APPLICATION OF COMPLEX TDOFS IN FIXED BOUNDARY PAYLOADS VIBRATION TESTS

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Force Limit Vibration (FLV) is used to decrease the over-testing problem associated with traditional vibration testing. The complex two-degree-of-freedom system (TDOFS) seems to give the most reasonable conservative force limits. However, the complex TDOFS model is based on free boundary conditions for the mounting structure, which is true for spacecraft mounted on a launch vehicle, but not necessarily true for other cases such as a payload mounted on a spacecraft, which requires fixed boundary conditions. In this work we set an example of fixed boundary condition, and compare the force limit predicted by the complex TDOFS method regarding the free boundary condition and the fixed boundary condition. The result indicated the force limit suggested by free boundary condition CTDOFS is not conservative enough in certain frequency band, while the corrected CTDOFS concerning fix boundary condition gives reasonable conservative force limit prediction. Some experimental methods to derive the mass parameter for the estimation of force limit is also presented.
Keywords: TDOFS, VIBRATION TEST, FORCE LIMIT, FIX BOUNDARY

1. Introduction

Vibration testing is an important way to verify the design and manufactory of spacecraft and space hardware. Conventionally the acceleration of hardware-shaker interface is controlled to specifications, which are mainly obtained from the envelope of acceleration peak of coupled model analyse, tests and flight environment. These specifications are well known to cause over-testing problem at the resonance frequencies, leading to over-weight, over-design, high costs and long schedule [1].

One of the ways to ease the over-testing problem is the Force Limit Method (FLM). Based on the based fact that the anti-resonance points of interface acceleration and force would be strictly at different frequency and appear alternately, the FLM control the vibration with both interface acceleration and force, which appears to be real-time notching of the input acceleration, through this process the over-testing at acceleration resonance would be greatly reduced [2].

Proper force limit is crucial during Force Limit Vibration (FLV), yet it is difficult to measure the force at interface compare with the measurement of acceleration, for the complicit set up of force measurement, and it should be in series within the mounting surface and the hardware. There are mainly three ways to calculate the force limit: the semi-empirical method, the simple two-degree-of-freedom system (TDOFS) method, and the complex TDOFS method. The semi-empirical method derives the force limits without requiring specific information on the mounting structures. However, they are based on the extrapolation of interface force data for similar mounting structures and test articles and/or comparison with the TDOFS methods and other parameters. The simple TDOFS method uses a simple a spring–mass–damper model to predict the force limits. It also requires information on the mounting structure. This simple method generally gives reasonably conservative force limits. However, the model is sometimes deficient because it neglects the contribution of modes with natural frequencies away from the exciting frequency (residual mass effect). The complex TDOFS model uses a TDOF spring–mass–damper model to predict the force limits, and it appears to be the most complete and versatile model [3]. However, the complex TDOFS model assumes free boundary conditions for the mounting structure. Although these boundary conditions appear to be natural for a spacecraft mounted on a launch vehicle, this is not necessarily true for other cases. For example, in the case of an electronic component mounted on a spacecraft antenna, fixed boundary conditions are usually assumed [4]. Cote et al. [5] shows that given certain precautions, the complex TDOFS gives good estimates of the force limits of fixed boundary conditions. The parameters need to be carefully obtained in order to get proper force limit. This paper gives an example of fixed boundary hardware on a spacecraft, and shows how to get the parameters, gives the comparison of force limit with free-boundary and fixed-boundary TDOFS, also the predicted force limit with FEM.

2. Introduction to complex TDOFS

In the following contents, as well as in FLV, the test unit is named as load, such as the satellite or the hardware, the mounting structure is named source, such as the launching vehicle or the mounting structure.

2.1 Free-boundary complex TDOFS

Divide the load and source from the interface, with mode analyse theory, both of load and source could be simplified into asparagus model [6]. For a structure attached at its base, it can be demonstrated that the relation between the interface force and the interface acceleration is described with the asparagus model shown in Fig. 1(a) and expressed mathematically as:

$$F = \sum_{i=1}^{\infty} m_i H(\frac{\omega_i}{\omega})\ddot{U}$$

$$H(\frac{\omega_i}{\omega}) = \frac{\left(\omega_i / \omega\right)^2 + 2j\xi_i\left(\omega_i / \omega\right)}{\left(\omega_i / \omega\right)^2 - 1 + 2j\xi_i\left(\omega_i / \omega\right)} \tag{1}$$

where F is the total base force, ¨ $\ddot{U}$ is the base acceleration, m_i is the effective mass of mode i, H is a transfer function, ω and ω_i are the angular frequency of excitation and natural frequency of mode i, j is the complex number, and ξ_i is the damping ratio of mode i.

With given damping ratio, the relationship of transfer function H and frequency ω is shown in Fig. 2Figure **2**. H trends to 1when the natural frequency ω_i significantly higher than ω, H trends to 0 when the natural frequency ω_i significantly lower than ω. According to this fact, the mode with ω_i close with ω is left, the modes with ω_i lower than ω are neglected, the modes with ω_i higher than ω is condensed as a mass attached to the base, namely residual mass M_i. The asparagus patch model is simplified to a TDOF model as shown in Fig. 1(b).

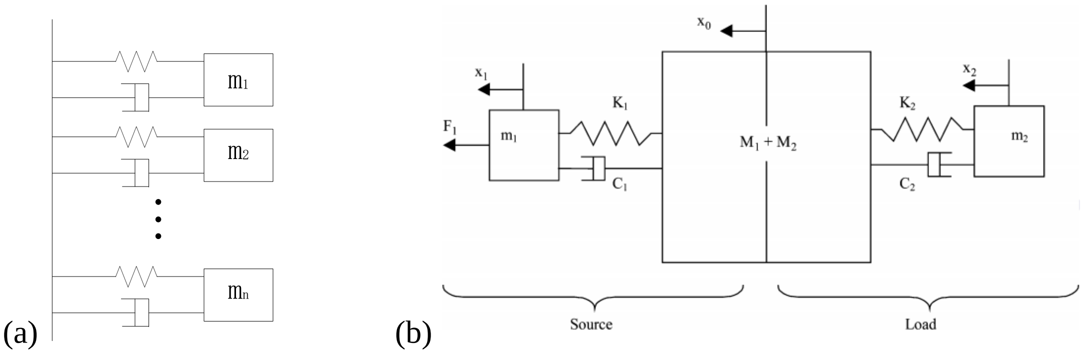

Figure 1: (a) Asparagus patch model of the structure and (b) complex TDOF model of load and source of free boundary condition.

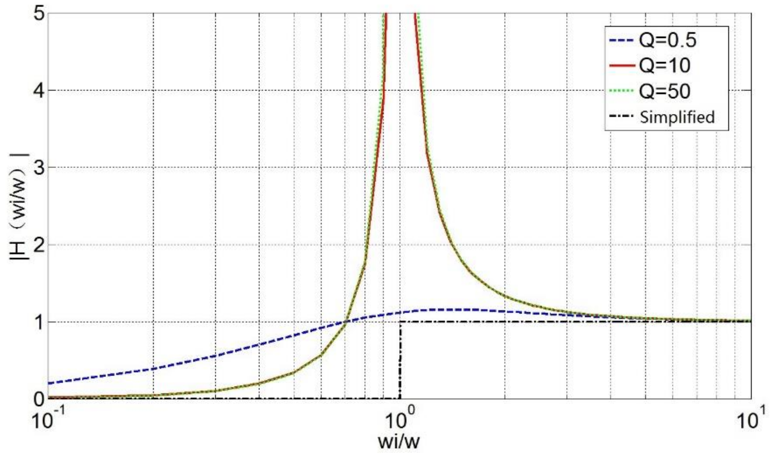

Figure 2: Frequency function vs frequency.

2.2 Fixed-boundary complex TDOFS

Compare with the free boundary condition, there are two interface of the source in fixed boundary condition, one is interface with the load, the other is the fixed boundary of source. The apparent mass of source would be different with the free boundary condition:

$$F = \sum_{i=1}^{n} \bar{m}_i \bar{H}(\frac{\omega_i}{\omega})\ddot{U}$$

$$\bar{H}(\frac{\omega_i}{\omega}) = \frac{(\omega_i/\omega)^2 \times (\omega_i/\omega)^2}{(\omega_i/\omega)^2 - 1 + 2j\xi_i(\omega_i/\omega)} \tag{2}$$

$\bar{m}_i$ is the **equivalent effective mass**, the way to obtain it is introduced below. $\bar{H}$ is the transfer function of fixed boundary condition, the different with *H* of free boundary condition is shown in Fig. 3. The transfer functions are similar at lower frequency band, but different at higher frequency band. $\bar{H}$ is not unit but trends to infinite at higher frequency band, so the residual mass is no longer the sum of m_i of higher frequency modes, and it is estimated with Eq. (3) :

$$M_b = \sum_{i=1}^{n} \bar{m}_i \bar{H}(\frac{\omega_i}{\omega_c}) \tag{3}$$

ω_c is the angular frequency at the center of current frequency band, it uses the average value over the defined frequency band to calculate the residual mass.

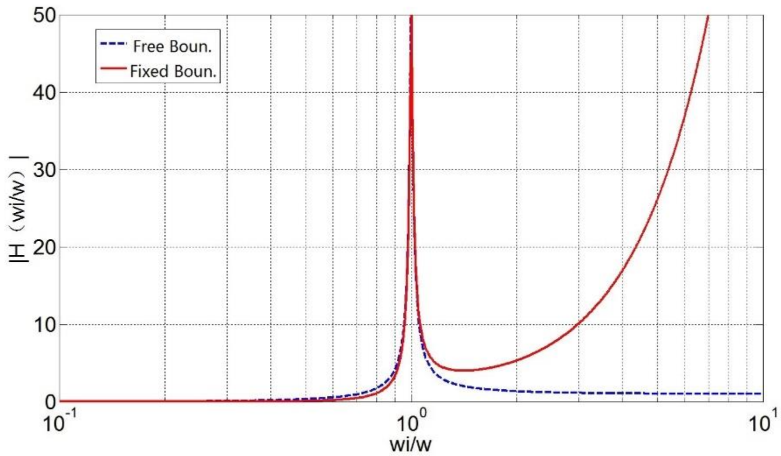

Figure 3: Comparison of transfer function of fixed and free boundary conditions.

3. Methods to obtain the effective mass

The effective mass indicates the sensitivity of each mode to a base excitation (or spatial coupling between each mode and the excitation), the accuracy of mass parameter determines the accuracy of the simplified model, which is crucial on the estimation of force limit. Here three different methods to obtain the mass parameter are presented: FEM methods, shaker experiment, and hammer test; and the first two methods are applied and verified.

3.1 FEM

Use big mass to simulate the fix boundary condition is a possible way to obtain the equivalent effective mass of source with fixed boundary condition. It was shown that the critically damped apparent mass is a good approximation of the lumped mass for a free source. The same approximation may be used for a fixed source [5].

Apparent mass is:

$$\frac{F}{\ddot{U}}(Q)=\sum_{i=1}^{v-1}\bar{m}_i\bar{H}(\frac{\omega_i}{\omega})+\bar{m}_v\bar{H}(\frac{\omega_v}{\omega})+\sum_{i=v+1}^{n}\bar{m}_i\bar{H}(\frac{\omega_i}{\omega})$$
$$\bar{H}(\frac{\omega_i}{\omega})=\frac{(\omega_i/\omega)^2\times(\omega_i/\omega)^2}{(\omega_i/\omega)^2-1+2j\xi_i(\omega_i/\omega)} \tag{4}$$

Given real damping factor, apparent mass at certain mode υ is:

$$\frac{F}{\ddot{U}}(Q)=\sum_{i=1}^{v-1}\bar{m}_i\bar{H}(\frac{\omega_i}{\omega})-Q\times j\bar{m}_v+\sum_{i=v+1}^{n}\bar{m}_i\bar{H}(\frac{\omega_i}{\omega}) \tag{5}$$

Apparent mass critically damped, Q=0.5:

$$\frac{F}{\ddot{U}}(Q=0.5)=\sum_{i=1}^{v-1}\bar{m}_i\bar{H}(\frac{\omega_i}{\omega})-0.5\times j\bar{m}_v+\sum_{i=v+1}^{n}\bar{m}_i\bar{H}(\frac{\omega_i}{\omega}) \tag{6}$$

Because the frequency function only slightly depends on the amplification factor for lower and higher modes, Eq. (5) and Eq. (6) may be combined to give:

$$F/\ddot{U}(Q)\approx F/\ddot{U}(Q=0.5)-j\bar{m}_v(Q-0.5) \tag{7}$$

The amplification factor is generally much larger than 0.5. Thus, Eq. (7) may be simplified to:

$$F/\ddot{U}(Q)\approx F/\ddot{U}(Q=0.5)-j\bar{m}_v(Q) \tag{8}$$

Hence, the equivalent effective mass could be obtained using the apparent mass at real damping and critical damping factor.

3.2 Shaker experiment

This method is mainly used for loads, and it is the same for free or fixed boundary conditions. The apparent mass around certain resonant frequency is [7]:

$$\begin{Bmatrix}\left(M_j^{app}\right)_1\\ \left(M_j^{app}\right)_2\\ \vdots\\ \left(M_j^{app}\right)_r\end{Bmatrix}=\begin{bmatrix}H_1(\frac{\omega_{n1}}{\omega_1},\xi_1) & \cdots & H_m(\frac{\omega_{nm}}{\omega_1},\xi_m) & 1\\ \vdots & \vdots & \vdots & \vdots\\ H_1(\frac{\omega_{n1}}{\omega_r},\xi_1) & \cdots & H_m(\frac{\omega_{nm}}{\omega_r},\xi_m) & 1\end{bmatrix}\begin{Bmatrix}M_{1j}^{eff}\\ \vdots\\ M_{mj}^{eff}\\ M_{mj}^{res}\end{Bmatrix} \tag{9}$$

Given damping parameters for each mode, the equation became system of linear equations of effective mass, residual mass, and apparent mass.

3.3 Hammer test

It is difficult to mount the source on shaker through the interface of source and load, so the mass parameter could be solved with hammer test. For fixed boundary condition, the source is usually defined as the mounting structure, and the boundary of mounting structure is fixed boundary. As we hammer each mounting point, the hammer force and acceleration at mounting points are measured. Assume the force and torque of each axial are decoupled, displacement in one axial is:

$$\{R_i\} = \begin{Bmatrix} A_i(y_1) \\ A_i(y_2) \\ A_i(y_3) \\ . \\ . \\ A_i(y_n) \end{Bmatrix} = [I_i] \times \begin{Bmatrix} F_i(x_1) \\ F_i(x_2) \\ F_i(x_3) \\ . \\ . \\ F_i(x_n) \end{Bmatrix} \tag{10}$$

In which:

$$[I_i] = \begin{bmatrix} \frac{A_i(y_1)}{F_i(x_1)} & \frac{A_i(y_1)}{F_i(x_2)} & \frac{A_i(y_1)}{F_i(x_3)} & . & . & . & \frac{A_i(y_1)}{F_i(x_n)} \\ \frac{A_i(y_2)}{F_i(x_1)} & \frac{A_i(y_2)}{F_i(x_2)} & & & & & . \\ & & . & & & & . \\ & & & . & & & . \\ & & & & . & & . \\ & & & & & . & . \\ \frac{A_i(y_n)}{F_i(x_1)} & . & . & . & . & . & \frac{A_i(y_n)}{F_i(x_n)} \end{bmatrix} \tag{11}$$

Namely Accelerance matrix [8].
Consider the condition as on the shaker:

$$\{R_i\} = R_i\{1\} \tag{12}$$

Total force at interface:

$$F_i = \sum_{n=1}^{N} F_{in} = (1)\{F_i\} = R_i(1)[I_i]^{-1}\{1\} \tag{13}$$

Total apparent mass:

$$W_i = F_i / R_i = (1)[I_i]^{-1}\{1\} \tag{14}$$

4. Methods application and verification

Test hardware as shown in Fig. 4 are designed to verify the methods for mass parameter obtainment. Mass of load is 4.9 kg, mass of source is 2 kg. Only sine sweep on Z direction is considered.

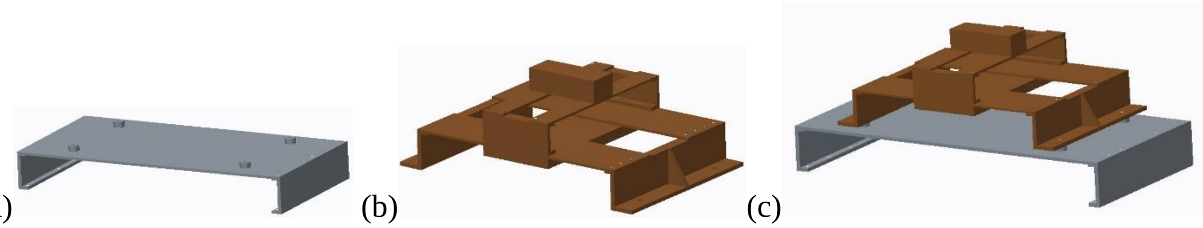

Figure 4: (a) source (b) load and (c) combined structure of test hardware.

4.1 Equivalent effective mass of source

4.1.1 Big mass to simulate fixed boundary

10^4 kg mass is attached to the mounting points of source, the effective mass of mode analyse are listed in Table 1. Comparison of fix boundary and free boundary also shown in Table 1.

Table 1: effective mass of source

	Mode	1	2	3	4
Fixed boundary	Frequency / Hz	2.3	181.1	512.8	878.2
	Effective mass / kg	10E4	1.08	3.6E-3	0.17
Free boundary	Frequency / Hz	139.9	188.2	463.2	935.6
	Effective mass / kg	0.09	0.99	0.34	0.20

The first mode of fixed boundary is the mode of attached mass, should be ignored here. The following 2 and 3 mode could compare with the free boundary condition.

4.1.2 Critical damping factor analyse

Apparent mass of source at different angular frequency are calculated.

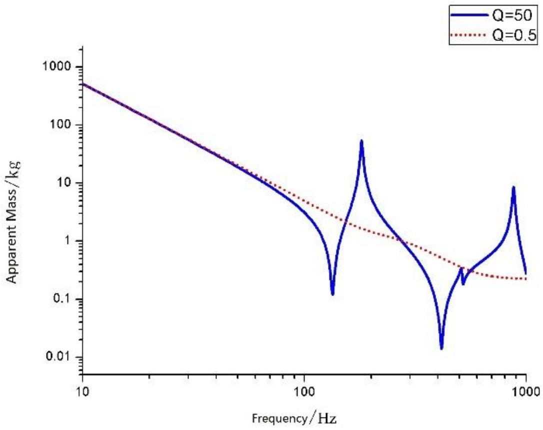

Figure 5: Source apparent mass at different frequency.

The equivalent effective mass thus can be calculated using Eq. (8):
Fist mode of natural frequency 181Hz: (54 - 1.65) / 50 = 1.05;
Second mode of natural frequency 512Hz: Critical damping lower than real damping, neglected;
Third mode of natural frequency 878Hz: (8.47 - 0.23) / 50 = 0.165.

4.1.3 Comparison of residual mass between free and fixed boundary condition

Source apparent mass is different for free and fixed boundary condition, the calculation for residual mass is also different, the comparison shown in Table 2:

Table 2: Residual mass of source

Frequency band	5~45	45~56	56~71	71~89	89~100
Free boundary	2.0	2.0	2.0	2.0	2.0
Fixed boundary	105.3	67.9	43.3	27.5	18.4

4.2 Effective mass of load

4.2.1 FEM modal analyse

Table 3: Effective mass of load

Mode	1	2	3	4	5
Frequency / Hz	51.8	75.3	93.9	94.2	111.9
Effective mass / kg	3.70	3.9E-20	0.53	2.7E-17	1.6E-19

4.2.2 Shaker test

Load mount on shaker, 5~1000Hz, 0.5g sine sweep test, measure the force and acceleration at mounting point, the apparent mass shown in Fig. 6.

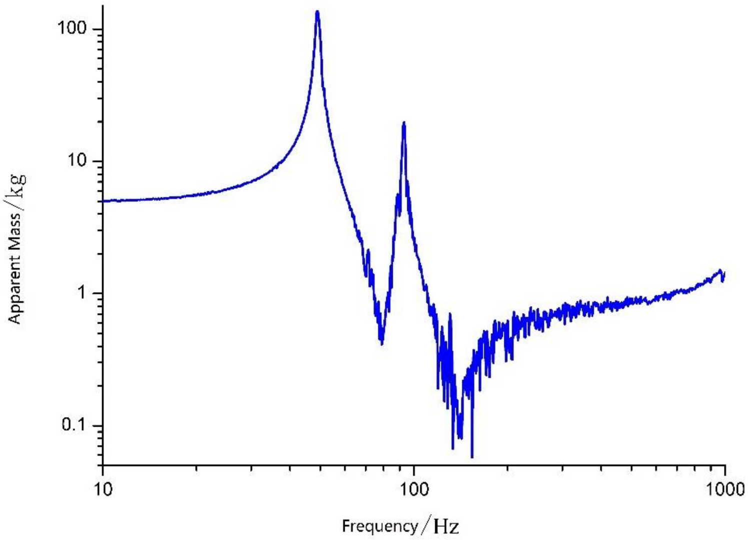

Figure 6: Load apparent mass at different frequency.

4.3 Comparison of force limit of free and fixed boundary conditions

As the mass parameters obtained, the force limit could be calculated using complex TDOFS method [6]. The interface force could be calculated with coupled FEM model, those force are compared in Fig. 7.

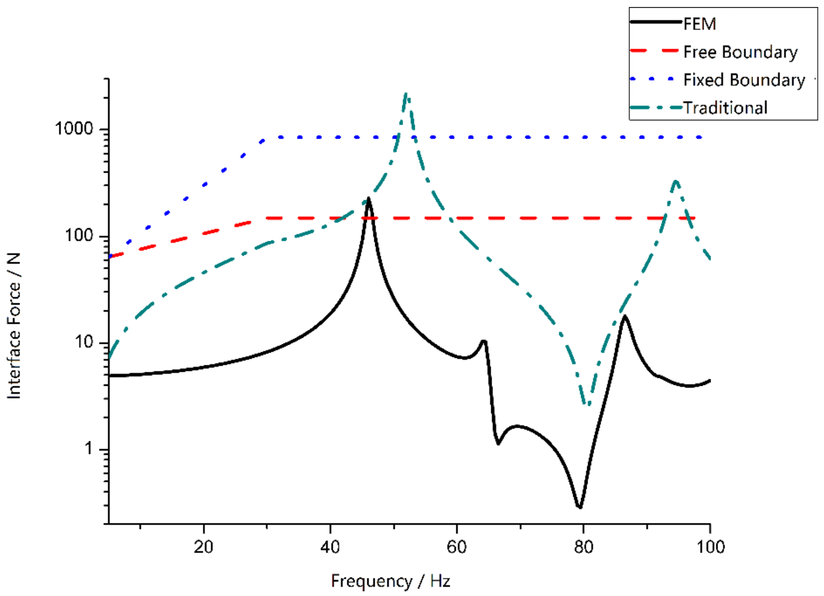

Figure 7: Comparison of force limit of free and fixed boundary condition.

The force limit predicted with free boundary condition leads to under-test, and the force limit predicted with fixed boundary condition is more conservative, compare with the force peak using traditional acceleration control test, the over-test is eased at 52Hz with 20×log(2286N/850N)=8.6dB.

5. Conclusion

(1) Application of complex TDOFS method in fixed boundary condition is introduced.
(2) Mass parameters are crucial in force limit calculation. Mass parameters obtaining methods are introduced, especially the equivalent effective mass of source. The methods are verified with a test hardware, from which we can find different methods gives similar mass parameter, so the effectiveness of the methods is verified for this sample.
(3) Force limit calculated with fixed boundary simplification complex TDOFS is very conservation, for some sample too conservation (not included in this paper). Force limit calculated with free boundary TDOFS, when applied in fixed boundary condition, is sometimes under-estimate. In actual applications, the envelop itself always includes margin. It is suggested to consider both methods and FEM simulation results, for a proper force limit.

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