EFFICIENT SIMULATION OF LARGE STRUCTURAL-ACOUSTIC SYSTEMS USING COMPONENT MODE SYNTHESIS METHOD

Yongzhen Mi

State Key Laboratory of Mechanical System and Vibration, Institute of Vibration, Shock and Noise, School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai 200240, China
email: miyongzhen@sjtu.edu.cn

Hui Zheng

Collaborative Innovation Center for Advanced Ship and Deep-Sea Exploration (CISSE), Shanghai Jiao Tong University, Shanghai 200240, China

Prohibitive computational cost has become the bottleneck of large-scale finite element simulations, especially for structural-acoustic systems in which the total degrees of freedom (DOFs) is substantially enlarged due to the presence of the acoustic cavity. In this paper, the classical component mode synthesis (CMS) method, which has been widely adopted as a powerful model reduction technique in the structural dynamics, is extended to the two-field problems with the aim of improving the efficiency of dynamic simulation of large structural-acoustic systems. Specifically, the coupled system is decomposed into structure domain and fluid media domain, and each domain is further divided into several subdomains. In each subdomain, the fixed-interface CMS procedure is applied to transform the interior DOFs to a set of much less modal DOFs while keeping the boundary DOFs as the original physical form, which steadily guarantees the velocity continuity on the coupling interface in the following assembling step. Particularly, because of the repetition of structure units and acoustic cavities in the coupled system formulating one unit is needed only, which considerably lessens the modelling work and therefore renders a significantly reduced model. Finally, as an example to demonstrate its computational efficiency, the structural-acoustic CMS method is applied to calculate the interior sound pressure level of an air-filled long elbow pipe. It is shown that an obviously shorter time is required to complete a computation compared with that of a full model. The two-filed CMS method presented in this paper can be further employed to alleviate the computational overburden in the simulation-intensive tasks like structural-acoustic optimization or uncertainty quantification problems.
Keywords: vibro-acoustic systems, component mode synthesis, computational efficiency

1. Introduction

In the 1960s, the component mode synthesis method (CMS) was proposed by Hurty [1], and further developed by Craig and Bampton [2~3]. To accommodate to the limited computational resources back then, this method seeks to divide a complex structure into several substructures, then cut down the number of degrees of freedom (DOFs) in each substructure by a modal superposition procedure, and finally assemble the substructures back to a complete but significantly reduced structure. As an efficient model reduction technique, the Craig-Bampton CMS method, together with its several variants [4~7], has been extensively employed in applications in which large-scale models or iterative computations are involved, like nonlinear dynamics [8], uncertainty quantifications [9] and optimization problems [10]. Recently, a thorough review is made by Gruber et al. in [11], in which the general concept of order reduction is summarized after a performance evaluation of four popular substructuring methods.

Another advantage of the CMS method is its independent and parallel manner to process each subsystem in a complex built-up system, which make it an appealing modelling framework in problems that include several segments or components, such as the multi-body dynamics and multi-physics problems. Among various multi-physics problems, the dynamic analysis of a large vibro-acoustic system remains as a challenge due to its inherent complexity and the corresponding computational difficulties. Traditional methods, like the finite element method and the boundary element method, are still the most widely used methods for complex dynamic analysis in industrial occasions, but these methods are only confined to low frequency and become quite cumbersome in higher frequency ranges due to the prohibitive amount of DOFs caused by mesh refinements; more recently, the wave-based method proposed by Desmet delivers an improved computational accuracy of vibro-acoustic analysis and a finer adaptivity to irregular domains, but the exponential growth of the number of oscillating basis functions makes it become obviously inefficient in complex systems.

The combination of order reduction ability and component-wise processing flexibility makes the CMS method a promising candidate for large vibro-acoustic system analysis. Actually, this has been noticed by several researchers and some preliminary studies have been done where only simple geometries and analytical modes are involved. Motivated by these practices, this paper investigates the application of vibro-acoustic CMS method in more complex systems in which the structural parts and acoustical cavities are modelled by the finite element method, to further validate the applicability of the proposed vibro-acoustic CMS method.

2. Theoretical Formulation

2.1 Problem description

The internal vibro-acoustic problem which is considered in this paper is presented Fig. 1. The system consists of several identical subsystems and thus only one subsystem will be formulated. Let V_f be the fluid domain, V_s the structural domain, S_c the fluid-structure coupling interface, and S_f the edge of structural domain which is subjected to an external distributed force denoted by f. It is assumed that there is no acoustical source in the fluid media.

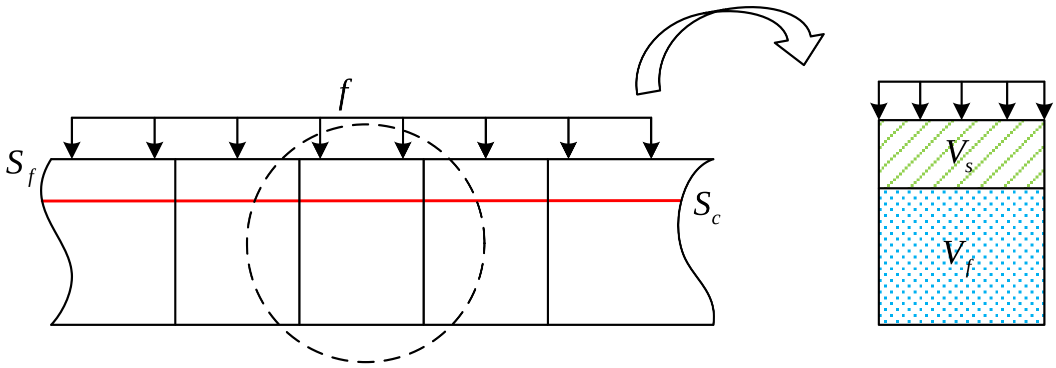

Fig. 1. Description of vibroacoustic problem.

The equations describing the harmonic response at frequency ω of structural domain can be written as

$$\begin{cases} \nabla \cdot \sigma + \rho_s \omega^2 u = 0 & \text{in } \mathrm{V_s} \\ \sigma \cdot \mathbf{n}^s = f & \text{on } \mathrm{S_f} \\ \sigma \cdot \mathbf{n}^c = p & \text{on } \mathrm{S_c} \end{cases} \tag{1}$$

and the equations of the acoustic pressure in the fluid domain are

$$\begin{cases} \Delta p + \dfrac{\omega^2}{c_0{}^2} p = 0 & \text{in } \mathrm{V_f} \\ \dfrac{\partial p}{\partial \mathbf{n}^c} = \rho_0 \omega^2 u \cdot \mathbf{n}^c = \rho_0 \omega^2 u_n & \text{on } \mathrm{V_f} \end{cases} \tag{2}$$

where ρ_s is the structural density; ρ_f and c_0 are the density of the fluid media and the speed of sound in it.

A weak form formulation of the vibro-acoustic problem can be obtained on the basis of Eq. (1) and (2) as

$$\begin{cases} \int_{V_s} \sigma(u) : \varepsilon(\delta u) dV - \omega^2 \int_{V_s} \rho_s u \cdot \delta u dV - \int_{S_c} p \cdot \delta u dS = \int_{S_f} \delta u \cdot f dS \\ \int_{V_f} \dfrac{1}{\rho_0} \nabla p \cdot \nabla \delta p dV - \omega^2 \int_{V_f} \dfrac{1}{\rho_f c_0^2} p \delta p dV - \omega^2 \int_{S_c} u_n \delta p dS = 0 \end{cases} \tag{3}$$

for any admissible δu and δp .

By introducing the standard finite element procedure, Eq. (3) is discretized to

$$\begin{cases} \left(k_s - \omega^2 m_s\right) u - Cp = f \\ \left(k_f - \omega^2 m_f\right) p - \omega^2 \rho_f C^T u = 0 \end{cases} \tag{4}$$

where k_s , m_s are structural stiffness and mass matrices, k_f and m_f are respectively matrices corresponding to the discretization of kinematic energy and compressional energy of acoustic fluid, and will be referred to as acoustical stiffness and mass matrices in the following; C is the coupling matrix between structure and fluid media. Eq. (4) can be organized to a matrix form as

$$\left(\begin{bmatrix} k_s & -C \\ 0 & k_f \end{bmatrix} - \omega^2 \begin{bmatrix} m_s & 0 \\ \rho_f C^T & m_f \end{bmatrix} \right) \begin{Bmatrix} u \\ p \end{Bmatrix} = \begin{Bmatrix} f \\ 0 \end{Bmatrix} \tag{5}$$

Through Eq. (5), we get the classical $u-p$ formulation of a vibro-acoustic problem based on the finite element method.

2.2 Vibro-acoustic component mode synthesis

The Craig-Bampton component mode synthesis method will be extended to solve the vibro-acoustic problems in this section. The primary step of this extension is to decompose the coupled system to a structural domain and an acoustical domain. To facilitate the re-assembly of the two domains at the final step, the DOFs on the coupling interface in each domain, i.e. the coupling nodal displacement and pressure variables denoted by subscript "*b*", are chosen to be the boundary DOFs in the component mode synthesis context. Correspondingly, the left DOFs of both domains, which are denoted by subscript "*i*", will be the inner DOFs. Therefore, Eq. (5) can be rewritten as

$$\left(\begin{bmatrix} k_{ii}^s & k_{ib}^s & 0 & 0 \\ k_{bi}^s & k_{bb}^s & 0 & -C_{bb} \\ 0 & 0 & k_{ii}^f & k_{ib}^f \\ 0 & 0 & k_{bi}^f & k_{bb}^f \end{bmatrix} - \omega^2 \begin{bmatrix} m_{ii}^s & m_{ib}^s & 0 & 0 \\ m_{bi}^s & m_{bb}^s & 0 & 0 \\ 0 & 0 & m_{ii}^f & m_{ib}^f \\ 0 & \rho_f C_{bb}^T & m_{bi}^f & m_{bb}^f \end{bmatrix} \right) \begin{Bmatrix} u_i \\ u_b \\ p_i \\ p_b \end{Bmatrix} = \begin{Bmatrix} f_i^s \\ 0 \\ 0 \\ 0 \end{Bmatrix} \tag{6}$$

A reduction basis can be constructed based on the classical CMS method as

$$\begin{Bmatrix} u_i \\ u_b \\ p_i \\ p_b \end{Bmatrix} = \begin{bmatrix} \varphi_i^s & \varphi_c^s & 0 & 0 \\ 0 & I & 0 & 0 \\ 0 & 0 & \varphi_i^f & \varphi_i^f \\ 0 & 0 & 0 & I \end{bmatrix} \begin{Bmatrix} q_i^s \\ u_b \\ q_i^f \\ p_b \end{Bmatrix} = T \begin{Bmatrix} q_i^s \\ u_b \\ q_i^f \\ p_b \end{Bmatrix} \tag{7}$$

where φ_i^s is the retained part of the fixed-interface mode shape matrix of structure derived by solving the following eigenvalue problem

$$k_{ii}^s \varphi_i^s = \Lambda_i^s m_{ii}^s \varphi_i^s \tag{8}$$

φ_c^s is the constraint mode shape matrix derived by successively imposing a unit displacement on each boundary DOF of the structure, thus leads to

$$\varphi_c^s = -\left(k_{ii}^s\right)^{-1} k_{ib}^s \tag{9}$$

The fixed-interface mode shape matrix and constraint mode shape matrix of the fluid media, i.e. φ_i^f and φ_c^f, can be defined likewise as Eq. (8) and (9).

Substituting Eq. (7) into Eq. (6) and pre-multiply by T^T, the reduced equation of a vibro-acoustic system is obtained

$$\left(\begin{bmatrix} \bar{k}_{ii}^f & 0 & 0 & 0 \\ 0 & \bar{k}_{bb}^f & 0 & -C_{bb} \\ 0 & 0 & \bar{k}_{ii}^f & 0 \\ 0 & 0 & 0 & \bar{k}_{bb}^f \end{bmatrix} - \omega^2 \begin{bmatrix} \bar{m}_{ii}^s & \bar{m}_{ib}^s & 0 & 0 \\ \bar{m}_{bi}^s & \bar{m}_{bb}^s & 0 & 0 \\ 0 & 0 & \bar{m}_{ii}^f & \bar{m}_{ib}^f \\ 0 & \rho_0 C_{bb}^T & \bar{m}_{bi}^f & \bar{m}_{bb}^f \end{bmatrix} \right) \begin{Bmatrix} q_i^s \\ u_b \\ q_i^f \\ p_b \end{Bmatrix} = \begin{Bmatrix} \left(\varphi_i^s\right)^T f_i^s \\ \left(\varphi_c^s\right)^T f_i^s \\ 0 \\ 0 \end{Bmatrix} \tag{10}$$

The sub-matrices of the structure part in Eq. (10) are defined as

$$\begin{cases} \bar{k}_{ii}^s = \Lambda_{ii}^s, \ \bar{k}_{bb}^s = k_{bi}^s \varphi_c^s + k_{bb}^s \\ \bar{m}_{ii}^s = I_{ii}^s, \ \bar{m}_{ib}^s = \left(\varphi_i^s\right)^T \left(m_{ii}^s \varphi_c^s + m_{ib}^s\right), \ \bar{m}_{bi}^s = \left(\bar{m}_{ib}^s\right)^T \\ \bar{m}_{bb}^s = \left(\varphi_c^s\right)^T m_{ii}^s \varphi_c^s + m_{bi}^s \varphi_c^s + \left(\varphi_c^s\right)^T m_{ib}^s + m_{bb}^s \end{cases} \tag{11}$$

The sub-matrices of the fluid part in Eq. (10) can be obtained by substituting the superscript "*s*" in Eq. (11) by "*f*".

3. Numerical Results and Discussion

An elbow tube is considered in this section as a numerical example to evaluate the computational performance of the proposed method. The geometrical dimensions of this tube is depicted in Fig. 2. The tube is made of aluminium with the Young's modulus $E = 71GPa$, Poisson's ratio $\nu = 0.33$ and density $\rho_s = 2700\,kg/m^3$. The interior fluid media of this tube is air with density $\rho_f = 1.225\,kg/m^3$ and speed of sound $c_0 = 340\,m/s$. The left end of the tube is clamped and a unit point force is applied on the surface of the tube as illustrated in Fig. 2. Both the structure and the fluid media are modelled by 8-node hexahedron elements. The whole vibro-acoustic system is divided to five subsystems in such a manner that four subsystems are identical. Therefore, only the first and fourth subsystems need to be actually modelled in the vibro-acoustic CMS procedure. Total number of DOFs of the full model and the reduced model is shown in Table 1. It should be pointed out that the rule of thumb, which requires the modes whose frequencies are within 2.5 times of the highest interested frequency should be kept, is used for modal truncation of the structure and fluid media.

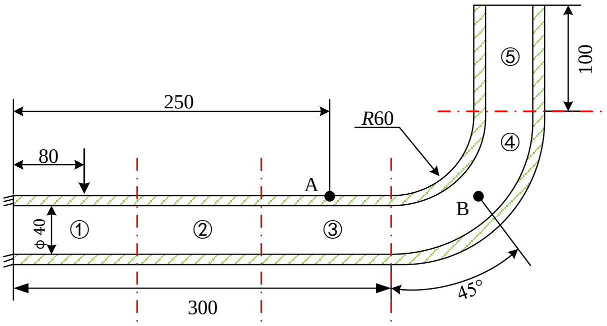

Fig. 2. Dimensions of the air-filled elbow tube.

Table 1 Total number of DOFs of the full model and the reduced model

	inner DOFs	boundary DOFs	total DOF
full model	15316	1296	16612
reduced model	100	1296	1396

It can be seen from Table 1 that although boundary DOFs remain unaltered, the total DOF of reduced model is still obviously decreased by 91.6%, rendering a significantly shortened CPU time to conduct a dynamic analysis. To further evaluate the computational accuracy of the proposed method, the displacement response of point A on the structure and the sound pressure level response of point B in the fluid media are shown in Fig. 3, and are compared with the corresponding results delivered by the full model.

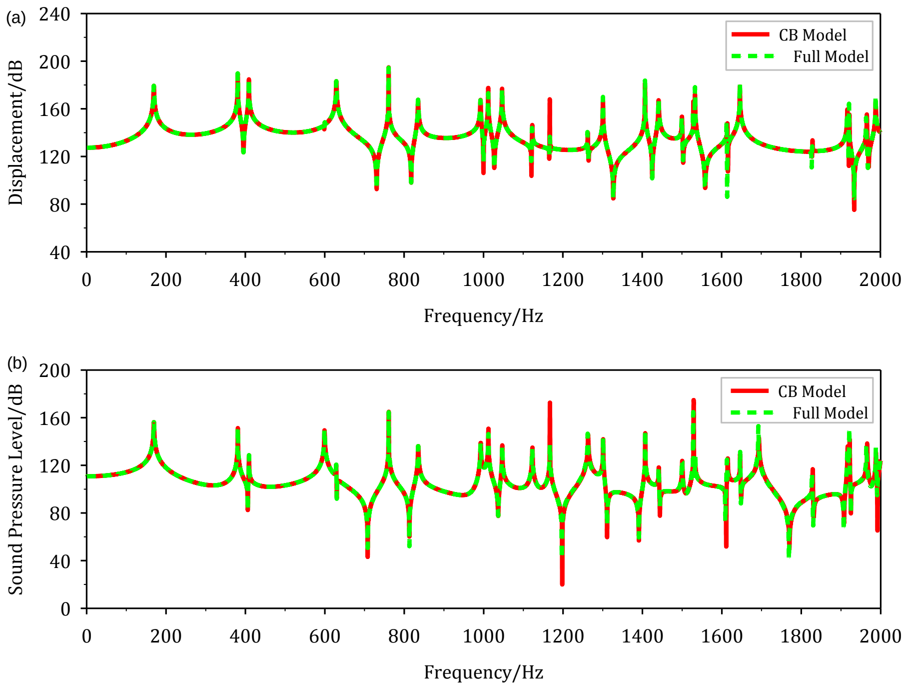

Fig. 2. Comparison of vibro-acoustic responses between full model and reduced model: (a) displacement response of point A; (b) sound pressure level response of point B.

From Fig. 2 a consistent agreement of present results and the referential full-model results is seen. Therefore, the vibro-acoustic CMS method is proven to be adequately accurate for dynamic simulations.

4. Conclusions

It is shown in this paper that the analysis of dynamic characteristics for a complex vibro-acoustic system can be carried out mainly at the component level and then assembled to form a complete model with the only requirement of verification, without going through the conventional extensive computations in which enormous degrees of freedom are involved. A number of steps in the theoretical modelling and numerical implementation have been explained, including component-wise modal transformation, subsystem re-assembly, and results verification. Overall, the numerical examples have demonstrated the effectiveness of the proposed method.

REFERENCES

1 Hurty W. Dynamic analysis of structural systems using component modes. *AIAA Journal*, Vol. 3, No. 4, 1965, pp. 678 – 685.

2 R.R. Craig Jr., Substructure methods in vibration, *ASME J. Vib. Acoust*. 117 (B) (1995) 207–213.

3 M.C.C. Bampton, R.R. Craig Jr., Coupling of substructures for dynamic analyses, *AIAA J*. 6 (7) (1968) 1313–1319.

4 Macneal, R.H. (1971). A hybrid method of component mode synthesis. *Computers & Structures*, vol. 1, no. 4, p. 581-601.

5 Rubin, S. (1975). Improved component-mode representation for structural dynamic analysis. *AIAA Journal*, vol. 13, no. 8, p. 995-1006.

6 Voormeeren, S.N., Van Der Valk, P.L.C., Rixen, D.J. (2011). Generalized methodology for assembly and reduction of component models for dynamic substructuring. *AIAA Journal*, vol. 49, no. 5, p. 1010-1020.

7 Jin-Gyun Kim, Phill-Seung Lee. (2015) An enhanced Craig–Bampton method. *International Journal for Numerical Methods in Engineering*, vol. 103, p. 79~93.

8 L. Hinke, F. Dohnal, B.R. Mace, et al. Component mode synthesis as a framework for uncertainty analysis. *Journal of Sound and Vibration*, vol. 324, p.161~178.

9 Colas Joannin, Benjamin Chouvion, Fabrice Thouverez, et al. A nonlinear component mode synthesis method for the computation of steady-state vibrationsin non-conservative systems. *Mechanical Systems and Signal Processing*, vol. 83, p. 75~92.

10 Papadimitriou C, Papadioti D C. Component mode synthesis techniques for finite element model updating[J]. *Computers & structures*, 2013, 126: 15-28.

11 Gruber F M, Rixen D J. Evaluation of Substructure Reduction Techniques with Fixed and Free Interfaces[J]. Strojniški vestnik-*Journal of Mechanical Engineering*, 2016, 62(7-8): 452-462.