

# RESEARCH OF NEW COMBINED TRACK SLAB SYSTEM IN LOW FREQUENCY VIBRATION CONTROL

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Taking the phenomenon of vibration attenuation in low frequency domain of the combined track system, a tuned slab dampers are designed based on passive dynamic vibration absorber theory. According to the vehicle-track coupling dynamics, a finite element model was established and analysed. Track frequency response functions have been compared between a standard slab track and the track with optimized tuned slab damper. The results show that although the slab vibration level of a standard track is lower than combined one due to the rigid connection between track structure and ground-base, but results in the higher the base vibration than combined track. Analysis has found that tuned slab dampers on a metro track could effectively reduce slab vibration in low frequency range of 20~40Hz, and the insertion loss was up to 15dB when mass ratio is 0.2.

## 1. Introduction

With the rapid development of rail transportation, the vibration and noise pollution caused by rail traffic has become increasingly prominent. Vehicles running on the tracks cause rail and track vibration, which potentially not only would affect the surrounding environment, and also shorten the service life of fasteners and rails, a direct impact on vehicle traffic safety. Therefore, the vibration and noise reduction and safe operation are important issues faced by the city subways. For this reason, Many analysis and research work were carried out and various vibration isolation measures were employed to achieve better results.

The combined track slab system consists of rails, damping fasteners, pre-stressed track slabs, rubber pads, mortar layer and baseplate. The system is designed based on vibration isolation and resonance mechanism, and adoption of several vibration reduction measures. By adjusting the relationship of stiffness and mass distribution between different coupling subsystems, the best attenuation of vibration energy in the transmission path is achieved. The vibration isolation frequency of the combined track slab system can be low to 15Hz and the vibration attenuation can reach more than 20dB. However in the low frequency range (<30Hz), the combined track slab system is not effective in reduce the vibration caused by excitation forces generated by train running on the tracks and in fact it may resonate and amplify the low frequency vibration.

This paper reports a tuned slab damper design to solve the problem of combined track system in low frequency domain. A passive dynamic vibration absorber<sup>1-3</sup> is attached to the slab bed. The parameters of the absorber are calculated and optimised based on the finite element simulation and

experimental validation on track lines.

## 2. Theory of tuned slab dampers on metro track

### 2.1 Concept of dynamic vibration absorber

According to the theory of dynamic vibration absorber<sup>3</sup>, a tuned mass-spring-damp system is added as the dynamic vibration absorber to the main vibration system. When the main system resonates, the reacting force of the added tuned system can reduce the vibration of the main system.

To design a dynamic vibration absorber with main vibration system a single degree freedom, the whole system can be simplified as a two degree freedom vibration system with adding a vibration absorber ( $k_2$ - $m_2$ - $c_2$ ) and the main vibration system ( $k_1$ - $m_1$ - $c_1$ ) as shown in Figure 1. A harmonic force  $Fe^{i\omega t}$ , is applied on the main vibration system.

The equation of motion based on Newton's laws can be established as below:

$$m_1\ddot{x}_1 + c_2(\dot{x}_1 - \dot{x}_2) + k_2(x_1 - x_2) + c_1\dot{x}_1 + k_1x_1 = Fe^{i\omega t} \quad (1)$$

$$m_2\ddot{x}_2 + c_2(\dot{x}_2 - \dot{x}_1) + k_2(x_2 - x_1) = 0 \quad (2)$$

Where  $m_1$ ,  $k_1$  and  $c_1$  are mass, stiffness, and damping respectively for the main vibration system;  $m_2$ ,  $k_2$  and  $c_2$  are for the tuned system;  $x_1$  is displacement of  $m_1$  and  $x_2$  is displacement of  $m_2$ .

Supposing the solution of the steady forced vibration displacement is

$$\{x\} = \{X\}e^{i\omega t} \quad (3)$$

Substituting this form of solution into equation (1) and (2) gives the complex displacement response amplitude of the main system.

$$\bar{X}_1 = \frac{(k_1k_2 - m_2k_1\omega^2 + ik_1c_2\omega)}{\left[m_1m_2\left(\frac{k_1}{m_1} - \omega^2\right)\left(\frac{k_2}{m_2} - \omega^2\right) - c_1c_2\omega^2\right] + i\omega[c_2k_1 + c_1k_2 - \omega^2(m_1c_2 + m_2c_1 + m_2c_2)]} \cdot \frac{F}{k_1} \quad (4)$$

Where,  $\delta_s = F/k_1$ ,  $\omega_{1n} = \sqrt{k_1/m_1}$ ,  $\omega_{2n} = \sqrt{k_2/m_2}$ ,  $\mu = m_2/m_1$ ,  $\alpha = \omega_{2n}/\omega_{1n}$ ,  $\gamma = \omega/\omega_{1n}$ ,  $\zeta_1 = c_1/2m_1\omega_{1n}$ ,  $\zeta_2 = c_2/2m_2\omega_{2n}$

The dimensionless form response amplitude for  $\bar{x}_1$  is

$$\frac{X_1}{\delta_s} = \sqrt{\frac{(\alpha^2 - \gamma^2)^2 + (2\alpha\gamma\zeta_2)^2}{(\alpha^2 - [1 + 4\alpha\zeta_1\zeta_2 + (1 + \mu)\alpha^2]\gamma^2 + \gamma^4)^2 + (2\gamma\{(\alpha^2 - \gamma^2)\zeta_1 + \alpha[1 - (1 + \mu)\gamma^2]\zeta_2\})^2}} \quad (5)$$

To illustrate this result, Figure 2 shows the dimensionless form of the amplitude plotted against frequency ratio  $\gamma$  ( $\alpha=1$ ,  $\mu=0.3$ ).

As shown in Figure 2, the displacement response amplitude of the system with a dynamic vibration absorber is lower than that of the original system in a frequency range nearby the frequency  $\omega_{1n}$ . It shows the dynamic vibration absorber system can reduce the vibration of the main system in a particular frequency range.

If the natural frequency of additional mass-spring-damp system is equal to the excitation

frequency of the main vibration system, the vibration amplitude is reduced where close to the excitation frequency.

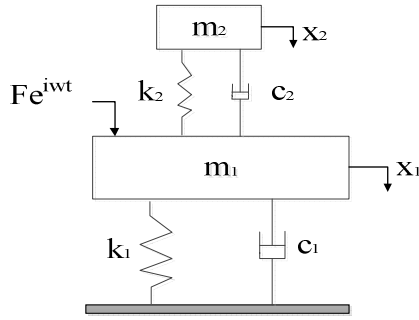


Figure 1. Model of dynamic vibration absorber

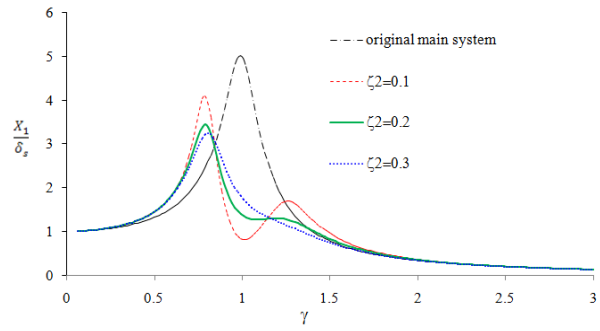


Figure 2. Displacement amplitude of main system

## 2.2 Application of tuned slab dampers on metro system

In order to reduce the vibration of the metro system at particular frequency range, a dynamic vibration absorber is introduced with a special design.

By adding optimized tuned slab dampers (the so called dynamic vibration absorber) for metro track system<sup>4</sup>, vibration energy of the metro slab is absorbed, and the vibration of the slab therefore reduced.

Figure 3 shows a combined track system, which consists of slabs, rubber isolation pads and fasteners. By the right combination of the stiffness of the rubber isolation pads, mass of the slabs and stiffness of the fasteners, the combination track system can achieve high-level vibration damping performance on the track ground.

As shown in Figure 3, tuned slab damper which includes rubber pads and concrete mass block is added on to the combined track system. Natural frequency of the tuned slab damper is specially designed to reduce the vibration of track slab.

For the purpose of easy installation, above tuned slab damper is divided the into 12 parts with 2 types, and each part constitutes a small tuned slab damper, as seen in Figure 4.

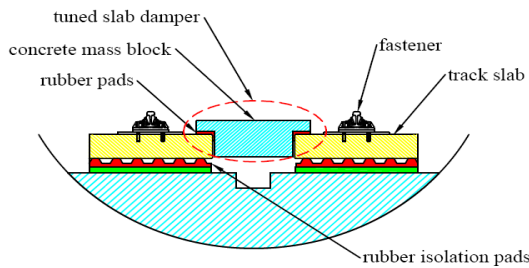


Figure 3. Section of a combination track system

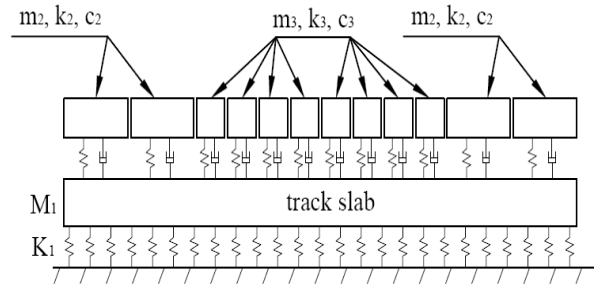


Figure 4. Tuned slab dampers on track lab

## 2.3 Principle of optimizing the tuned slab dampers

Mass, stiffness and damping of the added system are the main factors for designing a dynamic vibration absorber for main system. The best dynamic vibration absorber<sup>3</sup> solution and principles for designing a dynamic vibration absorber are:

$$m_2 = \mu m_1 \quad (6)$$

$$\zeta = \frac{c_2}{2m_2\omega_{1n}} = \sqrt{\frac{3\mu}{8(1+\mu)^3}} \quad (7)$$

$$\alpha = \frac{\omega_{2n}}{\omega_{1n}} = \sqrt{\frac{\zeta_2 - 2\zeta_1\zeta_2^2(1 + \mu) - 2\zeta_1^2\zeta_2}{\zeta_2(1 + \mu)^2 + \mu\zeta_1}} \quad (8)$$

## 2.4 Parameters study of the tuned slab damper

According to the modal analysis of combined track slab, the first order vertical mode frequency is 26Hz and the first torsional mode is 31Hz. Those modal frequencies are close to the train excitation frequency of 35Hz. Thus the dampers' parameters are designed for based on those two modal frequencies. The first-order vertical mode and the first-order lateral torsional mode mass are 5519kg and 5466kg respectively. The optimal design parameters for different mass ratios are calculated according to Equations 6 to 8 and given in Table 1, where  $\mu_1$  and  $\mu_2$  are the system mass ratios of the vertical and torsion modes.

Table 1. Tuned slab damper optimal design parameters

the first order vertical mode				the first torsional mode			
$\mu_1$	$m_1$	$k_1$	$c_1$	$\mu_2$	$m_2$	$k_2$	$c_1$
--	kg	kN/mm	kN·s/m	--	kg	kN/mm	kN·s/m
0.05	276.0	6.68	1.89	0.05	273.3	9.31	2.22
0.10	551.9	12.90	4.88	0.10	546.6	17.98	5.81
0.20	1103.9	24.96	10.82	0.20	1093.1	34.77	14.35
0.30	1655.8	36.83	20.06	0.30	1639.7	51.31	23.56

## 3. Theoretical calculations

### 3.1 Track models

In order to study the low-frequency vibration- dampening performance of the tuned slab dampers on the track slab, three finite element models are created for the general track system, the combined track system, and the combined track with tuned slab dampers ( $\mu_1 = 0.2$ ,  $\mu_2 = 0.1$ ) is created.

The finite element model of the general track system is as shown in Figure 5. The track slab bed is 25m long and 2.5m wide, and the height of the track structure is 0.5m. The sleeper spacing is 0.625m and the ordinary single-layer fasteners (DTVIII) with vertical stiffness of 60kN/mm are adopted. The finite element model of the combined track slab system is given in Figure 6. The model is included three track slabs. The base of the track bed is completely fixed without taking into account the deformation of the foundation. Each track slab is 5m long and 2.4m wide, and the height of the slab is 0.2m. The spacing of the sleeper is 0.625m and the fastener type used is GJ-4), which has a vertical stiffness of 8kN/mm. Fasteners are modeled using spring-damping elements. The stiffness of the rubber isolation pads is 30N/mm and the rigidity is 0.018N/mm<sup>3</sup>.

The model of tuned damper is added to the combined track<sup>6</sup> bed model, as shown in Figure 7. According to the tuned slab dampers designed in the previous section, the mass element is made of concrete and the elastic cushion is made of rubber damping material. The related material parameters are listed in Table 2. Constraints in horizontal and vertical degrees of freedom are applied to FE models.

Table 2. Material parameters

Item	Density/kg·m <sup>-3</sup>	Modulus of elasticity/MPa	Poisson ratio
Rail	7850	210000	0.3
Slab	2500	32500	0.2
Slab mat	1200	3	0.49
Foundation	1500	2000	0.25

### 3.2. Loading

Based on the equivalent lumped parameter method, the vehicle-track coupling model is established in SIMPACK by using the Level 5 track spectrum of the United States. In the calculation, it is assumed that excitation forces of the four wheels are all in phase, and track line is a straight. The vehicle used is the subway standard type A-model car, the main parameters<sup>5</sup> refer to reference 5. The corresponding wheel-rail loads are applied to the finite element models to analyze the low frequency vibration characteristics of the track system. Simulation model is shown in Figure 8. The response position of slab is on the center, and on the foundation is 300mm to the edge of slab.

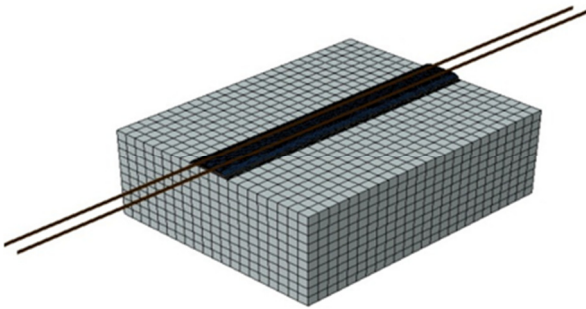


Figure 5. Model of general track system

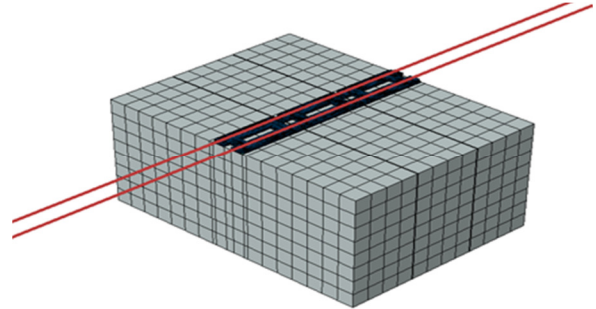


Figure 6. Model of combined track system

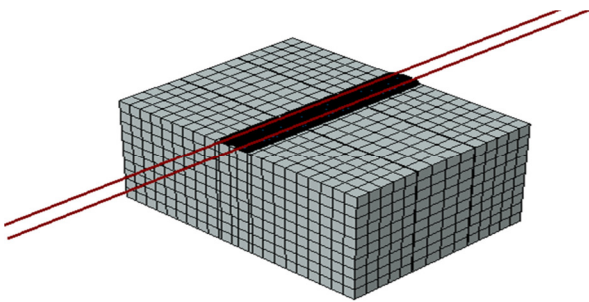


Figure 7. Model of combined track with tuned slab dampers

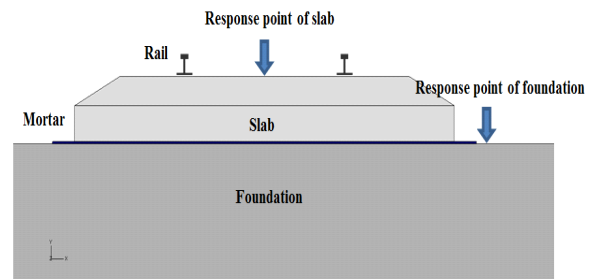


Figure 8. Response points of track system

### 3.3 Simulation results

#### 3.3.1. Vibration acceleration

The slab's vibration acceleration responses of the general track, combined track and combined track with the tuned slab dampers ( $\mu_1=0.2$ ,  $\mu_2=0.1$ ) are shown in Figure 9. It shows that the slab's vibration level of general track is much lower than the combined track in each frequency band, as the general track structure is rigidly connected with the ground. The acceleration of the combined

track slab has several high peaks between 25~35Hz. For the combined track slab with tuned damper, the slab's acceleration level is significantly reduced.

The acceleration responses of the foundation are shown in Figure 10. It demonstrates that ground vibration level of t of the general track is much higher than that of combined track slab system. This is because the general track structure is rigidly connected with ground foundation and there is no isolation between the slab and foundation;. while for the combined track slab system, there is an isolation mat between slab and foundation to provide good vibration isolation. However there is a high vibration peak close to 25Hz, the first vertical natural frequency of the combined track slab system, this indicates the resonance of the combined system to the wheel excitation. With tuned damper added to the combined track slab system, all foundation response peak levels are significantly reduced.

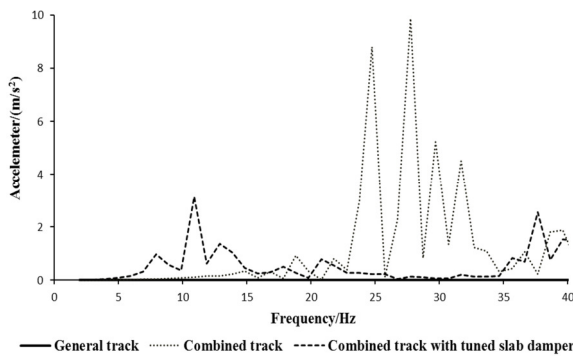


Figure 9. Vibration acceleration of track slab

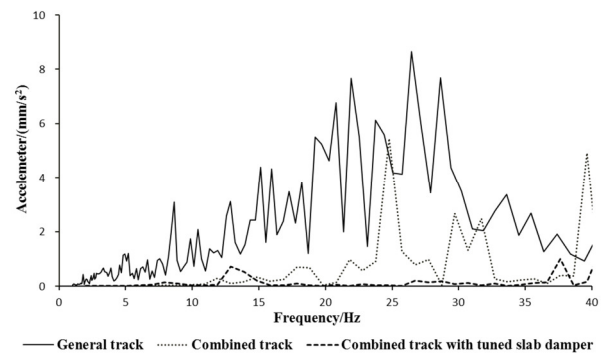
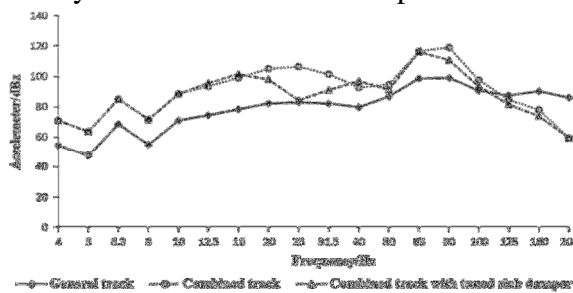


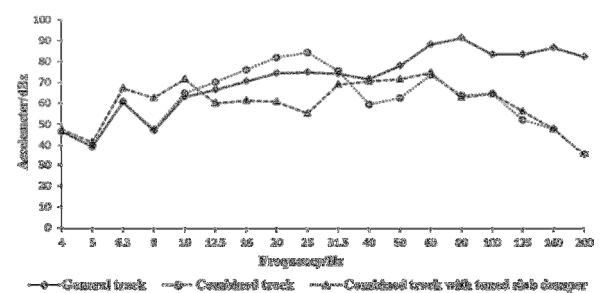
Figure 10. Vibration acceleration of track bed

### 3.3.2. 1/3 octave spectrum

The vertical vibration acceleration levels of the track slab and bed of the general track, combined track and combined track with the tuned slab dampers are shown in Figure 11. For the track slab vibration, general track system has the lowest acceleration level comparing with the combined tracks below 100Hz. For the combined track slab with tuned slab dampers, the slab acceleration level is reduced between 20Hz to 40Hz, and the maximum reduction is over 20dB at 25Hz. As for the foundation vibration, with tuned damper, the maximum acceleration response reduction in the frequency range 20Hz to 40Hz is over 25dB at 25Hz comparing with combined track system without tuned damper.



a) vibration acceleration level of track slab



b) vibration acceleration level of foundation

Figure 11. 1/3 octave spectrum

## 4. Frequency response function tests

### 4.1 Lab test method

To verify the performance of slab low frequency vibration attenuation by tuned slab dampers



on a metro track, laboratory tests are carried out in two conditions on a 1:1 size combined slab trials section. Test 1 is on a standard slab track setting, and Test 2 on the track with optimized tuned slab dampers.

Figure 12 shows the direction and position of exciting force and accelerometers. The excitation and 2 vibration response positions are above the cross-section of the fastener and 1/2 span between two fasteners respectively. Responses are measured at several different positions to eliminate the deviation of the system itself and experiments.

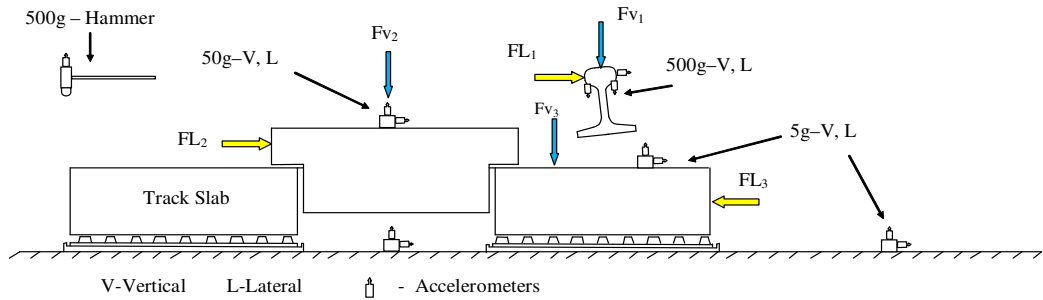


Figure 12. Schematic diagram of hammering test on rail

## 4.2 Test result

Track transfer response functions, vibration response functions of slab and track bed are discussed to illustrate the vibration attenuation influence of tuned slab dampers on a metro track.

### 4.2.1 The track slab transfer function analysis

Figure 13 shows the track slab vibration acceleration level and insertion loss of a standard slab track and the track with optimized tuned slab dampers. In the frequency range between 20Hz and 40Hz, the vibration levels in both vertical and lateral directions are significantly reduced on the track with optimized tuned slab dampers, especially at central frequency of 25Hz band, the vertical vibration amplitude was reduced up to about 15dB, and the lateral vibration amplitude reduced about 13dB.

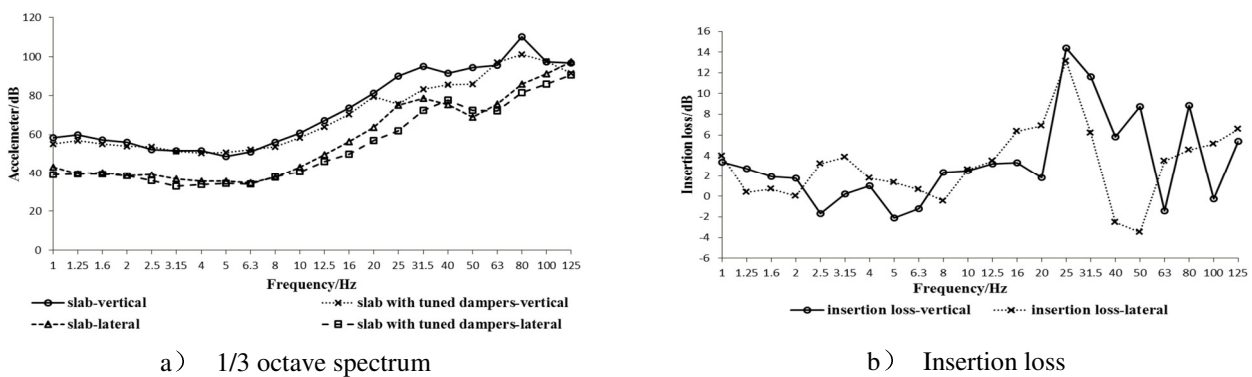


Figure 13. Track slab acceleration level and insertion loss

### 4.2.2 The slab bed transfer function analysis

Figure 14 shows the vibration acceleration level of slab bed with the standard slab track and the track with optimized tuned slab dampers and their insertion loss. In the low frequency range below 125Hz, the track bed vibration peak on the track with optimized tuned slab dampers is less than ordinary slab track, the vertical vibration amplitude has been reduced up to about 13dB, and the lateral vibration amplitude reduced about 10dB. The results demonstrate that tuned slab dampers

are very effective in reducing the transmission of vibration energy to from track to foundation.

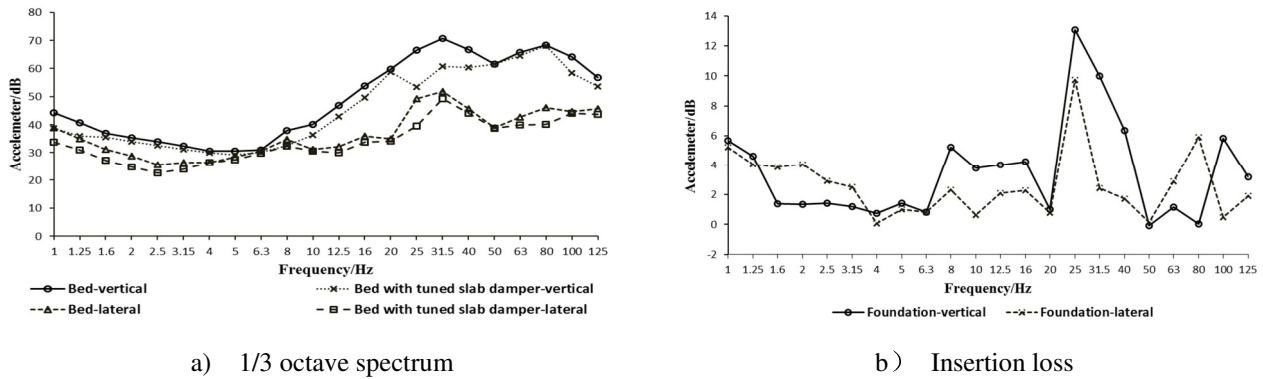


Figure 14. Track slab bed acceleration level and insertion loss

## 5. Conclusions

For low frequency ground vibration control on metro system, the effects of the combined slab track system with tuned slab dampers are analyzed, simulated, and measured in Lab, and the following conclusions are obtained:

- 1) The slab's vibration level of general track is much lower than the combined track in each frequency band as the general track structure is rigidly connected with the ground foundation.
- 2) The track bed's vibration level of the general track is much higher than the combined track, because the general track structure is rigidly connected with ground foundation and there is no isolation between the slab and foundation.
- 3) By introducing the tuned slab damper into a combined slab track system, the vibration of the slabs in the low frequency range is significantly reduced by about 15dB and the vibration of the track slab bed reduced about 10dB based on the simulation and Lab test.

This indicates that the tuned slab dampers are an effective mean to control the low frequency track vibration on metro system.

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