS algorithm is the modelling of the secondary paths. Since each controller needs to process the signal received by the corresponding error microphone from all the secondary sources, the centralized ANC system needs to process M^2 transfer functions (M is the number

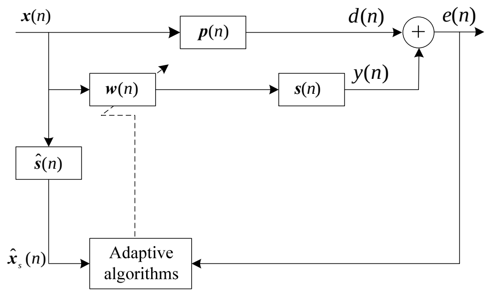
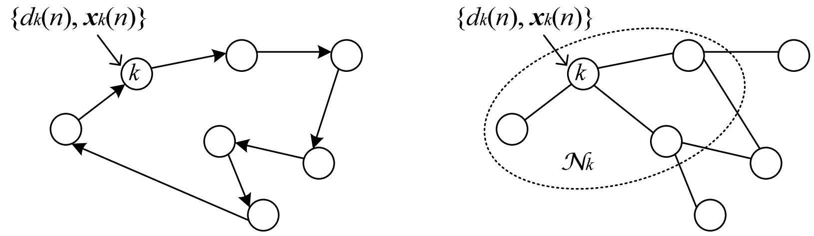


Figure 1: Block diagram of an ANC system.

of loudspeakers and error microphones) in order to minimize the global error, which is quite computationally consuming. A technique called the decentralized control has been developed to avoid this problem and facilitate the hardware and design of the multi-channel control system [3]. Instead of dealing with the global error that is received by all the M error microphones, each controller minimizes the error power at one cancellation point only. In this case, each independent controller only need to consider one transfer function such that only M direct transfer functions from each secondary source to the corresponding error microphone are needed for the multi-channel ANC system. The main advantage of the decentralized system is the ability of distributing the computational burden, but in most cases it cannot outperform the centralized system and the decentralized control brings in the major drawback of the increased risk for global instability [4]. A lot of effort has been taken to derive a sufficient stability condition for such systems [4][5] employing either feedforward [6] or feedback structures [7].

As an alternative, an implementation of the MEFxLMS algorithm over a network of distributed acoustic nodes is recently proposed in [8]. The formulation uses an incremental collaborative strategy [9][10] in the network as shown in Fig. 2(a) so that a distributed MEFxLMS algorithm is proposed. This algorithm includes secondary paths related to the controller's implementation on an incremental way, which is more robust than the decentralized ANC algorithm. The incremental version of MEFxLMS, however, suffers from several drawbacks for real-time adaptation over networks. First of all, the incremental structure is sensitive to the failure of a single node (the controller and the corresponding error microphone in the multi-channel ANC system) within the cyclic loop that visits the network incrementally. The cooperation between nodes is also limited since each node receives data from one preceding node and passes data to the successor. Moreover, for the update of controllers, it is difficult to determine a cyclic path for N incremental steps.



(a) (b)

Figure 2: Distributed network of node k, at time n: (a) an incremental structure where nodes update along a cyclic loop, and (b) a diffusion structure where nodes communicate within a neighborhood $\mathcal{N}_k$. Node k collects the data set { $d_k(n)$, $\boldsymbol{x}_k(n)$ }.

We thus aim at the derivation of a diffusion [11][12] strategy as shown in Fig. 2(b) for the MEFx-like algorithm that do not suffer from these limitations. Diffusion techniques take advantage of the following flexibility. Instead of receiving information from one neighbour at a time and passing information to the other neighbour along a cyclic loop, a diffusion structure allows simultaneous communication between nodes within a neighbourhood. This structure can achieve low computational

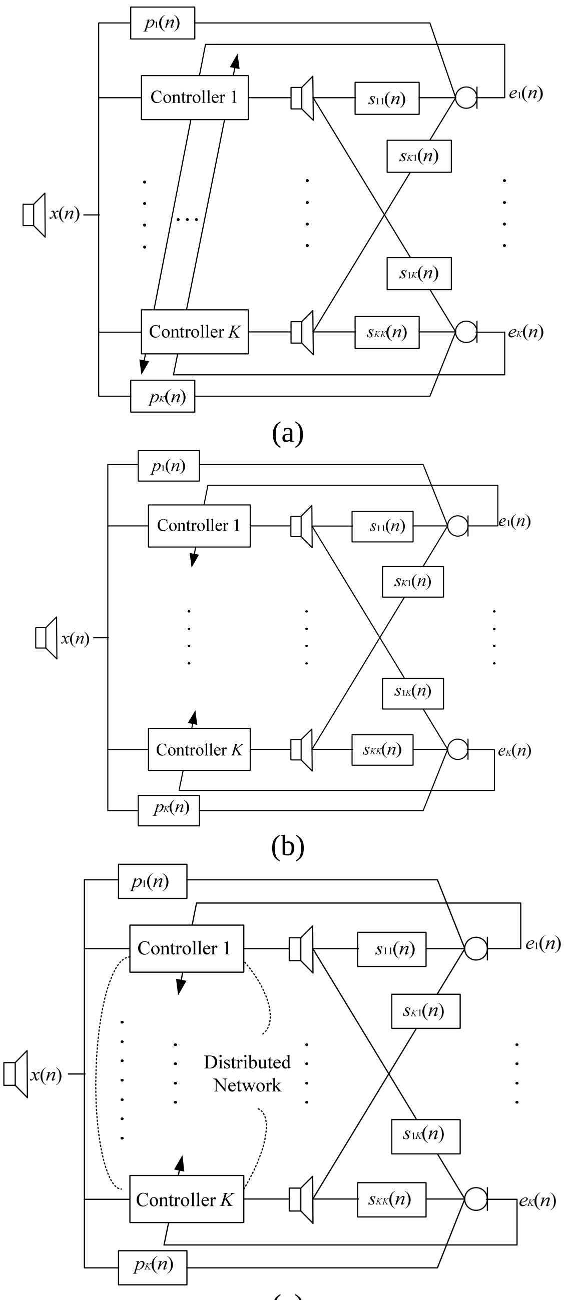


Figure 3: Block diagram of ANC systems with different control strategies: (a) centralized; (b) decentralized; and (c) distributed.

complexity that is comparable to the decentralized control, whereas it could also benefit from the advanced properties as the centralized system that is more stable and has better performance in terms of convergence speed and steady-state residual noises.

In particular, we conduct a detailed comparison between three existing multi-channel ANC structures, i.e. the centralized, decentralized and distributed ANC systems. Then, the diffusion multi-channel ANC system is introduced and the diffusion Fx- (Diff-Fx-) algorithms, with either single measurement or multiple measurement, are formulated. Compared to the single measurement algorithms such as the Diff-FxLMS, the multiple measurement algorithms, such as the Diff-Fx affine projection algorithm (Diff-FxAPA), have better convergence properties for coloured noises. The proposed Diff-Fx- algorithms update each ANC controller following a diffusion strategy, where the combination weights for the neighbouring nodes are chosen according to the commonly-used Metropolis rules [13]. Simulations are conducted to verify the improved performance of the proposed algorithms and it shows that the Diff-Fx- algorithms are particularly prominent due to the improved robustness with enlarged stable region [1] and could provide comparable performance with the centralized ANC at a much lower computational cost.

The rest of the paper is organized as follows: in Section 2 we compare three MEFxLMS algorithms for centralized, decentralized and distributed ANC. In Section 3, the Diff-FxLMS and Diff-FxAPA are proposed. Simulations are conducted in Section 4. Finally conditions are drawn in Section 5.

2. The Proposed Diff-Fx-like Algorithms

2.1 Centralized, Decentralized and Distributed ANC

The implementation of the centralized multi-channel ANC with K error microphones and K controllers, as shown in Fig. 3(a), requires the controllers to be updated in response to all the available error signals so as to minimize a global cost function [1], such as the LMS error

$$J = \sum_{k=1}^{K} J_k = E[\sum_{k=1}^{K} e_k^2(n)] \text{, with } J_k = E[e_k^2(n)] \tag{1}$$

where $e_k(n)$ is the residue received by the error microphone at the kth node. The centralized strategy thus has the following learning rule for the kth controller

$$\boldsymbol{w}_k(n+1) = \boldsymbol{w}_k(n) - \mu \nabla_{\boldsymbol{w}_k} J \tag{2}$$

where $\boldsymbol{w}(n) = [w_1(n), w_2(n), \ldots, w_L(n)]^T$ is a finite impulse response (FIR) filter with length L, and the gradient of J involves all the K secondary paths from the kth loudspeaker to all the error microphones $\boldsymbol{s}_{kl}$ for $l = 1,\ldots,K$.

In the decentralized ANC system, the schematic diagram of which is depicted in Fig. 3(b), each error signal is used to update the weights of only one controller. Therefore, the weight update scheme of the controllers is independent with each other. For the kth controller, the learning rule reads

$$\boldsymbol{w}_k(n+1) = \boldsymbol{w}_k(n) - \mu \nabla_{\boldsymbol{w}_k} J_k \,. \tag{3}$$

Compared to (2), the decentralized control only involves the secondary path from the kth loudspeaker to the kth error microphone $\boldsymbol{s}_{kk}$.

Unlike the passive control methods [14]–[16], the ANC algorithms have higher probability to diverge. To deal with the instability problems of the decentralized control, distributed strategies can be applied to the individual decentralized controllers such that the control status can be justified and rearranged. For example, the incremental structure passes the local control information from one controller to another along a cyclic path as shown in Fig. 3(c), while the diffusion structure provides more flexibility for the network topology.

2.2 The Diff-FxLMS Algorithm

To address the optimization problem in (1), the distributed network could employ either the combine-then-adapt (CTA) diffusion or adapt-then-combine (ATC) type, which are fundamentally identical to each other [17]. In this subsection, we develop the ATC-FxLMS algorithm for the multi-channel ANC system, which takes the form as:

$$\boldsymbol{w}_k(n) = \sum_{l \in \mathcal{N}_k} a_{lk} \boldsymbol{\psi}_l(n) \tag{4}$$

$$\boldsymbol{\psi}_k(n+1) = \boldsymbol{w}_k(n) - \mu_k e_k(n) \hat{\boldsymbol{x}}_k(n) \tag{5}$$

where $\mu_k > 0$ is the local step-size, $\hat{\boldsymbol{x}}_k(n) = [\hat{x}_k(n), \hat{x}_k(n-1), \ldots, \hat{x}_k(n-L+1)]^T$ is the filtered input vector of length L, and the cluster averaged estimate, $\boldsymbol{w}_k(n)$, combines the local solutions $\boldsymbol{\psi}_k(n)$ over $l \in \mathcal{N}_k$ using weights a_{lk} as shown in (4). The nonnegative scalars { a_{lk} } are selected such that $\boldsymbol{A} = \{a_{lk}\}$ is left stochastic **Error! Reference source not found.**4]. This algorithm could be implemented in either time or frequency domain.

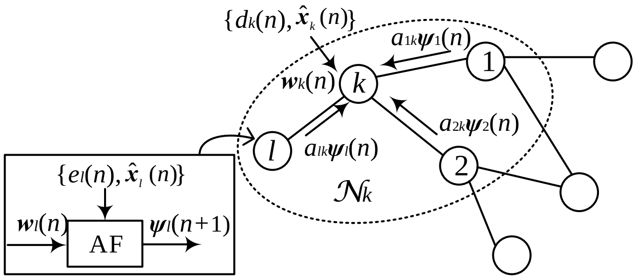


Figure 4: A diagram of diffusion strategy of the adaptive filter (AF) at node *k* and time *n*.

For algorithms employing multiple measurements, such as APA and subband adaptive filters (SAF), the local estimate can be updated as

$$\boldsymbol{\psi}_k(n+1) = \boldsymbol{w}_k(n) + \mu_k \hat{\boldsymbol{X}}_k(n)\boldsymbol{P}_k(n)\boldsymbol{e}_k(n) \quad (6)$$

where $\boldsymbol{e}_k(n)$ ($M \times 1$) is the multiple measurements for the *k*th error microphone, $\hat{\boldsymbol{X}}_k(n)$ ($L \times M$) is the filtered signal matrix, and $\boldsymbol{P}_k(n)$ is chosen as $E[\hat{\boldsymbol{X}}_k^T(n)\hat{\boldsymbol{X}}_k(n) + \xi \boldsymbol{I}_M]^{-1}$ with $\xi \boldsymbol{I}_M$ a small positive multiple of the $M \times M$ identity matrix. It should be noted that the multiple error and the signal matrix could either be time domain signal for APA or subband signal for the SAF.

3. Simulation Results

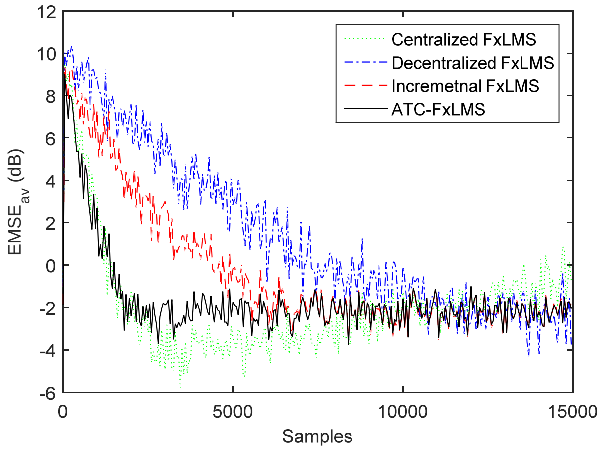
In this section, computer simulations are conducted to study the performance of the multi-channel ANC system using different control strategies or adaptive filtering algorithms.

The following simulations are based on the feedforward ANC structure using Fx-like algorithms. In experiments 1, different control strategies are compared, including the conventional centralized, decentralized and incrementally distributed control. In experiment 3, the proposed Diff-Fx-like algorithms are compared with each other.

The primary path is generated randomly as FIR filters. The multi-channel ANC system is supposed to be equipped in a small cabinet, say an automobile, and the noise source is close to the error microphones. Thus, a short primary path of length 12 is considered and the secondary path is assumed to have a length of 5. The length of the controllers is set to be 10, which is slightly shorter than that of the primary path. The background noise variance is set to $\sigma_\eta^2 = 0.001$. Both white and coloured noise sources are considered and their variances are normalized to be $\sigma_x^2 = 1$. For the coloured input, a first order autoregressive (AR) process is considered and it is given by $x(n) = 0.5x(n-1) + v(n)$, where $v(n)$ is zero-mean and white Gaussian distributed. All simulation results are averaged over 100 Monte Carlo runs if not specified.

3.1 Comparison of different control strategies

In this experiment, the MEFxLMS algorithm for three different control strategies is considered. The multi-channel ANC system under study has 5 loudspeaker and 5 error microphones. For the white input, the step-sizes of the ANC controllers are, respectively, $\mu_{Cen} = \mu_{Dec} =$ 0.0001, $\mu_{Inc} =$ 0.00015, and $\mu_{ATC} =$ 0.0004 for the centralized, decentralized, incrementally distributed and diffusion control strategies. Metropolis weights [13] are used for the combination matrix $\boldsymbol{A}$ of the ATC-FxLMS algorithm. Fig. 5(a) shows the averaged EMSE curves over the nodes for the ANC controller. The convergence speed of the decentralized FxLMS algorithm is decreased significantly compared to the centralized version. The proposed ATC-FxLMS algorithm, on the other hand, has similar convergence performance as the centralized one at a computational cost that is comparable to the decentralized control. Next, the above experiment is repeated for the coloured source. The step-sizes for different algorithms are set as $\mu_{Cen} = \mu_{Dec} = 0.0003$, $\mu_{Inc} = 0.00045$, and $\mu_{ATC} = 0.0012$ for the related control strategies. As shown in Fig. 5(b), the decentralized FxLMS algorithm is no longer stable. The incrementally distributed control achieves similar performance with the centralized one while the proposed ATC-FxLMS algorithm even has faster convergence speed than the other algorithms, especially when the noise sources emit coloured signals.

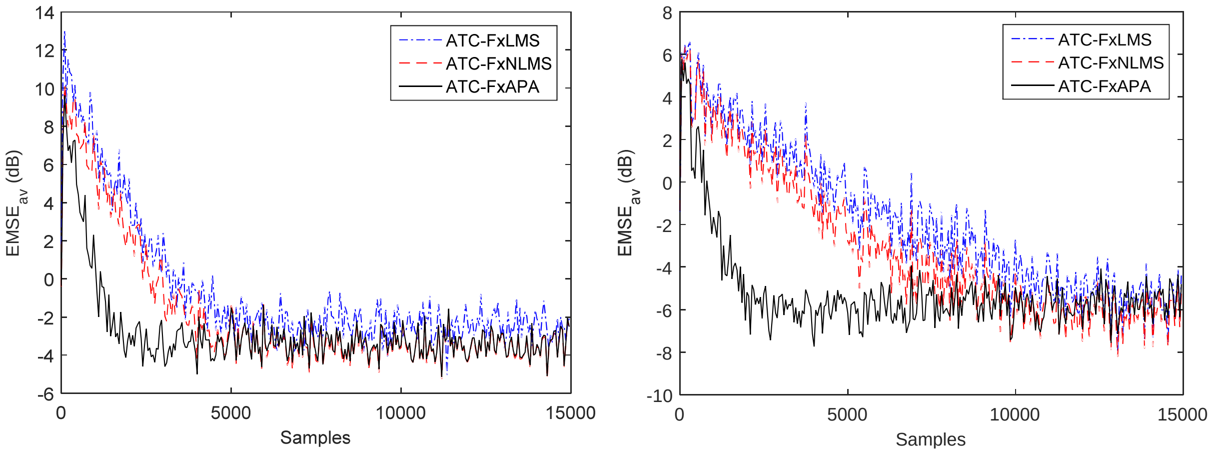


(a) (b)

Figure 5: Convergence curves of the controllers using different strategies: (a) white noise sources; and (b) colored noise sources.

3.2 Comparison of different diffusion Fx- adaptive algorithms

In this experiment, the Diff-Fx-based algorithms are compared with each other. The algorithms under study include the ATC-FxLMS, ATC-FxNLMS that employ single measurement and the ATC-FxAPA that employs multi-measurement. The settings of the ANC system is identical to that in Section 3.1. For both of the white and coloured noise sources, the step-sizes of the ANC controllers are chosen as $\mu_{LMS} = 0.0002$, $\mu_{NLMS} = 0.0015$, and $\mu_{APA} = 0.001$, respectively, for the ATC-FxLMS, ATC-FxNLMS and ATC-FxAPA algorithms so as to achieve similar steady-state EMSEs. For the ATC-FxAPA algorithm, the projection number is set to $M = 3$. The corresponding EMSE curves are shown in Fig. 6. It can be seen that when the noise source becomes coloured, the ATC-FxLMS and ATC-FxNLMS algorithms converge at a much slower speed due to the spread eigenvalues of the input covariance matrix, while the ATC-FxAPA algorithm maintains similar convergence speed.



(a) (b)

Figure 6: Convergence curves of the controllers using different adaptive filtering algorithms: (a) white noise sources; and (b) colored noise sources.

4. Conclusions

In this paper, a group of Diff-Fx-based adaptive algorithms for ANC has been proposed, where the diffusion control has been applied to the controller network. Compared to the conventional centralized and decentralized control, the diffusion control has a favourable global stability as the centralized control and a decreased the computational cost as low as the decentralized system. To further increase the convergence speed of the multi-channel ANC controller, we could explore in our future study the advanced single-measure or multi-measure adaptive algorithms, such as frequency domain or subband AFs. Another issue that needs further study is how to design the network for distributed control that involves the combination weight design and extended diffusion strategy design. This is especially important for networks with multiple tasks.

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